

Exercise 9.1

Q.1 Find the distance between the following pairs of points

Solution:

(a) $A(9, 2), B(7, 2)$

$$\text{Distance} = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

$$|AB| = \sqrt{|7 - 9|^2 + |2 - 2|^2}$$

$$|AB| = \sqrt{(-2)^2 + (0)^2}$$

$$|AB| = \sqrt{4}$$

$$|AB| = 2$$

(b) $A(2, -6), B(3, -6)$

$$\text{Distance} = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

$$|AB| = \sqrt{|3 - 2|^2 + |-6 - (-6)|^2}$$

$$|AB| = \sqrt{(1)^2 + (-6 + 6)^2}$$

$$|AB| = \sqrt{1 + (0)^2}$$

$$|AB| = \sqrt{1}$$

$$AB = 1$$

(c) $A(-8, 1), B(6, 1)$

$$\text{Distance} = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

$$|AB| = \sqrt{|6 - (-8)|^2 + |1 - 1|^2}$$

$$|AB| = \sqrt{(6 + 8)^2 + (0)^2}$$

$$|AB| = \sqrt{(14)^2}$$

$$|AB| = 14$$

(d) $A(-4, \sqrt{2}), B(-4, -3)$

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

$$|AB| = \sqrt{|-4 - (-4)|^2 + |-3 - \sqrt{2}|^2}$$

$$|AB| = \sqrt{(-4 + 4)^2 + (-(3 + \sqrt{2}))^2}$$

$$|AB| = \sqrt{(0)^2 + (3 + \sqrt{2})^2}$$

$$|AB| = \sqrt{(3 + \sqrt{2})^2}$$

$$|AB| = 3 + \sqrt{2}$$

(e) $A(3, -11), B(3, -4)$

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

$$|AB| = \sqrt{|3 - 3|^2 + |-4 - (-11)|^2}$$

$$|AB| = \sqrt{(0)^2 + (-4 + 11)^2}$$

$$|AB| = \sqrt{(7)^2}$$

$$|AB| = 7$$

(f) $A(0, 0), B(0, -5)$

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

$$|AB| = \sqrt{|0 - 0|^2 + |-5 - 0|^2}$$

$$|AB| = \sqrt{(-5)^2}$$

$$|AB| = \sqrt{25}$$

$$|AB| = 5$$

Q.2 Let P be the point on x -axis with x -coordinate a and Q be the point on y -axis with y coordinate b as given below. Find the distance between P and Q

Solution:

(i) $a = 9, b = 7$

P is $(9, 0)$ and Q is $(0, 7)$

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

$$|PQ| = \sqrt{|0 - 9|^2 + |7 - 0|^2}$$

$$|P Q| = \sqrt{(-9)^2 + (7)^2}$$

$$|P Q| = \sqrt{81 + 49}$$

$$|P Q| = \sqrt{130}$$

(ii) $a = 2, b = 3$

$$P(2, 0), Q(0, 3)$$

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

$$|P Q| = \sqrt{|0 - 2|^2 + |3 - 0|^2}$$

$$|P Q| = \sqrt{(-2)^2 + (3)^2}$$

$$|P Q| = \sqrt{4 + 9}$$

$$|P Q| = \sqrt{13}$$

(iii) $a = -8, b = 6$

$$P(-8, 0), Q(0, 6)$$

$$|d| = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

$$|P Q| = \sqrt{|0 - (-8)|^2 + |6 - 0|^2}$$

$$|P Q| = \sqrt{(8)^2 + (6)^2}$$

$$|P Q| = \sqrt{64 + 36}$$

$$|P Q| = \sqrt{100}$$

$$|P Q| = 10$$

(iv) $a = -2, b = -3$

$$P(-2, 0), Q(0, -3)$$

$$|d| = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

$$d = \sqrt{|0 - (-2)|^2 + |-3 - 0|^2}$$

$$d = \sqrt{(2)^2 + (-3)^2}$$

$$d = \sqrt{4 + 9}$$

$$d = \sqrt{13}$$

(v) $a = \sqrt{2}, b = 1$

$$P(\sqrt{2}, 0), Q(0, 1)$$

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

$$d = \sqrt{|0 - \sqrt{2}|^2 + |1 - 0|^2}$$

$$d = \sqrt{(-\sqrt{2})^2 + (1)^2}$$

$$d = \sqrt{2 + 1}$$

$$d = \sqrt{3}$$

(vi) $a = -9, b = -4$

$$P(-9, 0), Q(0, -4)$$

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

$$|P Q| = \sqrt{|0 - (-9)|^2 + |-4 - 0|^2}$$

$$|P Q| = \sqrt{(9)^2 + (-4)^2}$$

$$|P Q| = \sqrt{81 + 16}$$

$$|P Q| = \sqrt{97}$$

Exercise 9.2

Q.1 Show whether the points with vertices $(5,-2), (5,4)$ and $(-4,1)$ are the vertices of equilateral triangle or an isosceles triangle

$$P(5,-2), Q(5,4), R(-4,1)$$

Solution:

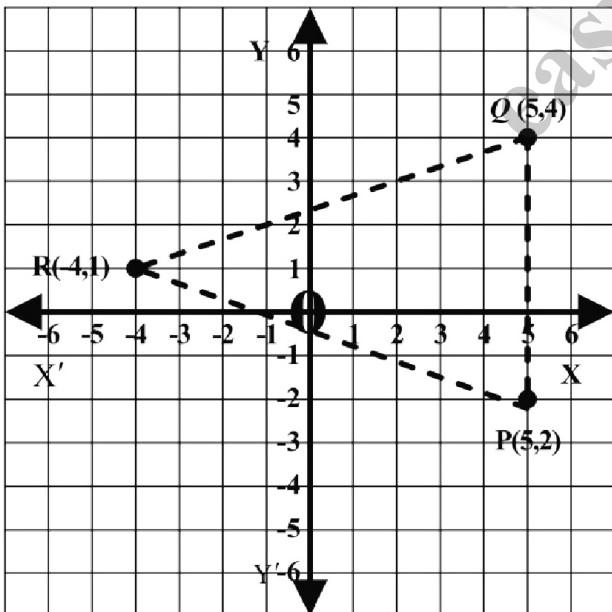
We know that the distance formula is

$$= \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

We have $P(5,-2), Q(5,4)$

$$|P Q| = \sqrt{|5-5|^2 + |4-(-2)|^2}$$

$$|P Q| = \sqrt{(0)^2 + (4+2)^2}$$



$$|P Q| = \sqrt{(6)^2}$$

$$|P Q| = 6$$

$$Q(5,4), R(-4,1)$$

$$|Q R| = \sqrt{|-4-5|^2 + |1-4|^2}$$

$$|Q R| = \sqrt{(-9)^2 + (-3)^2}$$

$$|Q R| = \sqrt{81+9}$$

$$|Q R| = \sqrt{90}$$

$$|Q R| = \sqrt{9 \times 10} = 3\sqrt{10}$$

$$R(-4,1), P(5,-2)$$

$$|R P| = \sqrt{|5-(-4)|^2 + |-2-1|^2}$$

$$|R P| = \sqrt{(5+4)^2 + (-3)^2}$$

$$|R P| = \sqrt{(9)^2 + 9}$$

$$|R P| = \sqrt{81+9}$$

$$|R P| = \sqrt{90}$$

$$|R P| = \sqrt{9 \times 10} = 3\sqrt{10}$$

$$|Q R| = |P R|$$

Two lengths of triangle are equal

So it is a isosceles triangle

Q.2 Show whether or not the points with vertices $(-1,1), (2,-2)$ and $(-4,1)$ form a Square

Solution:

$$P(-1,1)Q(5,4)R(2,-2)S(-4,1)$$

$$\text{Distance} = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

$$|P Q| = \sqrt{|5-(-1)|^2 + |4-1|^2}$$

$$|P Q| = \sqrt{|5+1|^2 + |3|^2}$$

$$|P Q| = \sqrt{6^2 + 9}$$

$$|P Q| = \sqrt{36+9}$$

$$|P Q| = \sqrt{45}$$

$$|P Q| = \sqrt{9 \times 5}$$

$$|P Q| = 3\sqrt{5}$$

$$|Q R| = \sqrt{|2-5|^2 + |-2-4|^2}$$

$$|QR| = \sqrt{(-3)^2 + (6)^2}$$

$$|QR| = \sqrt{9+36}$$

$$|QR| = \sqrt{45}$$

$$|QR| = \sqrt{9 \times 5}$$

$$|QR| = 3\sqrt{5}$$

$$|RS| = \sqrt{|-4-2|^2 + |1-(-2)|^2}$$

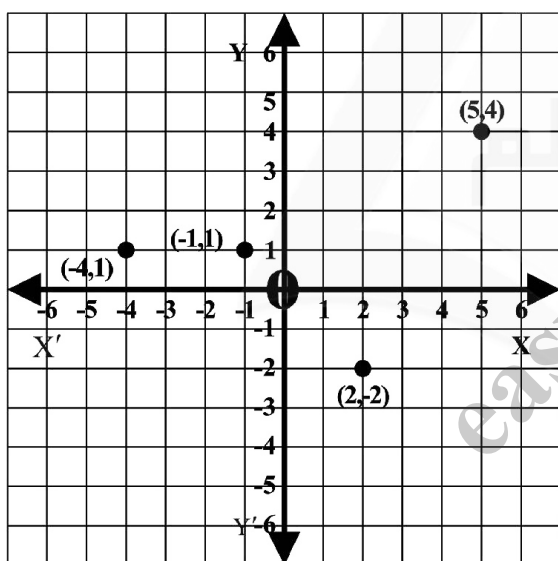
$$|RS| = \sqrt{(-6)^2 + (1+2)^2} = \sqrt{36+(3)^2}$$

$$|RS| = \sqrt{36+9}$$

$$|RS| = \sqrt{45}$$

$$|RS| = \sqrt{9 \times 5}$$

$$|RS| = 3\sqrt{5}$$



$$|SP| = \sqrt{|-4-(-1)|^2 + |1-1|^2}$$

$$|SP| = \sqrt{(-4+1)^2 + (0)^2}$$

$$|SP| = \sqrt{(-3)^2}$$

$$|SP| = \sqrt{9}$$

$$|SP| = 3$$

If all the length are same then it will be a Square all the length are not equal so it is not square.

$$|PQ| = |QR| = |RS| \neq |SP|$$

Q.3 Show whether or not the points with coordinates $(1,3)$, $(4,2)$ and $(-2,6)$ are vertices of a right triangle?

Solution:

$$A(1,3), B(4,2), C(-2,6)$$

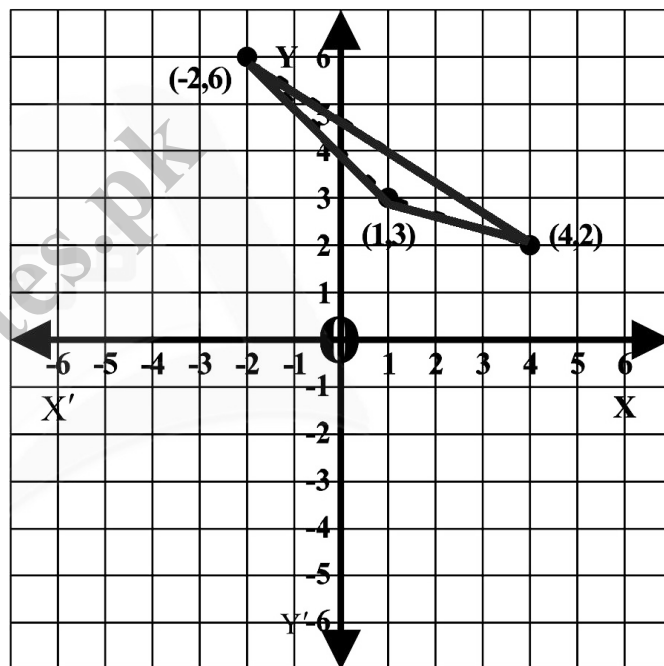
$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

$$|AB| = \sqrt{|4-1|^2 + |2-3|^2}$$

$$|AB| = \sqrt{(3)^2 + (-1)^2}$$

$$|AB| = \sqrt{9+1}$$

$$|AB| = \sqrt{10}$$



$$|BC| = \sqrt{|-2-4|^2 + |6-2|^2}$$

$$|BC| = \sqrt{(-6)^2 + (4)^2}$$

$$|BC| = \sqrt{36+16}$$

$$|BC| = \sqrt{52}$$

$$|CA| = \sqrt{|-2-1|^2 + |6-3|^2} = \sqrt{(-3)^2 + (3)^2}$$

$$|CA| = \sqrt{9+9}$$

$$|CA| = \sqrt{18}$$

By Pythagoras theorem

$$(\text{Hyp})^2 = (\text{Base})^2 + (\text{Perp})^2$$

$$(\sqrt{52})^2 = (\sqrt{18})^2 + (\sqrt{10})^2$$

$$52 = 18 + 10$$

$$52 \neq 28$$

Since $52 \neq 28$

So it not right angle triangle.

Q.4 Use distance formula to prove whether or not the points (1,1), (-2,-8) and (4,10) lie on a straight line?

Solution:

$$A(1,1), B(-2,-8), C(4,10)$$

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

$$|AB| = \sqrt{|-2 - 1|^2 + |-8 - 1|^2}$$

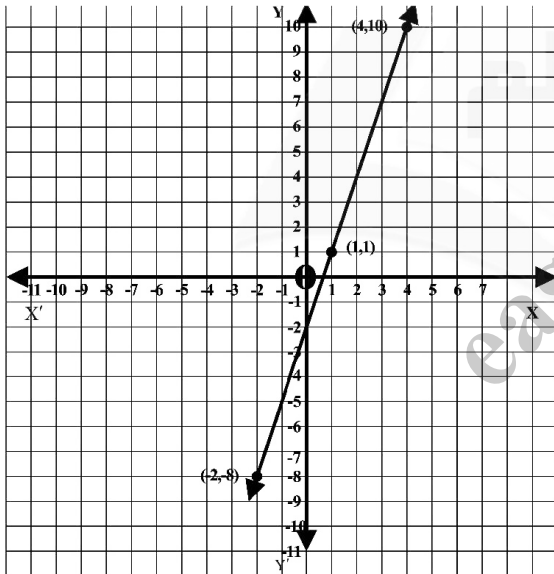
$$|AB| = \sqrt{(-3)^2 + (-9)^2}$$

$$|AB| = \sqrt{9 + 81}$$

$$|AB| = \sqrt{90}$$

$$|AB| = \sqrt{9 \times 10}$$

$$|AB| = 3\sqrt{10}$$



$$|BC| = \sqrt{|4 - (-2)|^2 + |10 - (-8)|^2}$$

$$|BC| = \sqrt{(4+2)^2 + (10+8)^2}$$

$$|BC| = \sqrt{(6)^2 + (18)^2}$$

$$|BC| = \sqrt{36 + 324}$$

$$|BC| = \sqrt{360}$$

$$|BC| = \sqrt{36 \times 10}$$

$$|BC| = 6\sqrt{10}$$

$$|AC| = \sqrt{|4 - 1|^2 + |10 - 1|^2}$$

$$|AC| = \sqrt{(3)^2 + (9)^2}$$

$$|AC| = \sqrt{9 + 81}$$

$$|AC| = \sqrt{90}$$

$$|AC| = \sqrt{9 \times 10}$$

$$|AC| = 3\sqrt{10}$$

$$|AC| + |AB| = |BC|$$

$$3\sqrt{10} + 3\sqrt{10} = 6\sqrt{10}$$

$$6\sqrt{10} = 6\sqrt{10}$$

It means that they lie on same line so they are collinear.

Q.5 Find K given that point (2, K) is equidistance from (3, 7) and (9, 1)

Solution: $M(2, K), A(3, 7)$ and $B(9, 1)$

$$\begin{array}{ccc} (3,7) & (2,K) & (9,1) \\ A & M & B \end{array}$$

$$|AM| = |BM|$$

$$\sqrt{|2 - 3|^2 + |K - 7|^2} = \sqrt{|9 - 2|^2 + |1 - K|^2}$$

$$\sqrt{(-1)^2 + (K - 7)^2} = \sqrt{(7)^2 + (1 - K)^2}$$

Taking square on both Side

$$\left(\sqrt{1 + K^2 + 49 - 14K}\right)^2 = \left(\sqrt{49 + 1 + K^2 - 2K}\right)^2$$

$$K^2 - 14K + 50 = 50 + K^2 - 2K$$

$$\cancel{K^2} - 14K \cancel{+50} \cancel{-50} \cancel{-K^2} + 2K = 0$$

$$-12K = 0$$

$$K = \frac{0}{-12}$$

$$K = 0$$

Q.6 Use distance formula to verify that the points

$A(0,7), B(3,-5), C(-2,15)$ are

Collinear.

Solution:

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

$$|AB| = \sqrt{|3 - 0|^2 + |-5 - 7|^2}$$

$$|AB| = \sqrt{(3)^2 + (-12)^2}$$

$$|AB| = \sqrt{9 + 144}$$

$$|AB| = \sqrt{153}$$

$$|AB| = \sqrt{9 \times 17}$$

$$52 = 18 + 10$$

$$52 \neq 28$$

Since $52 \neq 28$

So it not right angle triangle.

Q.4 Use distance formula to prove whether or not the points (1,1), (-2,-8) and (4,10) lie on a straight line?

Solution:

$$A(1,1), B(-2,-8), C(4,10)$$

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

$$|AB| = \sqrt{|-2 - 1|^2 + |-8 - 1|^2}$$

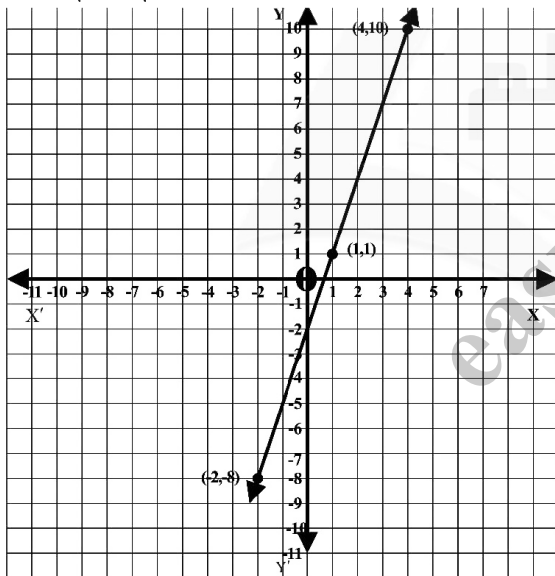
$$|AB| = \sqrt{(-3)^2 + (-9)^2}$$

$$|AB| = \sqrt{9 + 81}$$

$$|AB| = \sqrt{90}$$

$$|AB| = \sqrt{9 \times 10}$$

$$|AB| = 3\sqrt{10}$$



$$|BC| = \sqrt{|4 - (-2)|^2 + |10 - (-8)|^2}$$

$$|BC| = \sqrt{(4+2)^2 + (10+8)^2}$$

$$|BC| = \sqrt{(6)^2 + (18)^2}$$

$$|BC| = \sqrt{36 + 324}$$

$$|BC| = \sqrt{360}$$

$$|BC| = \sqrt{36 \times 10}$$

$$|BC| = 6\sqrt{10}$$

$$|AC| = \sqrt{|4 - 1|^2 + |10 - 1|^2}$$

$$|AC| = \sqrt{(3)^2 + (9)^2}$$

$$|AC| = \sqrt{9 + 81}$$

$$|AC| = \sqrt{90}$$

$$|AC| = \sqrt{9 \times 10}$$

$$|AC| = 3\sqrt{10}$$

$$|AC| + |AB| = |BC|$$

$$3\sqrt{10} + 3\sqrt{10} = 6\sqrt{10}$$

$$6\sqrt{10} = 6\sqrt{10}$$

It means that they lie on same line so they are collinear.

Q.5 Find K given that point (2, K) is equidistance from (3, 7) and (9, 1)

Solution: $M(2, K), A(3, 7)$ and $B(9, 1)$

$$\begin{array}{ccc} (3,7) & (2,K) & (9,1) \\ A & M & B \end{array}$$

$$|AM| = |BM|$$

$$\sqrt{|2 - 3|^2 + |K - 7|^2} = \sqrt{|9 - 2|^2 + |1 - K|^2}$$

$$\sqrt{(-1)^2 + (K - 7)^2} = \sqrt{(7)^2 + (1 - K)^2}$$

Taking square on both Side

$$(\sqrt{1 + K^2 + 49 - 14K})^2 = (\sqrt{49 + 1 + K^2 - 2K})^2$$

$$K^2 - 14K + 50 = 50 + K^2 - 2K$$

$$\cancel{K^2} - 14K + \cancel{50} - \cancel{50} - \cancel{K^2} + 2K = 0$$

$$-12K = 0$$

$$K = \frac{0}{-12}$$

$$K = 0$$

Q.6 Use distance formula to verify that the points

$A(0,7), B(3,-5), C(-2,15)$ are

Collinear.

Solution:

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

$$|AB| = \sqrt{|3 - 0|^2 + |-5 - 7|^2}$$

$$|AB| = \sqrt{(3)^2 + (-12)^2}$$

$$|AB| = \sqrt{9 + 144}$$

$$|AB| = \sqrt{153}$$

$$|AB| = \sqrt{9 \times 17}$$

$$|A B| = 3\sqrt{17}$$

$$|B C| = \sqrt{|-2-3|^2 + |15-(-5)|^2}$$

$$|B C| = \sqrt{(-5)^2 + (15+5)^2}$$

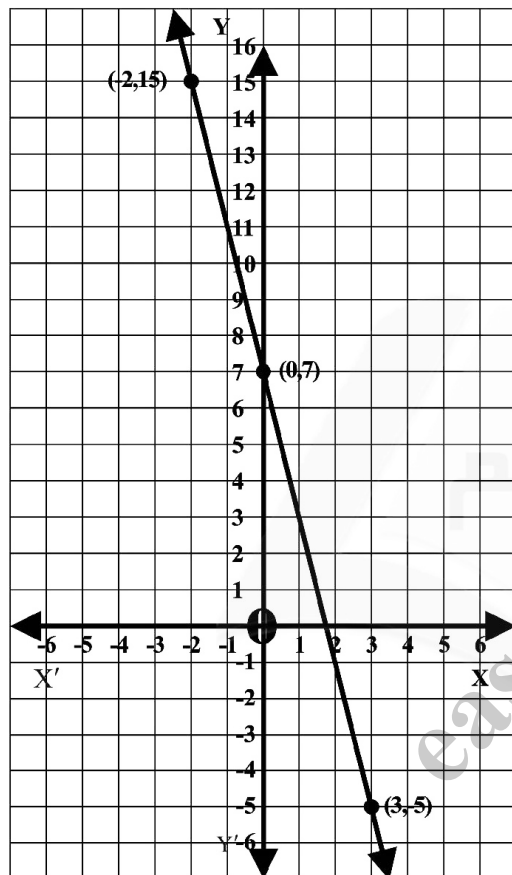
$$|B C| = \sqrt{25 + (20)^2}$$

$$|B C| = \sqrt{25 + 400}$$

$$|B C| = \sqrt{425}$$

$$|B C| = \sqrt{25 \times 17}$$

$$|B C| = 5\sqrt{17}$$



$$|A C| = \sqrt{|-2-0|^2 + |15-7|^2}$$

$$|A C| = \sqrt{(-2)^2 + (8)^2}$$

$$|A C| = \sqrt{4 + 64}$$

$$|A C| = \sqrt{68}$$

$$|A C| = \sqrt{4 \times 17}$$

$$|A C| = 2\sqrt{17}$$

$$|A B| + |A C| = |B C|$$

$$3\sqrt{17} + 2\sqrt{17} = 5\sqrt{17}$$

$$5\sqrt{17} = 5\sqrt{17}$$

L.H.S = R.H.S So

They lie on same line and they are collinear.

Q.7 Verify whether or not the points $O(0,0)$, $A(\sqrt{3},1)$, $B(\sqrt{3},-1)$ are the vertices of an equilateral triangle

Solution:

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

$$|O A| = \sqrt{|\sqrt{3}-0|^2 + |0-1|^2}$$

$$|O A| = \sqrt{(\sqrt{3})^2 + (-1)^2}$$

$$|O A| = \sqrt{3+1}$$

$$|O A| = \sqrt{4}$$

$$|O A| = 2$$

$$|O B| = \sqrt{|\sqrt{3}-0|^2 + |-1-0|^2}$$

$$|O B| = \sqrt{(\sqrt{3})^2 + (-1)^2}$$

$$|O B| = \sqrt{3+1}$$

$$|O B| = \sqrt{4}$$

$$|O B| = 2$$

$$|A B| = \sqrt{|\sqrt{3}-\sqrt{3}|^2 + |-1-1|^2}$$

$$|A B| = \sqrt{0 + (-2)^2}$$

$$|A B| = \sqrt{4}$$

$$|A B| = 2$$

All the sides are same in length so it is equilateral triangle

Q.8 Show that the points $A(-6,-5)$, $B(5,-5)$, $C(5,-8)$ and $D(-6,-8)$ are the vertices of a rectangle find the length of its diagonals are equal

Solution:

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

$$A(-6,-5), B(5,-5)$$

$$|A B| = 3\sqrt{17}$$

$$|B C| = \sqrt{|-2-3|^2 + |15-(-5)|^2}$$

$$|B C| = \sqrt{(-5)^2 + (15+5)^2}$$

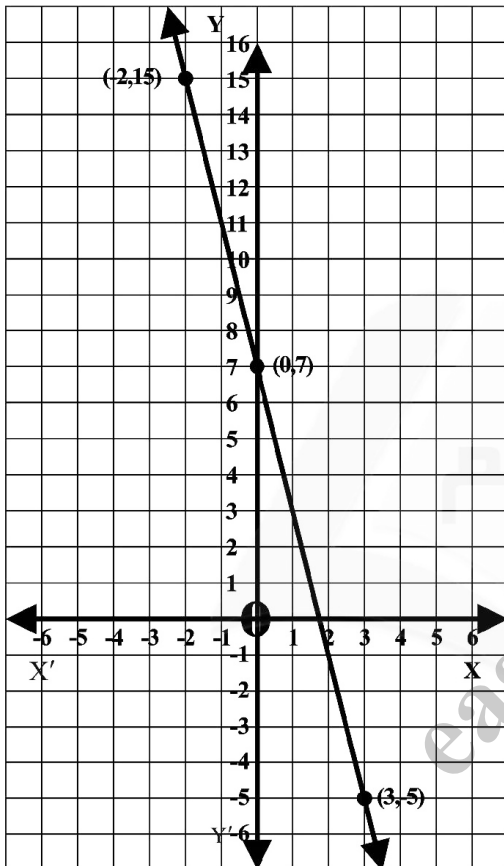
$$|B C| = \sqrt{25 + (20)^2}$$

$$|B C| = \sqrt{25 + 400}$$

$$|B C| = \sqrt{425}$$

$$|B C| = \sqrt{25 \times 17}$$

$$|B C| = 5\sqrt{17}$$



$$|A C| = \sqrt{|-2-0|^2 + |15-7|^2}$$

$$|A C| = \sqrt{(-2)^2 + (8)^2}$$

$$|A C| = \sqrt{4 + 64}$$

$$|A C| = \sqrt{68}$$

$$|A C| = \sqrt{4 \times 17}$$

$$|A C| = 2\sqrt{17}$$

$$|A B| + |A C| = |B C|$$

$$3\sqrt{17} + 2\sqrt{17} = 5\sqrt{17}$$

$$5\sqrt{17} = 5\sqrt{17}$$

L.H.S = R.H.S So

They lie on same line and they are collinear.

Q.7 Verify whether or not the points $O(0,0)$, $A(\sqrt{3},1)$, $B(\sqrt{3},-1)$ are the vertices of an equilateral triangle

Solution:

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

$$|O A| = \sqrt{|\sqrt{3} - 0|^2 + |0 - 1|^2}$$

$$|O A| = \sqrt{(\sqrt{3})^2 + (-1)^2}$$

$$|O A| = \sqrt{3 + 1}$$

$$|O A| = \sqrt{4}$$

$$|O A| = 2$$

$$|O B| = \sqrt{|\sqrt{3} - 0|^2 + |-1 - 0|^2}$$

$$|O B| = \sqrt{(\sqrt{3})^2 + (-1)^2}$$

$$|O B| = \sqrt{3 + 1}$$

$$|O B| = \sqrt{4}$$

$$|O B| = 2$$

$$|A B| = \sqrt{|\sqrt{3} - \sqrt{3}|^2 + |-1 - 1|^2}$$

$$|A B| = \sqrt{0 + (-2)^2}$$

$$|A B| = \sqrt{4}$$

$$|A B| = 2$$

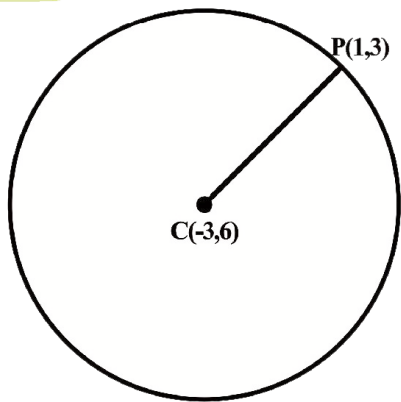
All the sides are same in length so it is equilateral triangle

Q.8 Show that the points $A(-6,-5)$, $B(5,-5)$, $C(5,-8)$ and $D(-6,-8)$ are the vertices of a rectangle find the length of its diagonals are equal

Solution:

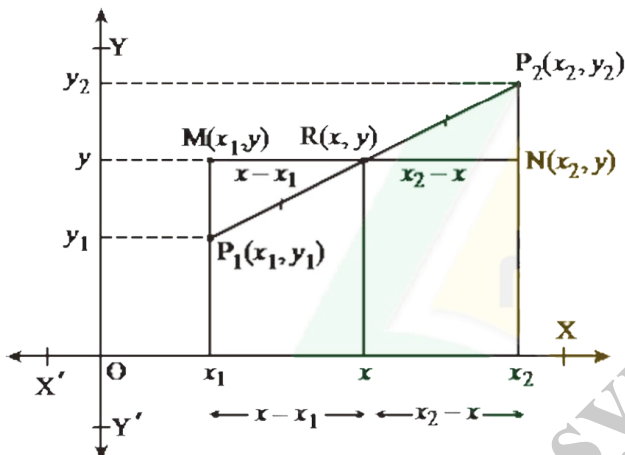
$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

$$A(-6,-5), B(5,-5)$$



Recognition of the midpoint formula for any two points in the plane

Let $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ be any two points in the plane and $R(x, y)$ be midpoint of point P_1 and P_2 on the line segment P_1P_2 as shown in the figure.



If the line segment MN , parallel to x -axis has its midpoint $R(x, y)$, then, $x_2 - x = x - x_1$

$$x_2 + x_1 = x + x$$

$$2x = x_1 + x_2 \Rightarrow x = \frac{x_1 + x_2}{2}$$

$$\text{Similarly, } y = \frac{y_1 + y_2}{2}$$

Thus the point $R(x, y) = R\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ is the

midpoint of the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$

Verification of the midpoint formula

$$|P_1R| = \sqrt{\left(\frac{x_1 + x_2}{2} - x_1\right)^2 + \left(\frac{y_1 + y_2}{2} - y_1\right)^2}$$

$$|P_1R| = \sqrt{\left(\frac{x_1 + x_2 - 2x_1}{2}\right)^2 + \left(\frac{y_1 + y_2 - 2y_1}{2}\right)^2}$$

$$|P_1R| = \sqrt{\left(\frac{x_2 - x_1}{2}\right)^2 + \left(\frac{y_2 - y_1}{2}\right)^2}$$

$$|P_1R| = \sqrt{\frac{(x_2 - x_1)^2}{4} + \frac{(y_2 - y_1)^2}{4}}$$

$$|P_1R| = \sqrt{\frac{(x_2 - x_1)^2 + (y_2 - y_1)^2}{4}}$$

$$|P_1R| = \sqrt{\frac{(x_2 - x_1)^2 + (y_2 - y_1)^2}{2}}$$

OR

$$|P_1R| = \frac{1}{2} \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \frac{1}{2} |P_1P_2|$$

$$\text{and } |P_2R| = \sqrt{\left(\frac{x_1 + x_2}{2} - x_2\right)^2 + \left(\frac{y_1 + y_2}{2} - y_2\right)^2}$$

$$|P_2R| = \sqrt{\left(\frac{x_1 + x_2 - 2x_2}{2}\right)^2 + \left(\frac{y_1 + y_2 - 2y_2}{2}\right)^2}$$

$$|P_2R| = \sqrt{\left(\frac{x_1 - x_2}{2}\right)^2 + \left(\frac{y_1 - y_2}{2}\right)^2}$$

$$|P_2R| = \sqrt{\frac{(x_1 - x_2)^2}{4} + \frac{(y_1 - y_2)^2}{4}}$$

$$|P_2R| = \sqrt{\frac{(x_1 - x_2)^2 + (y_1 - y_2)^2}{4}}$$

$$|P_2R| = \sqrt{\frac{(x_1 - x_2)^2 + (y_1 - y_2)^2}{2}}$$

OR

$$|P_2R| = \frac{1}{2} \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$\Rightarrow |P_2R| = |P_1R| = \frac{1}{2} |P_1P_2|$$

Thus it verifies that

$R\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ is the midpoint

of the line segment P_1RP_2 which lies on the line segment since

$$|P_1R| + |P_2R| = |P_1P_2|$$

Exercise 9.3

Q.1 Find the midpoint of the line Segments joining each of the following pairs of points

Solution:

(a) $A(9,2), B(7,2)$

Let $M(x, y)$ the midpoint of AB

$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Midpoint formula

$$M(x, y) = M\left(\frac{9+7}{2}, \frac{2+2}{2}\right)$$

$$= M\left(\frac{16}{2}, \frac{4}{2}\right)$$

$$= M(8, 2)$$

(b) $A(2, -6), B(3, -6)$

Let $M(x, y)$ the point of AB

$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Midpoint formula

$$M(x, y) = M\left(\frac{2+3}{2}, \frac{-6-6}{2}\right)$$

$$M(x, y) = M\left(\frac{5}{2}, \frac{-12}{2}\right)$$

$$M(x, y) = M(2.5, -6)$$

(c) $A(-8, 1), B(6, 1)$

Let $M(x, y)$ midpoint of AB

$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Formula

$$M(x, y) = M\left(\frac{-8+6}{2}, \frac{1+1}{2}\right)$$

$$M(x, y) = M\left(\frac{-2}{2}, \frac{2}{2}\right)$$

$$M(x, y) = M(-1, 1)$$

(d) $A(-4, 9), B(-4, -3)$

Let $M(x, y)$ midpoint of AB

$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \text{ Formula}$$

$$M(x, y) = M\left(\frac{-4-4}{2}, \frac{9-3}{2}\right)$$

$$M(x, y) = M\left(\frac{-8}{2}, \frac{6}{2}\right)$$

$$M(x, y) = M(-4, 3)$$

(e) $A(3, 11), B(3, -4)$

Let $M(x, y)$ is the midpoint of AB

$$M(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$M(x, y) = M\left(\frac{3+3}{2}, \frac{-11-4}{2}\right)$$

$$M(x, y) = M\left(\frac{6}{2}, \frac{-15}{2}\right)$$

$$M(x, y) = M(3, -7.5)$$

(f) $A(0, 0), B(0, -5)$

Let $M(x, y)$ is the midpoint of AB

$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

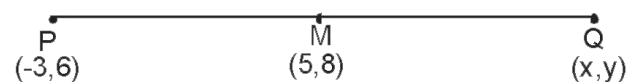
$$M(x, y) = M\left(\frac{0+0}{2}, \frac{0-5}{2}\right)$$

$$M(x, y) = M\left(\frac{0}{2}, \frac{-5}{2}\right)$$

$$= M(0, -2.5)$$

Q.2 The end point of line segment PQ is $(-3, 6)$ and its midpoint is $(5, 8)$ find the coordinates of the end point Q

Solution:



Let Q be the point (x, y) , $M(5, 8)$ is the midpoint of PQ

$$M(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$x = \frac{x_1 + x_2}{2}$$

$$5 = \frac{-3 + x}{2}$$

$$5 \times 2 = -3 + x$$

$$10 + 3 = x$$

$$x = 13$$

$$y = \frac{y_1 + y_2}{2}$$

$$8 = \frac{6 + y}{2}$$

$$2 \times 8 = 6 + y$$

$$16 - 6 = y$$

$$y = 10$$

Hence point Q is $(13, 10)$

Q.3 Prove that midpoint of the hypotenuse of a right triangle is equidistance from it three vertices

$P(-2, 5), Q(1, 3)$ and $R(-1, 0)$

Solution:

$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

$P(-2, 5), Q(1, 3)$

$$|PQ| = \sqrt{|-2 - 1|^2 + |5 - 3|^2}$$

$$|PQ| = \sqrt{(-3)^2 + (2)^2}$$

$$|PQ| = \sqrt{9 + 4}$$

$$|PQ| = \sqrt{13}$$

$Q(1, 3), R(-1, 0)$

$$|QR| = \sqrt{|1 - (-1)|^2 + |3 - 0|^2}$$

$$|QR| = \sqrt{(1+1)^2 + (3)^2}$$

$$|QR| = \sqrt{(2)^2 + 9} = \sqrt{4+9}$$

$$|QR| = \sqrt{13}$$

$P(-2, 5), R(-1, 0)$

$$|PR| = \sqrt{|-2 - (-1)|^2 + |5 - 0|^2}$$

$$|PR| = \sqrt{|-2 + 1|^2 + |5|^2}$$

$$|PR| = \sqrt{(-1)^2 + (5)^2} = \sqrt{1+25}$$

$$|PR| = \sqrt{26}$$

To find the length of hypotenuse and whether it is right angle triangle we use the Pythagoras theorem

$$(PR)^2 = (PQ)^2 + (QR)^2$$

$$(\sqrt{26})^2 = (\sqrt{13})^2 + (\sqrt{13})^2$$

$$26 = 13 + 13$$

$$26 = 26$$

It is a right angle triangle and PR is hypotenuse

$P(-2, 5), R(-1, 0)$

Midpoint of PR

$$M(x, y) = \left(\frac{-2 - 1}{2}, \frac{5 + 0}{2} \right)$$

$$M(x, y) = \left(\frac{-3}{2}, \frac{5}{2} \right)$$

$$MP = MR$$

$$M\left(\frac{-3}{2}, \frac{5}{2}\right), P(-2, 5), R(-1, 0)$$

$$|MP| = |MR|$$

$$|MP| = \sqrt{\left| \frac{-3}{2} - (-2) \right|^2 + \left| \frac{5}{2} - 5 \right|^2}$$

$$= \sqrt{\left(\frac{-3}{2} + 2 \right)^2 + \left(\frac{5 - 10}{2} \right)^2}$$

$$|MP| = \sqrt{\left(\frac{-3 + 4}{2} \right)^2 + \left(\frac{-5}{2} \right)^2}$$

$$= \sqrt{\left(\frac{1}{2} \right)^2 + \frac{25}{4}}$$

$$|MP| = \sqrt{\frac{1}{4} + \frac{25}{4}} = \sqrt{\frac{1+25}{4}}$$

$$|MP| = \sqrt{\frac{26}{4}}$$

$$|MP| = \frac{\sqrt{26}}{2}$$

(ii) $M\left(\frac{-3}{2}, \frac{5}{2}\right), R(-1, 0)$

$$|MR| = \sqrt{\left| \frac{-3}{2} - (-1) \right|^2 + \left| \frac{5}{2} - 0 \right|^2}$$

$$|MR| = \sqrt{\left(\frac{-3}{2} + 1\right)^2 + \left(\frac{5}{2}\right)^2}$$

$$|MR| = \sqrt{\left(\frac{-3+2}{2}\right)^2 + \frac{25}{4}}$$

$$= \sqrt{\left(\frac{-1}{2}\right)^2 + \frac{25}{4}}$$

$$|MR| = \sqrt{\frac{1}{4} + \frac{25}{4}}$$

$$|MR| = \sqrt{\frac{1+25}{4}} = \sqrt{\frac{26}{4}}$$

$$|MR| = \frac{\sqrt{26}}{2}$$

(iii) $M\left(\frac{-3}{2}, \frac{5}{2}\right)$

$Q(1,3)$

$$|MQ| = \sqrt{\left(\frac{-3}{2} - 1\right)^2 + \left(\frac{5}{2} - 3\right)^2}$$

$$= \sqrt{\left(\frac{-3-2}{2}\right)^2 + \left(\frac{5-6}{2}\right)^2}$$

$$= \sqrt{\left(\frac{-5}{2}\right)^2 + \left(\frac{-1}{2}\right)^2}$$

$$= \sqrt{\frac{25}{4} + \frac{1}{4}} = \sqrt{\frac{26}{4}}$$

Hence proved $MP = MR = |MQ|$

Q.4 If $O(0,0)$, $A(3,0)$ and $B(3,5)$ are three points in the plane find M_1 and M_2 as the midpoint of the line segments AB and OB respectively find $|M_1M_2|$

Solution:

M_1 is the midpoint of AB

$$M_1(x, y) = M_1\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$A(3,0), B(3,5)$

$$M_1\left(\frac{3+3}{2}, \frac{0+5}{2}\right)$$

$$M_1\left(\frac{6}{2}, \frac{5}{2}\right)$$

$$M_1\left(3, \frac{5}{2}\right)$$

M_2 is the midpoint of OB

$$M_2\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$O(0,0), B(3,5)$

$$M_2\left(\frac{0+3}{2}, \frac{0+5}{2}\right)$$

$$M_2\left(\frac{3}{2}, \frac{5}{2}\right)$$

$$M_1\left(3, \frac{5}{2}\right) M_2\left(\frac{3}{2}, \frac{5}{2}\right)$$

$$|M_1M_2| = \sqrt{\left|\frac{3}{2} - 3\right|^2 + \left|\frac{5}{2} - \frac{5}{2}\right|^2}$$

$$|M_1M_2| = \sqrt{\left(\frac{3-6}{2}\right)^2 + (0)^2}$$

$$= \sqrt{\left(\frac{-3}{2}\right)^2 + 0}$$

$$|M_1M_2| = \sqrt{\frac{9}{4}}$$

$$|M_1M_2| = \frac{3}{2}$$

Q.5 Show that the diagonals of the parallelogram having vertices $A(1,2), B(4,2), C(-1,-3)$ and $D(-4,-3)$ bisect each other.

Solution:

$ABCD$ is parallelogram which vertices are

$A(1,2), B(4,2), C(-1,-3), D(-4,-3)$

Let \overline{BD} and \overline{AC} the diagonals of parallelogram they intersect at point M

$A(1,2), C(-1,-3)$ midpoint of AC

Midpoint formula

$$M_1(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$M_1(x, y) = M_1\left(\frac{1-1}{2}, \frac{2-3}{2}\right)$$

$$M_1(x, y) = M_1\left(\frac{0}{2}, \frac{-1}{2}\right) = \left(0, \frac{-1}{2}\right)$$

Midpoint of BD ,

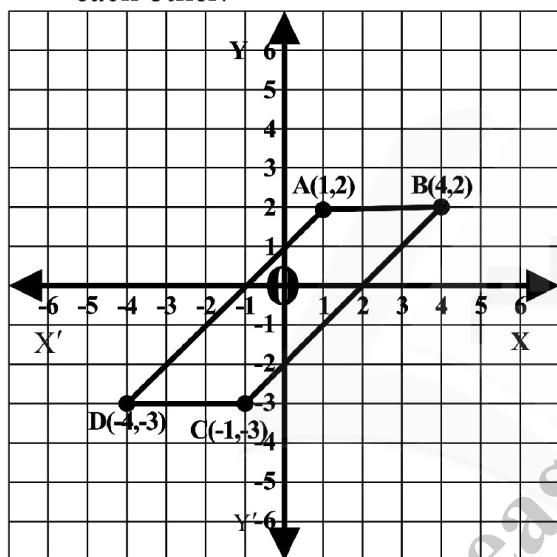
$$M_2(x, y) = M_2\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$M_2(x, y) = M_2\left(\frac{4-4}{2}, \frac{2-3}{2}\right)$$

$$M_2(x, y) = M_2\left(\frac{0}{2}, \frac{-1}{2}\right)$$

$$M_2(x, y) = M_2\left(0, \frac{-1}{2}\right)$$

As M_1 and M_2 coincide the diagonals of the parallelogram bisect each other.



- Q.6** The vertices of a triangle are $P(4,6)$, $Q(-2,-4)$ and $R(-8,2)$. Show that the length of the line segment joining the midpoints of the line segments \overline{PR} , \overline{QR} is

$$\frac{1}{2}\overline{PQ}$$

Solution:

M_1 the midpoint of QR is

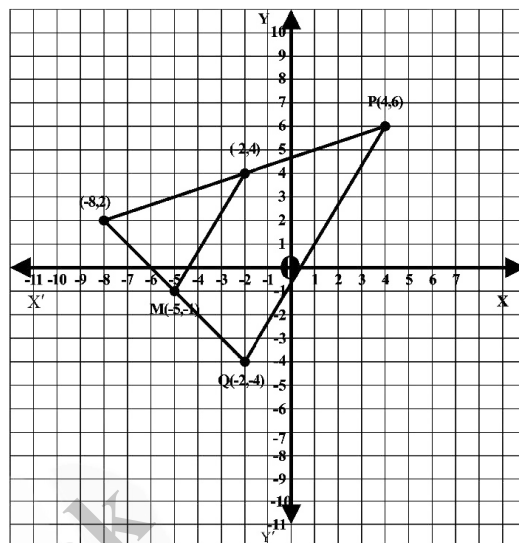
$$Q(-2,-4), R(-8,2)$$

$$M_1(x, y) = M_1\left(\frac{-2-8}{2}, \frac{-4+2}{2}\right)$$

$$= M_1\left(\frac{-10}{2}, \frac{-2}{2}\right)$$

$$= M_1(-5, -1)$$

$$M_1(-5, -1)$$



M_2 the midpoint of PR is

$$P(4,6), Q(-8, +2)$$

$$M_2(x, y) = M\left(\frac{4-8}{2}, \frac{6+2}{2}\right)$$

$$M_2(x, y) = M_2\left(\frac{-4}{2}, \frac{8}{2}\right)$$

$$M_2(x, y) = M_2(-2, 4)$$

$$M_2(-2, 4)$$

$$|M_1M_2| = \sqrt{(-5+2)^2 + |4+1|^2}$$

$$|M_1M_2| = \sqrt{(-3)^2 + (5)^2}$$

$$|M_1M_2| = \sqrt{9+25}$$

$$|M_1M_2| = \sqrt{34}$$

$$|PQ| = \sqrt{|4+2|^2 + |6+4|^2}$$

$$|PQ| = \sqrt{(6)^2 + (10)^2} = \sqrt{36+100}$$

$$|PQ| = \sqrt{136}$$

$$|PQ| = \sqrt{4 \times 34}$$

$$|PQ| = 2\sqrt{34}$$

$$\frac{|PQ|}{2} = \sqrt{34}$$

OR

$$\frac{1}{2}|PQ| = \sqrt{34}$$

Hence we proved that

$$|M_1M_2| = \frac{1}{2}|PQ|$$

easynotes.pk

Review Exercise 9

Q.1 Choose the Correct answer

(i) Distance between point (0 , 0) and (1, 1) is

- (a) 0 (b) 1
(c) 2 (d) $\sqrt{2}$

(ii) Distance between the point (1 , 0) and (0 ,1) is

- (a) 0 (b) 1
(c) $\sqrt{2}$ (d) 2

(iii) Midpoint of the (2, 2) and (0, 0) is

- (a) (1, 1) (b) (1, 0)
(c) (0, 1) (d) (-1, -1)

(iv) Midpoint of the points (2, -2) and (-2 , 2) is

- (a) (2, 2) (b) (-2, -2)
(c) (0 , 0) (d) (1, 1)

(v) A triangle having all sides equal is called

- (a) Isosceles (b) Scalene
(c) Equilateral (d) None of these

(vi) A triangle having all sides different is called

- (a) Isosceles (b) Scalene
(c) Equilateral (d) None of these

ANSWER KEYS

i	ii	iii	iv	v	vi
d	c	a	c	c	b

Q.2 Answer the following which is true and which is false

- (i)** A line has two end points (False)
(ii) A line segment has one end point (False)
(iii) A triangle is formed by the three collinear points (False)
(iv) Each side of triangle has two collinear vertices. (True)
(v) The end points of each side of a rectangle are Collinear (True)
(vi) All the points that lie on the x-axis are Collinear (True)
(vii) Origin is the only point Collinear with the points of both axis separately (True)

Q.3 Find the distance between the following pairs of points

Solution:

(i) $(6,3)(3,-3)$

$$A(6,3), B(3,-3)$$

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

$$|AB| = \sqrt{|3-6|^2 + |-3-3|^2}$$

$$|AB| = \sqrt{(-3)^2 + (-6)^2}$$

$$|AB| = \sqrt{9+36}$$

$$|AB| = \sqrt{45}$$

$$|AB| = \sqrt{9 \times 5}$$

$$|AB| = 3\sqrt{5}$$

(ii) $(7,5),(1,-1)$

$$A(7,5), B(1,-1)$$

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

$$|AB| = \sqrt{|7-1|^2 + |5-(-1)|^2}$$

$$|AB| = \sqrt{(6)^2 + (5+1)^2}$$

$$|AB| = \sqrt{36 + (6)^2} = \sqrt{36+36}$$

$$|AB| = \sqrt{72} = \sqrt{36 \times 2}$$

$$|AB| = 6\sqrt{2}$$

(iii) $(0,0),(-4,-3)$

$$A(0,0), B(-4,-3)$$

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

$$|AB| = \sqrt{|0-4|^2 + |0-(-3)|^2}$$

$$|AB| = \sqrt{(-4)^2 + (3)^2}$$

$$|AB| = \sqrt{16+9}$$

$$|AB| = \sqrt{25}$$

$$|AB| = 5$$

Q.4 Find the midpoint between the following pairs of points

Solution:

(i) $(6,6),(4,-2)$

$$M(x,y) = M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$M(x,y) = M\left(\frac{6+4}{2}, \frac{6-2}{2}\right)$$

$$M(x,y) = M\left(\frac{10}{2}, \frac{4}{2}\right)$$

$$M(x,y) = M(5,2)$$

(ii) $(-5,-7),(-7,-5)$

$$M(x,y) = M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$M(x,y) = M\left(\frac{-5-7}{2}, \frac{-7-5}{2}\right)$$

$$M(x,y) = M\left(\frac{-12}{2}, \frac{-12}{2}\right)$$

$$M(x,y) = M(-6,-6)$$

(iii) $(8,0),(0,-12)$

$$M(x,y) = M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$M(x,y) = M\left(\frac{8+0}{2}, \frac{0-12}{2}\right)$$

$$M(x,y) = M\left(\frac{8}{2}, \frac{-12}{2}\right)$$

$$M(x,y) = M(4,-6)$$

Q.5 Define the following

Solution:

(i) **Co-ordinate Geometry:-**

Co-ordinate geometry is the study of geometrical shapes in the Cartesian plane (or coordinate plane)

(ii) Collinear:-

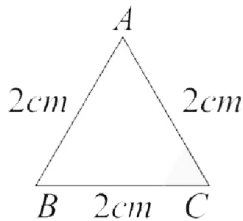
Two or more than two points which lie on the same straight line are called collinear points with respect to that line.

(iii) Non- Collinear:-

The points which do not lie on the same straight line are called non-collinear.

(iv) Equilateral Triangle:-

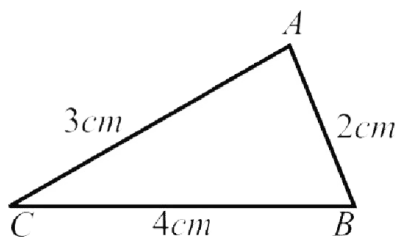
If the length of all three sides of a triangle are same then the triangle is called an equilateral triangle.



ΔABC is an equilateral triangle.

(v) Scalene Triangle:-

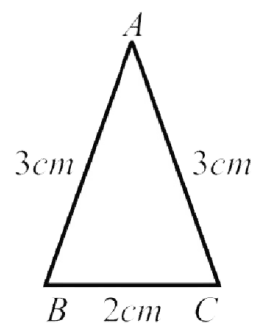
A triangle is called a scalene triangle if measure of all sides are different.



ΔABC is a Scalene triangle.

(vi) Isosceles Triangle:-

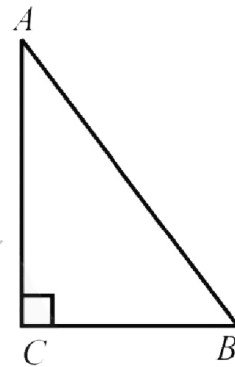
An isosceles triangles is a triangle which has two of its sides with equal length while the third side has different length.



ΔABC is an isosceles triangle

(vii) Right Triangle:-

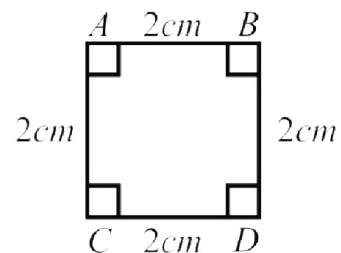
A triangle in which one of the angles has measure equal to 90° is called a right triangle.



ΔABC is a right angled triangle.

(viii) Square:-

A Square is closed figure formed by four non- collinear points such that lengths of all sides are equal and measure of each angles is 90° .



$ABCD$ is a square.