

## Exercise 2.1

**Q.1** Identify which of the following are rational and irrational numbers?

- (i)  $\sqrt{3}$  Irrational number
- (ii)  $\frac{1}{6}$  Rational number
- (iii)  $\pi$  Irrational number
- (iv)  $\frac{15}{2}$  Rational number
- (v) 7.25 Rational number
- (vi)  $\sqrt{29}$  Irrational number

**Q.2** Convert the following fractions into decimal fractions.

(i)  $\frac{17}{25}$

**Solution:**  $\frac{17}{25}$

$$\begin{array}{r} 0.68 \\ 25 \overline{) 170} \\ \underline{-150} \\ 200 \\ \underline{-200} \\ 0 \end{array}$$

$\frac{17}{25} = 0.68$  **Ans**

(ii)  $\frac{19}{4}$

**Solution:**  $\frac{19}{4}$

$$\begin{array}{r} 4.75 \\ 4 \overline{) 19.000} \\ \underline{16} \\ 30 \\ \underline{28} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

$= \frac{19}{4}$   
 $= 4.75$  **Ans**

(iii)  $\frac{57}{8}$

**Solution:**  $\frac{57}{8}$

$$\begin{array}{r} 7.125 \\ 8 \overline{) 57} \\ \underline{-56} \\ 10 \\ \underline{8} \\ 20 \\ \underline{-16} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

$= \frac{57}{8}$   
 $= 7.125$  **Ans**

(iv)  $\frac{205}{18}$

**Solution:**  $\frac{205}{18}$

$$\begin{array}{r} 11.388 \\ 18 \overline{) 205.000} \\ \underline{25} \end{array}$$

$$\begin{array}{r}
 18 \\
 \hline
 70 \\
 -54 \\
 \hline
 160 \\
 -144 \\
 \hline
 160 \\
 -144 \\
 \hline
 16 \\
 \\
 208 \\
 \hline
 18 \\
 = 11.3888 \\
 = 11.3889 \text{ Ans}
 \end{array}$$

(v)  $\frac{5}{8}$

Solution:  $\frac{5}{8}$

$$\begin{array}{r}
 .625 \\
 8 \overline{) 5.000} \\
 \underline{48} \\
 20 \\
 \underline{-16} \\
 40 \\
 \underline{-40} \\
 0
 \end{array}$$

$$\begin{array}{r}
 5 \\
 \hline
 8 \\
 = 0.625 \text{ Ans}
 \end{array}$$

(vi)  $\frac{25}{38}$

Solution:  $\frac{25}{38}$

$$\begin{array}{r}
 0.65789\dots \\
 38 \overline{) 250} \\
 \underline{-228} \\
 220 \\
 \underline{-190} \\
 300 \\
 \underline{-266} \\
 340 \\
 \underline{-304} \\
 360 \\
 \underline{-342} \\
 18
 \end{array}$$

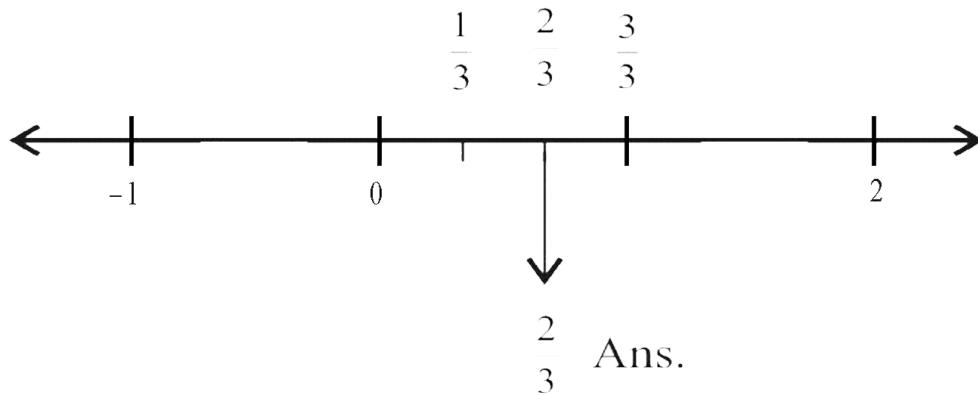
$$\begin{array}{r}
 25 \\
 \hline
 38 \\
 = 0.65789 \text{ Ans}
 \end{array}$$

**Q.3 Which of the following statements are true and which are false?**

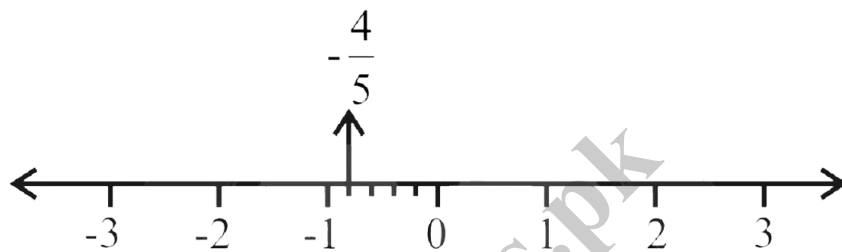
- |       |  |       |
|-------|--|-------|
| (i)   | $\frac{2}{3}$ is an irrational number.   | False |
| (ii)  | $\pi$ is an irrational number.           | True  |
| (iii) | $\frac{1}{9}$ is a terminating fraction. | False |
| (iv)  | $\frac{3}{4}$ is a terminating fraction. | True  |
| (v)   | $\frac{4}{5}$ is a recurring fraction.   | False |

**Q.4** Represent the following numbers on the number line.

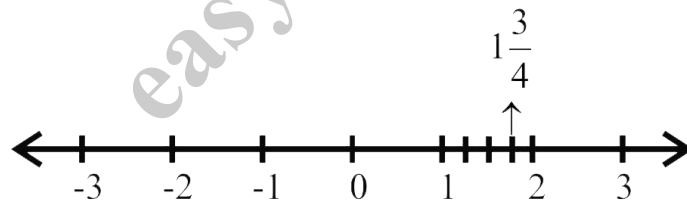
(i)  $\frac{2}{3}$



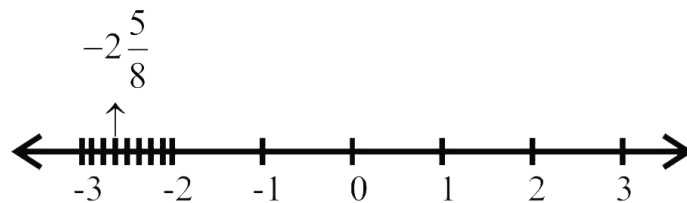
(ii)  $-\frac{4}{5}$



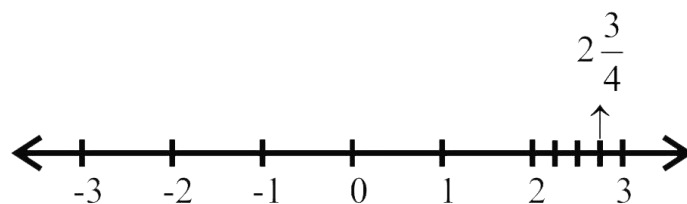
(iii)  $1\frac{3}{4}$



(iv)  $-2\frac{5}{8}$



(v)  $2\frac{3}{4}$



(vi)  $\sqrt{5}$

By Pythagoras theorem

$$(\text{Hypotenusus})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$$

$$(\overline{OB})^2 = (2)^2 + (1)^2$$

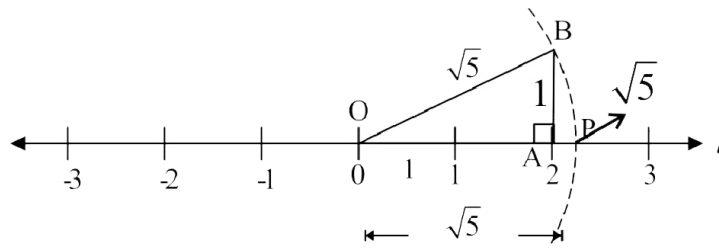
$$(\overline{OB})^2 = 4 + 1$$

$$(\overline{OB})^2 = 5$$

Taking square root on both sides

$$\sqrt{(\overline{OB})^2} = \sqrt{5}$$

$$\overline{OB} = \sqrt{5}$$



**Q.5** Give a rational number between

$$\frac{3}{4} \text{ and } \frac{5}{9}$$

**Solution:**

Required No between

$$\frac{3}{4} \text{ and } \frac{5}{9}$$

$$= \left[ \frac{3}{4} + \frac{5}{9} \right] \div 2$$

$$= \left[ \frac{27 + 20}{36} \right] \div 2$$

$$= \left[ \frac{47}{36} \right] \div 2$$

$$= \frac{47}{36} \times \frac{1}{2}$$

$$= \frac{47}{72} \text{ Ans}$$

**Q.6** Express the following recurring decimals as the rational number

$\frac{p}{q}$  where  $p, q$  are integer

and  $q \neq 0$ .

(i)  $0.\overline{5}$

**Solution:**

$$x = 0.\overline{5}$$

$$x = 0.555\dots$$

$$10 \times x = 10 \times 0.555\dots$$

$$10x = 5.555\dots$$

$$10x = 5 + 0.555\dots$$

$$10x = 5 + x$$

$$10x - x = 5$$

$$9x = 5$$

$$x = \frac{5}{9}$$

$$\therefore 0.\overline{5} = \frac{5}{9} \text{ Ans}$$

(ii)  $0.\overline{13}$

**Solutions:**

Suppose

$$x = 0.\overline{13}$$

$$x = 0.131313\dots$$

$$100x = 100 \times 0.131313\dots$$

$$100x = 13.1313\dots$$

$$100x = 13 + 0.1313\dots$$

$$100x = 13 + x$$

$$100x - x = 13$$

$$99x = 13$$

$$x = \frac{13}{99}$$

$$\therefore 0.\overline{13} = \frac{13}{99} \text{ Ans}$$

(iii)  $0.\overline{67}$

**Solutions:**

Suppose

$$x = 0.\overline{67}$$

$$x = 0.676767\dots$$

$$100x = 100 \times 0.676767\dots$$

$$100x = 67.6767\dots$$

$$100x = 67 + 0.6767\dots$$

$$100x = 67 + x$$

$$100x - x = 67$$

$$99x = 67$$

$$x = \frac{67}{99}$$

$$\therefore 0.\overline{67} = \frac{67}{99} \text{ Ans}$$

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## Exercise 2.2

**Q.1 Identify the property used in the following.**

- (i)  $a + b = b + a$  Commutative Property *w.r.t* addition  
(ii)  $(ab)c = a(bc)$  Associative Property *w.r.t* multiplication  
(iii)  $7 \times 1 = 7$  Multiplicative Identity  
(iv)  $x > y$  or  $x = y$  or  $x < y$  Trichotomy  
(v)  $ab = ba$  Commutative *w.r.t* multiplication  
(vi)  $a + c = b + c = a + b$  Cancellation Property of addition  
(vii)  $5 + (-5) = 0$  Additive Inverse  
(viii)  $7 \times \frac{1}{7} = 1$  Multiplicative inverse  
(ix)  $a > b \Rightarrow ac > bc (c > 0)$  Multiplicative property

**Q.2 Fill in the following blanks by stating the properties of real numbers used.**

$$\begin{aligned} & 3x + 3(y - x) \\ &= 3x + 3y - 3x, \dots \text{Distributive property} \\ &= 3x - 3x + 3y, \dots \text{Commutative} \\ &= 0 + 3y, \dots \text{Additive Inverse} \\ &= 3y, \dots \text{Additive identity} \end{aligned}$$

**Q.3 Give the name of property used in the following.**

- (i)  $\sqrt{24} + 0 = \sqrt{24}$  Additive Identity  
(ii)  $-\frac{2}{3} \left[ 5 + \frac{7}{2} \right] = \left[ -\frac{2}{3} \right] (5) + \left[ -\frac{2}{3} \right] \left[ \frac{7}{2} \right]$  Distributive Property  
(iii)  $\pi + (-\pi) = 0$  Additive Inverse  
(iv)  $\sqrt{3} \cdot \sqrt{3}$  is a real number. Closure property *w.r.t*  $x$ .  
(v)  $\left[ -\frac{5}{8} \right] \left[ -\frac{8}{5} \right] = 1$  Multiplicative Inverse.

## Exercise 2.3

**Q.1** Write each radical expression in exponential notation and each exponential expression in radical notation. Do not simplify.

(i)  $\sqrt[3]{-64}$   
 $= (-64)^{\frac{1}{3}}$

(ii)  $2^{\frac{3}{5}}$   
 $= \sqrt[5]{2^3}$

(iii)  $-7^{\frac{1}{3}}$   
 $= -\sqrt[3]{7}$

(iv)  $y^{\frac{2}{3}}$   
 $= \sqrt[3]{y^{-2}}$

**Q.2** Tell whether the following statements are true or false?

(i)  $5^{\frac{1}{5}} = \sqrt{5}$  **False**

(ii)  $2^{\frac{2}{3}} = \sqrt[3]{4}$  **True**

(iii)  $\sqrt{49} = \sqrt{7}$  **False**

(iv)  $\sqrt[3]{x^{27}} = x^3$  **False**

**Q.3** Simplify the following radical expression.

(i)  $\sqrt[3]{-125}$

**Solution:**

$$\begin{aligned} &= \sqrt[3]{-125} \\ &= \sqrt[3]{-5 \times -5 \times -5} \\ &= \sqrt[3]{(-5)^3} \\ &= -5 \text{ Ans} \end{aligned}$$

$$(ii) \quad \sqrt[4]{32}$$

**Solution:**

$$\begin{aligned} &= \sqrt[4]{32} \\ &= \sqrt[4]{2 \times 2 \times 2 \times 2 \times 2} \\ &= \sqrt[4]{2^4 \times 2} \\ &= \sqrt[4]{2^4} \times \sqrt[4]{2} \\ &= 2\sqrt[4]{2} \text{ Ans} \end{aligned}$$

$$(iii) \quad \sqrt[5]{\frac{3}{32}}$$

**Solution:**

$$\begin{aligned} &= \sqrt[5]{\frac{3}{32}} \\ &= \frac{\sqrt[5]{3}}{\sqrt[5]{32}} \\ &= \frac{\sqrt[5]{3}}{\sqrt[5]{2 \times 2 \times 2 \times 2 \times 2}} \\ &= \frac{\sqrt[5]{3}}{\sqrt[5]{(2)^5}} \\ &= \frac{\sqrt[5]{3}}{2} \text{ Ans} \end{aligned}$$

$$(iv) \quad \sqrt[3]{-\frac{8}{27}}$$

**Solution:**

$$\begin{aligned} &= \sqrt[3]{-\frac{8}{27}} \\ &= \sqrt[3]{\left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right)} \\ &= \sqrt[3]{\left(-\frac{2}{3}\right)^3} \\ &= -\frac{2}{3} \text{ Ans} \end{aligned}$$

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## Exercise 2.4

**Q.1** Use laws of exponents to simplify.

(i) 
$$\frac{(243)^{\frac{2}{3}} (32)^{\frac{1}{5}}}{\sqrt{(196)^{-1}}}$$

**Solution:**

$$\begin{aligned} & \frac{(243)^{\frac{2}{3}} (32)^{\frac{1}{5}}}{\sqrt{(196)^{-1}}} \\ &= \frac{(243)^{\frac{2}{3}} (32)^{\frac{1}{5}}}{\sqrt{(196)^{-1}}} \\ &= \frac{(3^5)^{\frac{2}{3}} \times (2^5)^{\frac{1}{5}}}{\sqrt{[(14)^2]^{-1}}} \\ &= \frac{(3)^{\frac{10}{3}} \times 2^{-1}}{\sqrt{[(14)^{-1}]^2}} \\ &= \frac{(3)^{\frac{10}{3}} \times 2^{-1}}{(14)^{-1}} \\ &= \frac{14}{(3)^{\frac{10}{3}} \times 2} \\ &= \frac{7}{\frac{10}{3^3}} \\ &= \frac{7}{\sqrt[3]{3^{10}}} \\ &= \frac{7}{\sqrt[3]{3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3}} \\ &= \frac{7}{\sqrt[3]{3^3 \times 3^3 \times 3^3 \times 3}} \\ &= \frac{7}{\sqrt[3]{3^3 \times 3^3 \times 3^3 \times 3^3}} \end{aligned}$$

$$\begin{aligned} &= \frac{7}{3 \times 3 \times 3 \times \sqrt[3]{3}} \\ &= \frac{7}{27\sqrt[3]{3}} \text{ Ans} \end{aligned}$$

(ii)  $(2x^5y^{-4})(-8x^{-3}y^2)$

**Solution:**

$$\begin{aligned} & (2x^5y^{-4})(-8x^{-3}y^2) \\ &= -16x^{5-3}y^{-4+2} \\ &= -16x^2y^{-2} \\ &= \frac{-16x^2}{y^2} \text{ Ans} \end{aligned}$$

(iii) 
$$\left[ \frac{x^{-2}y^{-1}z^{-4}}{x^4y^{-3}z^0} \right]^{-3}$$

**Solution:**

$$\begin{aligned} & \left[ \frac{x^{-2}y^{-1}z^{-4}}{x^4y^{-3}z^0} \right]^{-3} \\ &= [x^{-2-4}y^{-1+3}z^{-4-0}]^{-3} \\ &= (x^{-6}y^{+2}z^{-4})^{-3} \\ &= (x^{-6})^{-3} (y^2)^{-3} (y^{-4})^{-3} \\ &= x^{18}y^{-6}z^{12} \\ &= \frac{x^{18}z^{12}}{y^6} \text{ Ans} \end{aligned}$$

(iv) 
$$\frac{(81)^n \cdot 3^5 - (3)^{4n-1} (243)}{(9^{2n})(3^3)}$$

**Solution:**

$$\begin{aligned} & \frac{(81)^n \cdot 3^5 - (3)^{4n-1} (243)}{(9^{2n})(3^3)} \\ &= \frac{(3^4)^n \cdot 3^5 - 3^{4n} \cdot 3^{-1} \cdot 3^5}{(3^2)^{2n} \cdot 3^3} \\ &= \frac{3^{4n} \cdot 3^5 - 3^{4n} \cdot 3^{-1+5}}{3^{4n} \cdot 3^3} \\ &= \frac{3^{4n} \cdot 3^5 - 3^{4n} \cdot 3^4}{3^{4n} \cdot 3^3} \end{aligned}$$

$$\begin{aligned}
 &= \frac{3^{4n} \cdot 3^4 (3-1)}{3^{4n} \cdot 3^3} \\
 &= 3^{4n-4n} \cdot 3^{4-3} \cdot (2) \\
 &= 3^0 \cdot 3^1 \cdot 2 \\
 &= 1 \times 3 \times 2 \\
 &= 6 \text{ Ans}
 \end{aligned}$$

**Q.2 Show that**

$$\left[ \frac{x^a}{x^b} \right]^{a+b} \times \left[ \frac{x^b}{x^c} \right]^{b+c} \times \left[ \frac{x^c}{x^a} \right]^{c+a} = 1$$

**Proof:**

L.H.S

$$\begin{aligned}
 &= \left[ \frac{x^a}{x^b} \right]^{a+b} \times \left[ \frac{x^b}{x^c} \right]^{b+c} \times \left[ \frac{x^c}{x^a} \right]^{c+a} \\
 &= (x^{a-b})^{a+b} \times (x^{b-c})^{b+c} \times (x^{c-a})^{c+a} \\
 &= x^{(a-b)(a+b)} \times x^{(b-c)(b+c)} \times x^{(c-a)(c+a)} \\
 &= x^{a^2-b^2} \times x^{b^2-c^2} \times x^{c^2-a^2} \\
 &= x^{a^2-b^2+b^2-c^2+c^2-a^2} \\
 &= x^0 \\
 &= 1 \\
 &1 = \text{R.H.S Ans}
 \end{aligned}$$

**Q.3 Simplify**

$$\text{(i)} \quad \frac{2^{\frac{1}{3}} \times (27)^{\frac{1}{3}} \times (60)^{\frac{1}{2}}}{(180)^{\frac{1}{2}} \times (4)^{\frac{1}{3}} \times (9)^{\frac{1}{4}}}$$

$$\text{Solution: } \frac{2^{\frac{1}{3}} \times (27)^{\frac{1}{3}} \times (60)^{\frac{1}{2}}}{(180)^{\frac{1}{2}} \times (4)^{\frac{1}{3}} \times (9)^{\frac{1}{4}}}$$

$$\begin{aligned}
 &= \frac{2^{\frac{1}{3}} \times (3^3)^{\frac{1}{3}} \times (2 \times 2 \times 3 \times 5)^{\frac{1}{2}}}{(2 \times 2 \times 3 \times 3 \times 5)^{\frac{1}{2}} \times (2^2)^{\frac{1}{3}} \times (3^2)^{\frac{1}{4}}} \\
 &= \frac{2^{\frac{1}{3}} \times 3 \times (2^2)^{\frac{1}{2}} \times 3^{\frac{1}{2}} \times 5^{\frac{1}{2}}}{(2^2)^{\frac{1}{2}} \times (3^2)^{\frac{1}{2}} \times (5)^{\frac{1}{2}} \times 2^{-\frac{2}{3}} \times 3^{\frac{1}{2}}} \\
 &= \frac{2^{\frac{1}{3}} \times 3 \times 2 \times 3^{\frac{1}{2}} \times 5^{\frac{1}{2}}}{2 \times 3 \times 5^{\frac{1}{2}} \times 2^{-\frac{2}{3}} \times 3^{\frac{1}{2}}}
 \end{aligned}$$

$$\begin{aligned}
 &= 2^{\frac{1}{3}} \times 2^{+1} \times 2^{-1} \times 2^{\frac{+2}{3}} \times 3^1 \times 3^{\frac{1}{2}} \times 3^{-1} \times 3^{\frac{-1}{2}} \times 5^{\frac{1}{2}} \times 5^{\frac{-1}{2}} \\
 &= 2^{\frac{1}{3}+1-1+\frac{2}{3}} \times 3^{1-\frac{1}{2}-1+\frac{1}{2}} \times 5^{\frac{1}{2}-\frac{1}{2}} \\
 &= 2^{\frac{1}{3}+\frac{2}{3}} \times 3^0 \times 5^0 \\
 &= 2^{\frac{1+2}{3}} \times 1 \times 1 \\
 &= 2^{\frac{3}{3}} \\
 &= 2 \text{ Ans}
 \end{aligned}$$

$$\text{(ii)} \quad \sqrt{\frac{(216)^{\frac{2}{3}} \times (25)^{\frac{1}{2}}}{(0.04)^{\frac{1}{2}}}}$$

$$\text{Solution: } \sqrt{\frac{(216)^{\frac{2}{3}} \times (25)^{\frac{1}{2}}}{(0.04)^{\frac{1}{2}}}}$$

$$= \sqrt{\frac{(6^3)^{\frac{2}{3}} \times (5^2)^{\frac{1}{2}}}{\left(\frac{4}{100}\right)^{\frac{1}{2}}}}$$

$$= \sqrt{\frac{6^2 \times 5}{\left(\frac{25 \cancel{100}}{100}\right)^{\frac{1}{2}}}}$$

$$= \sqrt{\frac{6^2 \times 5}{(25)^{\frac{1}{2}}}}$$

$$= \sqrt{\frac{6^2 \times 5}{(5^2)^{\frac{1}{2}}}}$$

$$= \sqrt{\frac{6^2 \times 5}{5}}$$

$$= \sqrt{6^2 \times 5^{+1} \times 5^{-1}}$$

$$= \sqrt{6^2 \times 5^{+1-1}}$$

$$= \sqrt{6^2 \times 5^0}$$

$$= \sqrt{6^2 \times 1}$$

$$= \sqrt{6^2}$$

$$= 6 \text{ Ans}$$

(iii)  $5^{2^3} \div (5^2)^3$

**Solution:**  $5^{2^3} \div (5^2)^3$

$$= 5^8 \div 5^6$$

$$= 5^{8-6}$$

$$= 5^2$$

$$= 25 \text{ Ans}$$

(iv)  $(x^3)^2 \div x^{3^2}, x \neq 0$

**Solution:**  $(x^3)^2 \div x^{3^2}, x \neq 0$

$$= x^6 \div x^9$$

$$= x^{6-9}$$

$$= x^{-3}$$

$$= \frac{1}{x^3} \text{ Ans}$$

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## Exercise 2.5

### Q.1 Evaluate

(i)  $i^7$

**Solution:**

$$\begin{aligned} &= i^7 \\ &= i^6 \cdot i \\ &= (i^2)^3 \cdot i \\ &= (-1)^3 \cdot i \\ &= -1 \times i \\ &= -i \\ &= -i \text{ Ans} \end{aligned}$$

(ii)  $i^{50}$

**Solution:**  $i^{50}$

$$\begin{aligned} &= (i^2)^{25} \\ &= (-1)^{25} \\ &= -1 \text{ Ans} \end{aligned}$$

(iii)  $i^{12}$

**Solution:**

$$\begin{aligned} &i^{12} \\ &= (i^2)^6 \\ &= (-1)^6 \\ &= 1 \text{ Ans} \end{aligned}$$

(iv)  $(-i)^8$

**Solution:**

$$\begin{aligned} &(-i)^8 \\ &= i^8 \\ &= (i^2)^4 \\ &= (-1)^4 \\ &= 1 \text{ Ans} \end{aligned}$$

(v)  $(-i)^5$

**Solution:**

$$\begin{aligned} &(-i)^5 \\ &= -i^5 \\ &= -i^4 \cdot i \\ &= -(i^2)^2 \cdot i \\ &= -(-1)^2 \cdot i \\ &= -(1)(i) \\ &= -i \text{ Ans} \end{aligned}$$

(vi)  $i^{27}$

**Solution:**  $i^{27}$

$$\begin{aligned} &= i^{26} \cdot i \\ &= (i^2)^{13} \cdot i \\ &= (-1)^{13} \cdot i \\ &= -1 \cdot i \\ &= -i \text{ Ans} \end{aligned}$$

### Q.2 Write the conjugate of the following numbers.

(i)  $2 + 3i$   
 $= 2 - 3i$

(ii)  $3 - 5i$   
 $= 3 + 5i$

(iii)  $-i$   
 $= i$

(iv)  $-3 + 4i$   
 $= -3 - 4i$

(v)  $-4 - i$   
 $= -4 + i$

(vi)  $i - 3$   
 $= -i - 3$

### Q.3 Write the real and imaginary part of the following numbers.

(i)  $1 + i$   
Real = 1  
Imaginary = 1

(ii)  $-1 + 2i$   
Real = -1  
Imaginary = 2

- (iii)  $-3i + 2$   
Real = 2  
Imaginary = - 3
- (iv)  $-2 - 2i$   
Real = -2  
Imaginary = - 2
- (v)  $-3i$   
Real = 0  
Imaginary = - 3
- (vi)  $2 + 0i$   
Real = 2  
Imaginary = 0

**Q.4 Find the value of  $x$  and  $y$  if**

$$x + iy + 1 = 4 - 3i$$

**Solution: Given that**

$$x + iy + 1 = 4 - 3i$$

$$x + iy = 4 - 3i - 1$$

$$x + iy = 3 - 3i$$

$$x = 3 \quad y = -3$$

$$x = 3, y = -3 \text{ Ans}$$

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## Exercise 2.6

**Q.1** Identify the following statement as true or false.

- (i)  $\sqrt{-3}\sqrt{-3} = 3$  **False**
- (ii)  $i^{73} = -i$  **False**
- (iii)  $i^{10} = -1$  **True**
- (iv) Complex conjugate of  $(-6i + i^2)$  is  $(-1 + 6i)$  **True**
- (v) Difference of a complex number  $z = a + bi$  and its conjugate is a real number. **False**
- (vi) If  $(a - 1) - (b + 3)i = 5 + 8i$ , then  $a = 6$  and  $b = -11$ . **True**
- (vii) Product of a complex number and its conjugate is always a non-negative real number. **True**

**Q.2** Express the each complex number in the standard form  $a + bi$ , where  $a$  and  $b$  are real number.

(i)  $(2 + 3i) + (7 - 2i)$

**Solution:**

$$\begin{aligned} &= 2 + 3i + 7 + 2i \\ &= 2 + 7 + 3i + 2i \\ &= 9 + i \text{ Ans} \end{aligned}$$

(ii)  $2(5 + 4i) - 3(7 + 4i)$

**Solution:**  $2(5 + 4i) - 3(7 + 4i)$

$$\begin{aligned} &= 10 + 8i - 21 - 12i \\ &= 10 - 21 + 8i - 12i \\ &= -11 - 4i \text{ Ans} \end{aligned}$$

(iii)  $(-3 + 5i) - (4 + 9i)$

**Solution:**  $(-3 + 5i) - (4 + 9i)$

$$\begin{aligned} &= +3 - 5i - 4 - 9i \\ &= 3 - 4 - 5i - 9i \\ &= -1 - 14i \text{ Ans} \end{aligned}$$

(iv)  $2i^2 + 6i^3 + 3i^{16} - 6i^{19} + 4i^{25}$

**Solution:**  $2i^2 + 6i^3 + 3i^{16} - 6i^{19} + 4i^{25}$

$$\begin{aligned} &= 2(-1) + 6i^2 \cdot i + 3(i^2)^8 - 6(i^2)^9 \cdot i + 4(i^2)^{12} \cdot i \\ &= -2 + 6(-1)i + 3(-1)^8 - 6(-1) \cdot i + 4(-1)^{12} \cdot i \\ &= -2 - 6i + 3 - 6(-1)i + 4(+1)i \\ &= 1 - \cancel{6i} + \cancel{6i} + 4i \\ &= 1 + 4i \text{ Ans} \end{aligned}$$

**Q.3** Simplify and write your answer in the form  $a + bi$

(i)  $(-7 + 3i)(-3 + 2i)$

**Solution:**  $(-7 + 3i)(-3 + 2i)$

$$\begin{aligned} &= -7(-3 + 2i) + 3i(-3 + 2i) \\ &= 21 - 14i - 9i + 6i^2 \\ &= 21 - 23i + 6(-1) \\ &= 21 - 23i - 6 \\ &= 21 - 6 - 23i \\ &= 15 - 23i \text{ Ans} \end{aligned}$$

(ii)  $(2 - \sqrt{-4})(3 - \sqrt{-4})$

**Solution:**  $(2 - \sqrt{-4})(3 - \sqrt{-4})$   
 $= (2 - \sqrt{4 \times -1})(3 - \sqrt{4 \times -1})$   
 $= (2 - \sqrt{4i^2})(3 - \sqrt{4i^2})$   
 $= (2 - 2i)(3 - 2i)$   
 $= 2(3 - 2i) - 2i(3 - 2i)$   
 $= 6 - 4i - 6i + 4i^2$   
 $= 6 - 10i + 4(-1)$   
 $= 6 - 10i - 4$   
 $= 2 - 10i$  **Ans**

(iii)  $(\sqrt{5} - 3i)^2$

**Solution:**  $(\sqrt{5} - 3i)^2$   
 $= (\sqrt{5})^2 + (3i)^2 - 2(\sqrt{5})(3i)$   
 $= 5 + 9i^2 - 6\sqrt{5}i$   
 $= 5 + 9(-1) - 6\sqrt{5}i$   
 $= 5 - 9 - 6\sqrt{5}i$   
 $= -4 - 6\sqrt{5}i$  **Ans**

(iv)  $(2 - 3i)(\overline{3 - 2i})$

**Solution:**  $(2 - 3i)(\overline{3 - 2i})$   
 $= (2 - 3i)(3 + 2i)$   
 $= 2(3 + 2i) - 3i(3 + 2i)$   
 $= 6 + 4i - 9i - 6i^2$   
 $= 6 - 5i - 6(-1)$   
 $= 6 - 5i + 6$   
 $= 6 + 6 - 5i$   
 $= 12 - 5i$  **Ans**

**Q.4** Simplify and write your answer in the form  $a + bi$ .

(i)  $\frac{-2}{1+i}$

**Solution:**  $\frac{-2}{1+i}$   
 $= \frac{-2}{1+i} \times \frac{1-i}{1-i}$   
 $= \frac{-2(1-i)}{(1)^2 - (i)^2}$   
 $= \frac{-2+2i}{1-i^2}$   
 $= \frac{-2+2i}{1-(-1)}$   
 $= \frac{-2+2i}{1+1}$   
 $= \frac{-2+2i}{2}$   
 $= -\frac{2}{2} + \frac{2i}{2}$   
 $= -1+i$  **Ans**

(ii)  $\frac{2+3i}{4-i}$

**Solution:**  $\frac{2+3i}{4-i}$   
 $= \frac{2+3i}{4-i} \times \frac{4+i}{4+i}$   
 $= \frac{(2+3i)(4+i)}{(4)^2 - (i)^2}$   
 $= \frac{2(4+i) + 3i(4+i)}{16 - (-1)}$   
 $= \frac{8+2i+12i+3i^2}{16+1}$   
 $= \frac{8+4i+3(-1)}{17}$   
 $= \frac{8+14i-3}{17}$   
 $= \frac{8-3+14i}{17}$   
 $= \frac{5+14i}{17}$   
 $= \frac{5}{17} + \frac{14}{17}i$  **Ans**

(iii)  $\frac{9-7i}{3+i}$

**Solution:**  $\frac{9-7i}{3+i}$

$$= \frac{9-7i}{3+i} \times \frac{3-i}{3-i}$$

$$= \frac{(9-7i)(3-i)}{(3)^2 - (i)^2}$$

$$= \frac{9(3-i) - 7i(3-i)}{9 - (-1)}$$

$$= \frac{27 - 9i - 21i + 7i^2}{9+1}$$

$$\frac{27 - 30i + 7(-1)}{10}$$

$$= \frac{27 - 30i - 7}{10}$$

$$= \frac{27 - 7 - 30i}{10}$$

$$= \frac{20 - 30i}{10}$$

$$= \frac{20}{10} - \frac{30i}{10}$$

$$= 2 - 3i \text{ Ans}$$

(iv)  $\frac{2-6i}{3+i} - \frac{4+i}{3+i}$

**Solution:**  $\frac{2-6i}{3+i} - \frac{4+i}{3+i}$

$$= \frac{2-6i - (4+i)}{3+i}$$

$$= \frac{2-6i-4-i}{3+i}$$

$$= \frac{2-4-6i-i}{3+i}$$

$$= \frac{-2-7i}{3+i}$$

$$= \frac{-2-7i}{3+i} \times \frac{3-i}{3-i}$$

$$= \frac{-2-7i}{3+i} \times \frac{3-i}{3-i}$$

$$= \frac{-2(3-i) - 7i(3-i)}{(3)^2 - (i)^2}$$

$$= \frac{-6+2i-21i+7i^2}{9-(-1)}$$

$$= \frac{-6-19i+7(-1)}{9+1}$$

$$= \frac{-6-19i-7}{10}$$

$$= \frac{-6-7-19i}{10}$$

$$= \frac{-13-19i}{10}$$

$$= \frac{-13}{10} - \frac{19i}{10} \text{ Ans}$$

(v)  $\left[ \frac{1+i}{1-i} \right]^2$

**Solution:**  $\left[ \frac{1+i}{1-i} \right]^2$

$$= \frac{(1+i)^2}{(1-i)^2}$$

$$= \frac{(1)^2 + (i)^2 + 2ab}{(1)^2 + (i)^2 - 2ab}$$

$$= \frac{(1)^2 + (i)^2 + 2(1)(i)}{(1)^2 + (i)^2 - 2(1)(i)}$$

$$= \frac{1+(-1)+2i}{+1+(-1)-2i}$$

$$= \frac{\cancel{1} - \cancel{1} + 2i}{\cancel{1} - \cancel{1} - 2i}$$

$$= \frac{2i}{-2i} = -1$$

$$= -1$$

$$= -1 + 0i \text{ Ans}$$

(vi)  $\frac{1}{(2+3i)(1-i)}$

**Solution:**  $\frac{1}{(2+3i)(1-i)}$

$$= \frac{1}{2(1-i) + 3i(1-i)}$$

$$= \frac{1}{2-2i+3i-3i^2}$$



$$\begin{aligned}
&= \frac{1}{2+i-3(-1)} \\
&= \frac{1}{2+i+3} \\
&= \frac{1}{2+3+i} \\
&= \frac{1}{5+i} \\
&= \frac{1}{5+i} \times \frac{5-i}{5-i} \\
&= \frac{1(5-i)}{(5)^2 - (i)^2} \\
&= \frac{5-i}{25 - (-1)} \\
&= \frac{5-i}{25+1} \\
&= \frac{5-i}{26} \\
&= \frac{5}{26} - \frac{1i}{26} \text{ Ans}
\end{aligned}$$

**Q.5 Calculate**

(a)  $\bar{z}$  (b)  $z + \bar{z}$  (c)  $z - \bar{z}$  (d)  $z\bar{z}$  for each of the following.

(i)  $z = -i$

**Solution:**  $z = -i$

(a)  $\bar{z} = +i$

(b)  $z + \bar{z} = -i + i$   
 $= 0$

(c)  $z - \bar{z} = (-i) - (i)$   
 $= -2i$

(d)  $z\bar{z} = (-i)(i)$   
 $= -i^2$   
 $= -(-1)$   
 $= 1 \text{ Ans}$

(ii)  $z = 2+i$

**Solution:**  $z = 2+i$   
 $z + 2i$

(a)  $\bar{z} = 2-i$

(b)  $z + \bar{z} = (2+i) + (2-i)$   
 $= 2 + \cancel{i} + 2 - \cancel{i}$   
 $= 2 + 2$   
 $= 4$

(c)  $z - \bar{z} = (2+i) - (2-i)$   
 $= \cancel{2} + i - \cancel{2} + i$   
 $= i + i$   
 $= 2i$

(d)  $z\bar{z} = (2+i)(2-i)$   
 $= (2)^2 - (i)^2$   
 $= 4 - i^2$   
 $= 4 - (-1)$   
 $= 4 + 1$   
 $= 5 \text{ Ans}$

(iii)  $z = \frac{1+i}{1-i}$

**Solution:**  $z = \frac{1+i}{1-i}$

$z = \frac{1+i}{1-i} \times \frac{1+i}{1+i}$   
 $= \frac{1(1+i) + i(1+i)}{(1-i)(1+i)}$

$= \frac{1+i+i+(-1)}{(1)^2 - (i)^2}$

$= \frac{1+2i+(-1)}{1 - (-1)}$

$= \frac{\cancel{1} + 2i - \cancel{1}}{1+1}$

$= \frac{2i}{2}$

$= i$

$z = i$

(a)  $\bar{z} = -i$

(b)  $z + \bar{z} = i + (-i)$   
 $= \cancel{i} - \cancel{i}$   
 $= 0$

(c)  $z - \bar{z} = i - (-i)$   
 $= i + i$

$$= 2i$$

$$\begin{aligned} \text{(d) } z\bar{z} &= (i)(-i) \\ &= -i^2 \\ &= -(-1) \\ &= +1 \text{ Ans} \end{aligned}$$

$$\text{(iv) } z = \frac{4-3i}{2+4i}$$

$$\text{Solution: } z = \frac{4-3i}{2+4i}$$

$$\begin{aligned} z &= \frac{4-3i}{2+4i} \times \frac{2-4i}{2-4i} \\ &= \frac{4(2-4i) - 3i(2-4i)}{(2+4i)(2-4i)} \end{aligned}$$

$$= \frac{8-16i-6i+12i^2}{(2)^2 - (4i)^2}$$

$$= \frac{8-22i+12(-1)}{4-16i^2}$$

$$= \frac{8-22i-12}{4-16(-1)}$$

$$= \frac{8-12-22i}{4+16}$$

$$= \frac{-4-22i}{20}$$

$$= \frac{-4}{20} - \frac{22}{20}i$$

$$= -\frac{1}{5} - \frac{11}{10}i$$

$$\text{(a) } \bar{z} = \frac{-1}{5} + \frac{11}{10}i$$

(b)

$$z + \bar{z} = \left(-\frac{1}{5} - \frac{11}{10}i\right) + \left(-\frac{1}{5} + \frac{11}{10}i\right)$$

$$= -\frac{1}{5} - \frac{11}{10}i - \frac{1}{5} + \frac{11}{10}i$$

$$= -\frac{1}{5} - \frac{1}{5}$$

$$= \frac{-1-1}{5}$$

$$= -\frac{2}{5}$$

(c)

$$z - \bar{z} = \left(-\frac{1}{5} - \frac{11}{10}i\right) - \left(-\frac{1}{5} + \frac{11}{10}i\right)$$

$$= -\frac{1}{5} - \frac{11}{10}i + \frac{1}{5} - \frac{11}{10}i$$

$$= -\frac{11}{10}i - \frac{11}{10}i = \frac{-11i-11i}{10}$$

$$= -\frac{22i}{10}$$

$$= -\frac{11}{5}i$$

$$\text{(d) } z\bar{z} = \left(-\frac{1}{5} - \frac{11}{10}i\right)\left(-\frac{1}{5} + \frac{11}{10}i\right)$$

$$= \left(-\frac{1}{5}\right)^2 - \left(\frac{11}{10}i\right)^2$$

$$= \frac{1}{25} - \frac{121}{100}i^2$$

$$= \frac{1}{25} - \frac{121}{100}(-1)$$

$$= \frac{1}{25} + \frac{121}{100}$$

$$= \frac{4+121}{100}$$

$$= \frac{125}{100}$$

$$= \frac{5}{4} \text{ Ans}$$

**Q.6** If  $z = 2 + 3i$  and show that.

$$\text{(i) } \overline{z+w} = \bar{z} + \bar{w}$$

$$\text{Solution: } z+w = \bar{z} + \bar{w}$$

$$z+w = 2+3i+5-4i$$

$$= 2+5+3i-4i$$

$$= 7-i$$

$$\text{L.H.S} = \overline{z+w}$$

$$= \overline{7-i}$$

$$= 7+i$$

... (i)

$$\text{R. H. S} = \bar{z} + \bar{w}$$

$$= \overline{(2+3i)} + \overline{(5-4i)}$$

$$= 2-3i+5+4i$$

$$= 2+5-3i+4i$$

$$= 7 + i$$

(ii)

From (i) and (ii) we get

$$\overline{\text{L.H.S}} = \overline{\text{R.H.S}}$$

$$\overline{z + w} = \overline{z} + \overline{w}$$

Hence proved

(ii)  $\overline{z - w} = \overline{z} - \overline{w}$

Solution:  $\overline{z - w} = \overline{z} - \overline{w}$

$$z - w = (2 + 3i) - (5 - 4i)$$

$$= 2 + 3i - 5 + 4i$$

$$= 2 - 5 + 3i + 4i$$

$$= -3 + 7i$$

$$\text{L.H.S} = \overline{z - w}$$

$$= \overline{-3 + 7i}$$

$$= -3 - 7i$$

... (i)

$$\text{R.H.S} = \overline{z} - \overline{w}$$

$$= \overline{(2 + 3i)} - \overline{(5 - 4i)}$$

$$= 2 + 3i - (5 + 4i)$$

$$= 2 - 3i - 5 - 4i$$

$$= -3 - 7i$$

From (i) and (ii) we get

$$\overline{\text{L.H.S}} = \overline{\text{R.H.S}}$$

$$\overline{z - w} = \overline{z} - \overline{w}$$

Hence proved

(iii)  $\overline{zw} = \overline{z} \overline{w}$

Solutions:  $\overline{zw} = \overline{z} \overline{w}$

$$zw = (2 + 3i)(5 + 4i)$$

$$= 2(5 + 4i) + 3i(5 + 4i)$$

$$= 10 - 8i + 15i - 12i^2$$

$$= 10 + 7i - 12(-1)$$

$$= 10 + 7i + 12$$

$$= 22 + 7i$$

$$\text{L.H.S} = \overline{zw}$$

$$= \overline{22 + 7i}$$

$$= 22 - 7i$$

$$\text{R.H.S} = \overline{z} \overline{w}$$

$$= \overline{(2 + 3i)} \overline{(5 - 4i)}$$

$$= (2 - 3i)(5 + 4i)$$

$$= 2(5 + 4i) - 3i(5 + 4i)$$

$$= 10 + 8i - 15i - 12i^2$$

$$= 10 - 7i - 12(-1)$$

$$= 10 - 7i + 12$$

$$= 22 - 7i$$

From (i) and (ii) we get

$$\overline{\text{L.H.S}} = \overline{\text{R.H.S}}$$

$$\overline{zw} = \overline{z} \overline{w}$$

Hence proved

(iv)  $\overline{\left(\frac{z}{w}\right)} = \frac{\overline{z}}{\overline{w}}$ , where  $w \neq 0$

Solutions:  $\overline{\left(\frac{z}{w}\right)} = \frac{\overline{z}}{\overline{w}}$

$$\frac{z}{w} = \frac{2 + 3i}{5 - 4i} \times \frac{5 + 4i}{5 + 4i}$$

$$= \frac{2(5 + 4i) + 3i(5 + 4i)}{(5 - 4i)(5 + 4i)}$$

$$= \frac{10 + 8i + 15i + 12i^2}{(5)^2 - (4i)^2}$$

$$= \frac{10 + 23i + 12(-1)}{25 - 16i^2}$$

$$= \frac{10 + 23i - 12}{25 - (6(-1))}$$

$$= \frac{10 + 23i - 12}{25 + 16}$$

$$= \frac{-2 + 23i}{41}$$

$$\text{L.H.S} = \overline{\left(\frac{z}{w}\right)}$$

$$= \overline{\left(\frac{-2 + 23i}{41}\right)}$$

$$= \frac{-2}{41} - \frac{23}{41}i$$

... (i)

$$\text{R.H.S} = \frac{\overline{z}}{\overline{w}}$$

$$= \frac{\overline{(2 + 3i)}}{\overline{(5 - 4i)}}$$

$$\begin{aligned}
&= \frac{2-3i}{5+4i} \\
&= \frac{2-3i}{5+4i} \times \frac{5-4i}{5-4i} \\
&= \frac{2(5-4i)-3i(5-4i)}{(5+4i)(5-4i)} \\
&= \frac{10-8i-15i+12i^2}{(5)^2-(4i)^2} \\
&= \frac{10-23i+12(-1)}{25-16i^2} \\
&= \frac{10-23i+12(-1)}{25-16(-1)} \\
&= \frac{10-23i-12}{25+16} \\
&= \frac{-2-23i}{41} \\
&= \frac{-2}{41} - \frac{23}{41}i \quad \dots \text{(ii)}
\end{aligned}$$

From (i) and (ii) we get

L.H.S=R.H.S

**Hence Proved**

$$\overline{\left[\frac{z}{w}\right]} = \frac{\bar{z}}{\bar{w}}$$

(v)  $\frac{1}{2}(z+\bar{z})$  is the real part of  $z$ .

$$\begin{aligned}
\text{Solution: } &\frac{1}{2}(z+\bar{z}) \\
&= \frac{1}{2}[(2+3i)+(\overline{2+3i})] \\
&= \frac{1}{2}[(2+3i)+(2-3i)] \\
&= \frac{1}{2}[2+\cancel{3i}+2-\cancel{3i}] \\
&= \frac{1}{2}[2+2] \\
&= \frac{1}{2}[4] \\
&= 2 = \text{Re}(z)
\end{aligned}$$

$\frac{1}{2}(z+\bar{z})$  is the real part of  $z$ . **Ans**

(vi)  $\frac{1}{2}(z-\bar{z})$  is the imaginary part of  $z$ .

$$\begin{aligned}
\text{Solution: } &\frac{1}{2}(z-\bar{z}) \\
&= \frac{1}{2}(z-\bar{z}) \\
&= \frac{1}{2}[(2+3i)-(\overline{2+3i})] \\
&= \frac{1}{2}[(2+3i)-(2-3i)] \\
&= \frac{1}{2}[\cancel{2}+3i-\cancel{2}+3i] \\
&= \frac{1}{2}[3i+3i] \\
&= \frac{1}{2}[6i] \\
&= 3i \\
&= \text{Imaginary}(z)
\end{aligned}$$

$\frac{1}{2}(z-\bar{z})$  is the imaginary part of  $z$ . **Ans**

**Q.7** Solve the following equations for real  $x$  and  $y$ .

(i)  $(2-3i)(x+yi) = 4+i$

**Solution:**  $(2-3i)(x+yi) = 4+i$

$$\begin{aligned}
x+yi &= \frac{4+i}{2-3i} \\
x+yi &= \frac{4+i}{2-3i} \times \frac{2+3i}{2+3i} \\
&= \frac{4(2+3i)+i(2+3i)}{(2-3i)(2+3i)} \\
&= \frac{8+12i+2i+3i^2}{(2)^2-(3i)^2} \\
&= \frac{8+14i+3(-1)}{4-9i^2} \\
&= \frac{8+14i-3}{4-9(-1)} \\
&= \frac{8-3+14i}{4+9}
\end{aligned}$$

$$= \frac{5+14i}{13}$$

$$x + yi = \frac{5}{13} + \frac{14}{13}i$$

$$x = \frac{5}{13}, y = \frac{14}{13} \text{ Ans}$$

$$(ii) \quad (3 - 2i)(x + yi) = 2(x - 2yi) + 2i - 1$$

**Solution:**

$$(3 - 2i)(x + yi) = 2(x - 2yi) + 2i - 1$$

$$3(x + yi) - 2i(x + yi) = 2x - 4yi + 2i - 1$$

$$3x + 3yi - 2xi - 2yi^2 = (2x - 1) + i(2 - 4y)$$

$$3x + (3x - 2x)i - 2y(-1) = (2x - 1) + i(2 - 4y)$$

$$3x + (3y - 2x)i + 2y = (2x - 1) + i(2 - 4y)$$

$$(3x + 2y) + (3y - 2x)i = (2x - 1) + (2 - 4y)i$$

Comparing the real and imaginary parts.

$$3x + 2y = 2x - 1 \quad ,$$

$$3y - 2x = 2 - 4y$$

$$3x - 2x + 2y = -1 \quad ,$$

$$3y - 2x = 2 - 4y$$

$$x + 2y = -1 \quad ,$$

$$-2x + 3y + 4y = 2$$

$$-2x + 7y = 2$$

$$x + 2y = -1 \quad \text{_____ (i)}$$

$$-2x + 7y = 2 \quad \text{_____ (ii)}$$

Multiply equation (i) with (2)

$$2(x + 2y) = -1 \times 2$$

$$2x + 4y = -2 \quad \text{_____ (iii)}$$

$$\underline{\cancel{2x} + 4y = \cancel{-2}}$$

$$\underline{\cancel{-2x} + 7y = \cancel{2}}$$

$$11y = 0$$

$$y = \frac{0}{11}$$

$$y = 0$$

Putting  $y = 0$  in equation (i)

$$x + 2y = -1$$

$$x + 2(0) = -1$$

$$x + 0 = -1$$

$$x = -1 + 0$$

$$x = -1 \text{ Ans}$$

$$(iii) \quad (3 + 4i)^2 - 2(x - yi) = x + yi$$

$$\text{Solution: } (3 + 4i)^2 - 2(x - yi) = x + yi$$

$$(3 + 4i)^2 - 2(x - yi) = x + yi$$

$$9 + 24i + 16i^2 - 2x + 2yi = x + yi$$

$$9 + 24i + 16(-1) - 2x + 2yi = x + yi$$

$$9 + 24i - 16 - 2x + 2yi = x + yi$$

$$9 + 24i - 16 - 2x = x + 2yi - yi = 0$$

$$9 + 24i - 16 - 3x + yi = 0$$

$$-3x + yi = -9 - 24i + 16$$

$$-3x + yi = 16 - 9 - 24i$$

$$-3x + yi = 7 - 24i$$

Comparing the real and imaginary parts.

$$-3x = 7$$

$$y = -24$$

$$x = \frac{-7}{3}$$

$$y = -24 \text{ Ans}$$

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## Review Exercise 2

**Q.1 Multiple choice questions. Choose the correct answer.**

(i)  $(27x^{-1})^{-\frac{2}{3}}$  \_\_\_\_\_

(a)  $\frac{\sqrt[3]{x^2}}{9}$

(b)  $\frac{\sqrt{x^3}}{9}$

(c)  $\frac{\sqrt[3]{x^2}}{8}$

(d)  $\frac{\sqrt{x^3}}{8}$

(ii) Write  $\sqrt[7]{x}$  in the exponential form \_\_\_\_\_

(a)  $x$

(b)  $x^7$

(c)  $x^{\frac{1}{7}}$

(d)  $x^{\frac{7}{2}}$

(iii) Write  $4^{\frac{2}{3}}$  with radical sign \_\_\_\_\_

(a)  $\sqrt[3]{4^2}$

(b)  $\sqrt[2]{4^3}$

(c)  $\sqrt[2]{4^3}$

(d)  $\sqrt{4^6}$

(iv) In  $\sqrt[3]{35}$  the radicand is;

(a) 3

(b)  $\frac{1}{3}$

(c) 35

(d) None

(v)  $\left(\frac{25}{16}\right)^{-\frac{1}{2}} =$  \_\_\_\_\_

(a)  $\frac{5}{4}$

(b)  $\frac{4}{5}$

(c)  $-\frac{5}{4}$

(d)  $-\frac{4}{5}$

(vi) The conjugate of  $5 + 4i$  is \_\_\_\_\_

(a)  $-5 + 4i$

(b)  $-5 - 4i$

(c)  $5 - 4i$

(d)  $5 + 4i$

(vii) The value of  $i^9$  is;

(a) 1

(b) -1

(c)  $i$

(d)  $-i$

- (viii) Every real number is \_\_\_\_\_  
 (a) Positive integer (b) A rational number  
 (c) A negative integer (d) A complex number
- (ix) Real point of  $2ab(i+i^2)$  is \_\_\_\_\_  
 (a)  $2ab$  (b)  $-2ab$   
 (c)  $2abi$  (d)  $-2abi$
- (x) Imaginary part of  $-i(3i+2)$  is \_\_\_\_\_  
 (a)  $-2$  (b)  $2$   
 (c)  $3$  (d)  $-3$
- (xi) Which of the following sets have the closure property w.r.t addition \_\_\_\_\_  
 (a)  $\{0\}$  (b)  $\{0,1\}$   
 (c)  $\{0,1\}$  (d)  $\left\{1, \sqrt{2}, \frac{1}{2}\right\}$
- (xii) Name the property of real number used in  $\left[-\frac{\sqrt{5}}{2}\right] \times 1 = -\frac{\sqrt{5}}{2}$  \_\_\_\_\_  
 (a) Additive identity (b) Additive inverse  
 (c) Multiplicative identity (d) Multiplicative inverse
- (xiii) If  $x, y, z \in R, z < 0$ , then  $x < y \Rightarrow \dots$   
 (a)  $xz < yz$  (b)  $xz > yz$   
 (c)  $xz = yz$  (d) None of these
- (xiv) IF  $a, b \in R$ , only one of  $a = b$  or  $a < b$  or  $a > b$  hold is called \_\_\_\_\_  
 (a) Trichotomy property (b) Transitive property  
 (c) Additive property (d) Multiplicative property
- (xv) A non-terminating, non-recurring decimal represents ...  
 (a) A natural number (b) A rational number  
 (c) An irrational number (d) A prime number

### ANSWER KEY

<b>i</b>	<b>a</b>	<b>vi</b>	<b>c</b>	<b>xi</b>	<b>a</b>
<b>ii</b>	<b>c</b>	<b>vii</b>	<b>c</b>	<b>xii</b>	<b>c</b>
<b>iii</b>	<b>a</b>	<b>viii</b>	<b>d</b>	<b>xiii</b>	<b>b</b>
<b>iv</b>	<b>c</b>	<b>ix</b>	<b>b</b>	<b>xiv</b>	<b>a</b>
<b>v</b>	<b>b</b>	<b>x</b>	<b>a</b>	<b>xv</b>	<b>c</b>



**Q.2 True or False? Identity**

- (i) Division is not an associative operation. **True**
- (ii) Every whole number is a natural number. **False**
- (iii) Multiplicative inverse of 0.02 is 50. **True**
- (iv)  $\pi$  is rational number. **False**
- (v) Every integer is a rational number. **True**
- (vi) Subtraction is a commutative operation. **False**
- (vii) Every real number is a rational number. **False**
- (viii) Decimal representation of a rational number is either terminating or recurring. **True**
- (ix)  $1.\bar{8} = 1 + \frac{8}{9}$  **True**

**Q.3 Simplify the following**

(i)  $\sqrt[4]{81y^{-12}x^{-8}}$

**Solution:**

$$\begin{aligned}
 &= (3^4 y^{12} x^{-8})^{\frac{1}{4}} \\
 &= 3^{4 \times \frac{1}{4}} y^{12 \times \frac{1}{4}} x^{-8 \times \frac{1}{4}} \\
 &= 3y^{-3}x^{-2} \\
 \sqrt[4]{81y^{-12}x^{-8}} &= \frac{3}{y^3x^2} \text{ Ans}
 \end{aligned}$$

(ii)  $\sqrt{25x^{10}y^{8m}}$

**Solution:**

$$\begin{aligned}
 &= \sqrt{25x^{10n}y^{8m}} \\
 &= (5^2 x^{10n} y^{8m})^{\frac{1}{2}} \\
 &= 5^{2 \times \frac{1}{2}} x^{10n \times \frac{1}{2}} y^{8m \times \frac{1}{2}} \\
 \sqrt{25x^{10}y^{8m}} &= 5x^{5n}y^{4m} \text{ Ans}
 \end{aligned}$$

(iii)  $\left[ \frac{x^3y^4z^5}{x^{-2}y^{-1}z^{-5}} \right]^{\frac{1}{5}}$

**Solution:**

$$\begin{aligned}
 &= (x^{3+2} \cdot y^{4+1} \cdot z^{5+5})^{\frac{1}{5}} \\
 &= (x^5 y^5 z^{10})^{\frac{1}{5}} \\
 &= x^{\frac{5}{5}} y^{\frac{5}{5}} z^{\frac{10}{5}} \\
 &= x^1 y^1 z^2
 \end{aligned}$$

$$\left[ \frac{x^3y^4z^5}{x^{-2}y^{-1}z^{-5}} \right]^{\frac{1}{5}} = x.y.z^2 \text{ Ans}$$

(iv)  $\left( \frac{32x^{-6}y^{-4}z}{625x^4yz^{-4}} \right)^{\frac{2}{5}}$

**Solution:**

$$\begin{aligned}
 &= \left( \frac{2^5 x^{-4} y^{-4} z}{5^4 x^4 y z^{-4}} \right)^{\frac{2}{5}} \\
 &= \left[ \frac{2^5 z^{1+4}}{5^4 x^{4+6} \times y^{1+4}} \right]^{\frac{2}{5}} \\
 &= \left[ \frac{2^5 z^5}{5^4 x^{10} y^5} \right]^{\frac{2}{5}} \\
 &= \frac{2^{\frac{5}{5} \times \frac{2}{5}} \times z^{\frac{5}{5} \times \frac{2}{5}}}{5^{4 \times \frac{2}{5}} \times x^{10 \times \frac{2}{5}} \times y^{\frac{5}{5} \times \frac{2}{5}}} \\
 &= \frac{2^2 \times z^2}{5^{\frac{8}{5}} \times x^4 \times y^2} \\
 &= \frac{4z^2}{5^{\frac{5}{5} + \frac{3}{5}} \times x^4 y^2} \\
 &= \frac{4z^2}{5^{1 + \frac{3}{5}} \times x^4 y^2} \\
 \left( \frac{32x^{-6}y^{-4}z}{625x^4yz^{-4}} \right)^{\frac{2}{5}} &= \frac{4z^2}{5 \times 5^{\frac{3}{5}} x^4 y^2} \text{ Ans}
 \end{aligned}$$

**Q.4 Simplify**

$$\sqrt{\frac{(216)^{\frac{2}{3}} \times (25)^{\frac{1}{2}}}{(0.04)^{\frac{3}{2}}}}$$

**Solution:**

$$\begin{aligned} & \sqrt{\frac{(216)^{\frac{2}{3}} \times (25)^{\frac{1}{2}}}{(0.04)^{\frac{3}{2}}}} \\ &= \sqrt{\frac{(6^3)^{\frac{2}{3}} \times (5^2)^{\frac{1}{2}}}{\left(\frac{4}{100}\right)^{\frac{3}{2}}}} \\ &= \sqrt{\frac{6^2 \times 5}{\left(\frac{100}{4}\right)^{\frac{3}{2}}}} \\ &= \sqrt{\frac{6^2 \times 5}{(5^2)^{\frac{3}{2}}}} \\ &= \sqrt{\frac{6^2 \times 5}{(5^3)}} \\ &= \sqrt{\frac{6^2}{5^3 \times 5^{-1}}} \\ &= \sqrt{\frac{6^2}{5^{3-1}}} \\ &= \sqrt{\frac{6^2}{5^2}} \\ &= \sqrt{\left(\frac{6}{5}\right)^2} \\ &= \left(\frac{6}{5}\right)^{2 \times \frac{1}{2}} \\ &= \frac{6}{5} \text{ Ans} \end{aligned}$$

**Q.5**

$$\left(\frac{a^p}{a^q}\right)^{p+q} \times \left(\frac{a^q}{a^r}\right)^{q+r} \div 5(a^q \cdot a^r)^{p-r}$$

**Solution:**

$$\begin{aligned} &= \frac{(a^{p-q})^{p+q} (a^{q-r})^{q+r}}{5(a^{p+r})^{p-r}} \\ &= \frac{a^{(p-q)(p+q)} a^{(q-r)(q+r)}}{5a^{(p+r)(p-r)}} \end{aligned}$$

$$\begin{aligned} &= \frac{a^{p^2-q^2} a^{q^2-r^2}}{5a^{p^2-r^2}} \\ &= \frac{a^{p^2-q^2+q^2-r^2}}{5a^{p^2-r^2}} \\ &= \frac{a^{p^2-r^2-p^2+r^2}}{5} \\ &= \frac{a^0}{5} \end{aligned}$$

$$\left(\frac{a^p}{a^q}\right)^{p+q} \times \left(\frac{a^q}{a^r}\right)^{q+r} \div 5(a^q \cdot a^r)^{p-r}$$

$$= \frac{1}{5} \text{ Ans}$$

**Q.6 Simplify**  $\left(\frac{a^{2l}}{a^{l+m}}\right) \left(\frac{a^{2m}}{a^{m+n}}\right) \left(\frac{a^{2n}}{a^{n+2}}\right)$

**Solution:**

$$\begin{aligned} &= a^{2l-l-m} a^{2m-m-n} a^{2n-n-n} \\ &= a^{l-m} a^{m-n} a^{n-l} \\ &= a^{l-m+m-n+n-l} \\ &= a^0 \end{aligned}$$

$$\left(\frac{a^{2l}}{a^{l+m}}\right) \left(\frac{a^{2m}}{a^{m+n}}\right) \left(\frac{a^{2n}}{a^{n+2}}\right) = 1 \text{ Ans}$$

**Q.7 Simplify**  $\sqrt[3]{\frac{a^l}{a^m}} \times \sqrt[3]{\frac{a^m}{a^n}} \times \sqrt[3]{\frac{a^n}{a^r}}$

**Solution:**

$$\begin{aligned} &= \sqrt[3]{a^{l-m}} \sqrt[3]{a^{m-n}} \sqrt[3]{a^{n-l}} \\ &= (a^{l-m})^{\frac{1}{3}} \times (a^{m-n})^{\frac{1}{3}} \times (a^{n-l})^{\frac{1}{3}} \\ &= a^{\frac{l-m}{3}} \times a^{\frac{m-n}{3}} \times a^{\frac{n-l}{3}} \\ &= a^{\frac{l-m}{3} + \frac{m-n}{3} + \frac{n-l}{3}} \\ &= a^{\frac{l-m+n-n+l}{3}} \\ &= a^{\frac{0}{3}} \end{aligned}$$

$$\begin{aligned} &= a^0 \\ &= \sqrt[3]{\frac{a^l}{a^m}} \times \sqrt[3]{\frac{a^m}{a^n}} \times \sqrt[3]{\frac{a^n}{a^r}} = 1 \text{ Ans} \end{aligned}$$