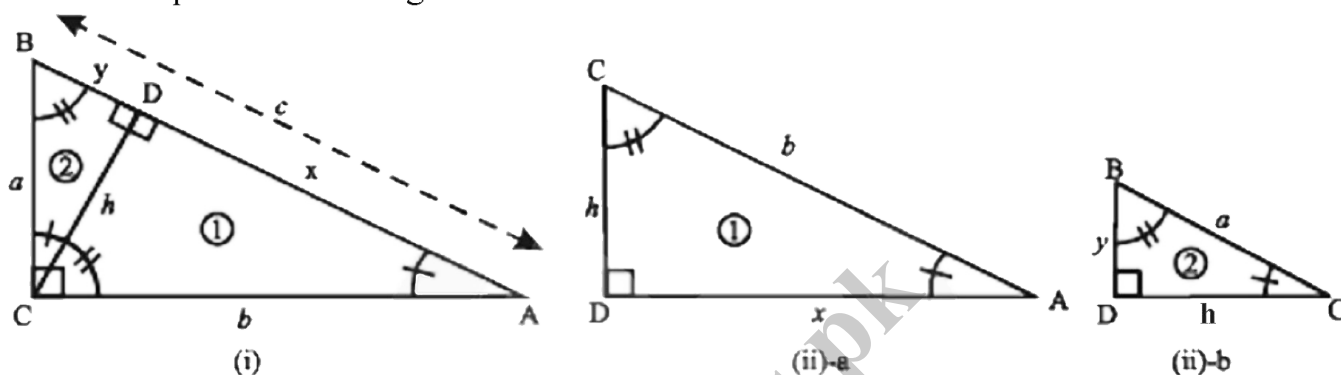


Unit 15: Pythagoras Theorem

Overview

Theorem 15.1.1

In a right angled triangle, the square of the length of hypotenuse is equal to the sum of the squares of the lengths of the other two sides



Given

ΔACB is a right angled triangle in which $m\angle C = 90^\circ$ and $m\overline{BC} = a$, $m\overline{AC} = b$ and $m\overline{AB} = c$

To prove

$$c^2 = a^2 + b^2$$

Construction

Draw \overline{CD} perpendicular from C on \overline{AB}

Let $m\overline{CD} = h$, $m\overline{AD} = x$ and $m\overline{BD} = y$. Line segment CD splits ΔABC into two Δ s ADC and BDC which are separately shown in the figures

(ii) –a and (ii) –b respectively

Proof (using similar Δ s)

Statements	Reasons
In $\Delta ADC \leftrightarrow \Delta ACB$	Refer to figure (ii)-a and (i)
$\angle A \cong \angle A$	Common – Self Congruent
$\angle ADC \cong \angle ACB$	Construction- given each angle = 90°
$\angle C \cong \angle B$	$\angle C$ and $\angle B$ complements of $\angle A$

$$\therefore \triangle ADC \sim \triangle ACB$$

$$\therefore \frac{x}{b} = \frac{b}{c}$$

$$\text{or } x = \frac{b^2}{c} \text{ ————— (i)}$$

Again in $\triangle BDC \leftrightarrow \triangle BCA$

$$\angle B \cong \angle B$$

$$\angle BDC \cong \angle BCA$$

$$\angle C \cong \angle A$$

$$\therefore \triangle BDC \sim \triangle BCA$$

$$\therefore \frac{y}{a} = \frac{a}{c}$$

$$\text{or } y = \frac{a^2}{c} \text{ ————— (ii)}$$

But $y + x = c$

$$\therefore \frac{a^2}{c} + \frac{b^2}{c} = c$$

$$\text{or } a^2 + b^2 = c^2$$

$$\text{i-e } c^2 = a^2 + b^2$$

Congruency of three angles

(Measures of corresponding sides of similar triangles are proportional)

Refer to figure (ii)-b and (i)

Common – self Congruent

Construction – given each angle = 90°

$\angle C$ and $\angle A$ complements of $\angle B$

Congruency of three angles

(Corresponding sides of similar triangles are proportional)

Supposition

By (i) and (ii)

Multiplying both side by c

Theorem 15.1.2 Converse of Pythagoras Theorem 15.1.1

If the Square of one side of a triangle is equal to the sum of the square of the other two sides then the triangle is a right angled triangle

Given

In a $\triangle ABC$, $m\overline{AB} = c, m\overline{BC} = a, m\overline{AC} = b$

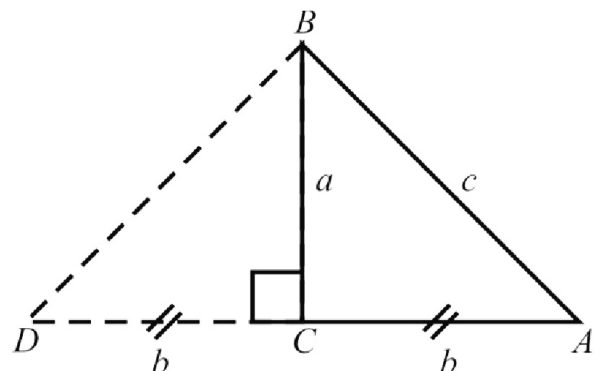
Such that $a^2 + b^2 = c^2$

To prove

$\triangle ACB$ is a right angled triangle

Construction

Draw \overline{CD} perpendicular to \overline{BC} Such that



$\overline{CD} \cong \overline{CA}$. Join the points B and D

Proof

Statements	Reasons
$\triangle DCB$ is a right angled triangle	Construction
$\therefore (m\overline{BD})^2 = a^2 + b^2$	Pythagoras theorem
But $a^2 + b^2 = c^2$	Given
$\therefore (m\overline{BD})^2 = c^2$	
or $m\overline{BD} = c$	Taking Square root on both sides
Now in $\triangle DCB \leftrightarrow \triangle ACB$	
$\overline{CD} \cong \overline{CA}$	Construction
$\overline{BC} \cong \overline{BC}$	Common
$\overline{DB} \cong \overline{AB}$	Each side = c
$\therefore \triangle DCB \cong \triangle ACB$	S.S.S \cong S.S.S
$\therefore \angle DCB \cong \angle ACB$	(Corresponding angles of congruent triangle)
But $m\angle DCB = 90^\circ$	Construction
$\therefore m\angle ACB = 90^\circ$	
Hence the $\triangle ACB$ is a Right angled triangle	

Exercise 15

Q.1 Verify that the Δ s having the following measures of sides are right-angled to verify whether the Δ s are right angled or not we use Pythagoras Theorem

$$(\text{Hypotenuse})^2 = (\text{base})^2 + (\text{Perpendicular})^2$$

(i) $a = 5\text{cm}$
 $b = 12\text{cm}$
 $c = 13\text{cm}$
 $a^2 = 25\text{cm}^2$
 $b^2 = 144\text{cm}^2$
 $c = 169\text{cm}^2$
 Larger Side is Hypotenuse So
 $169 = 25 + 144$
 $169 = 169$
 L.H.S = R.H.S
 So it is right angled triangle

(ii) $a = 1.5\text{cm}$
 $b = 2\text{cm}$
 $c = 2.5\text{cm}$
 $a^2 = 2.25\text{cm}^2$
 $b^2 = 4\text{cm}^2$
 $c^2 = 6.25$
 $6.25 = 2.25 + 4$
 $6.25 = 6.25$
 L.H.S = R.H.S
 So it is right-angled triangle

(iii) $a = 9\text{cm}$
 $b = 12\text{cm}$
 $c = 15\text{cm}$
 $a^2 = 81\text{cm}^2$
 $b^2 = 144\text{cm}^2$
 $c = 225\text{cm}^2$
 $225\text{cm}^2 = 81\text{cm} + 144\text{cm}$
 $225\text{cm}^2 = 225\text{cm}^2$
 L.H.S = R.H.S
 So it is right angled triangle

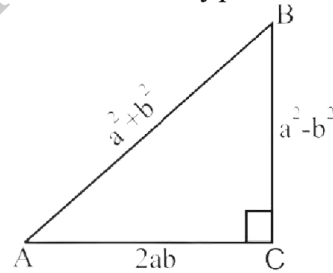
(iv) $a = 16\text{cm}$
 $b = 30\text{cm}$
 $c = 34\text{cm}$
 $a^2 = 256\text{cm}^2$
 $b^2 = 900\text{cm}$
 $c^2 = 1156\text{cm}^2$

$1156 = 256 + 900$
 $1156 = 1156$
 L.H.S = R.H.S
 It is right angled triangle

Q.2 Verify that $a^2 + b^2, a^2 - b^2$ and $2ab$ are the measures of the sides of a right angled Triangle where a and b are any two real numbers ($a > b$)

Let $a = z$ and $b = 1$
 $a^2 + b^2 = (z)^2 + (1)^2 = z^2 + 1 = 5$
 $a^2 - b^2 = (z)^2 - (1)^2 = z^2 - 1 = 3$
 $2ab = 2(z)(1) = 2z = 4$

Since $a^2 + b^2$ is the largest side so $a^2 + b^2$ will be hypotenuse



So

$$(\overline{AB})^2 = (\overline{AC})^2 + (\overline{BC})^2$$

$$(a^2 + b^2)^2 = (2ab)^2 + (a^2 - b^2)^2$$

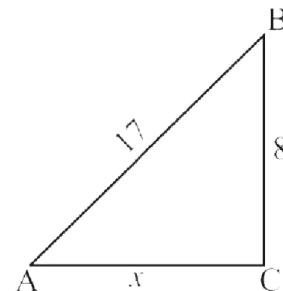
$$a^4 + b^4 + 2a^2b^2 = 4a^2b^2 + a^4 + b^4 - 2a^2b^2$$

$$a^4 + b^4 + 2a^2b^2 = a^4 + b^4 + 2a^2b^2$$

L.H.S = R.H.S

It is proved that it is a right angled triangle

Q.3 The three sides of a triangle are of measure 8, x and 17 respectively. For what value of x will it become base of right angled triangle by Pythagoras theorem



$$(\overline{AB})^2 = (\overline{AC})^2 + (\overline{BC})^2$$

$$(17)^2 = (x)^2 + (8)^2$$

$$289 = x^2 + 64$$

$$289 - 64 = x^2$$

$$x^2 = 225$$

Taking square root both side

$$\sqrt{x^2} = \sqrt{225}$$

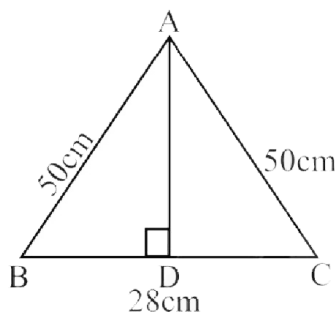
$$x = 15$$

Q.4 In an isosceles Δ the base

$\overline{BC} = 28\text{ cm}$ and

$\overline{AB} = \overline{AC} = 50\text{ cm}$

If $\overline{AD} \perp \overline{BC}$ then find



(i) Length of \overline{AD}

Solution:

$\overline{AD} \perp \overline{BC}$

So $\overline{BD} = \overline{CD}$

$$\frac{1}{2}\overline{BC} = \frac{1}{2}(28)$$

$$\frac{1}{2}\overline{BC} = 14$$

So

$$\overline{BD} = \overline{CD} = 14$$

$$(\overline{AB})^2 = (\overline{BD})^2 + (\overline{AD})^2$$

$$2500 = (14)^2 + (\overline{AD})^2$$

$$2500 = 196 + (\overline{AD})^2$$

$$2500 - 196 = (\overline{AD})^2$$

$$(\overline{AD})^2 = 2304$$

Taking square root on both side

$$\sqrt{(\overline{AD})^2} = \sqrt{2304}$$

$$\overline{AD} = 48\text{ cm}$$

(ii) Area of ΔABC

$$\text{Area of } \Delta ABC = \frac{1}{2} (\text{base})$$

(height)

$$= \frac{1}{2} (28)(48)$$

$$= (14)(48)$$

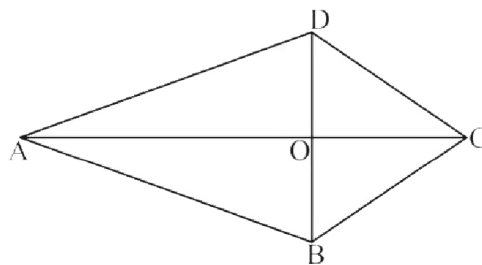
$$= 672\text{ cm}^2$$

Q.5 In a quadrilateral ABCD the diagonals \overline{AC} and \overline{BD} are perpendicular to each other.

Prove that

$$(\overline{AB})^2 + (\overline{CD})^2 = (\overline{AD})^2 + (\overline{BC})^2$$

ΔAOB



$$(\overline{AB})^2 = (\overline{OB})^2 + (\overline{OA})^2 \longrightarrow \text{(i)}$$

ΔBOC

$$(\overline{BC})^2 = (\overline{OB})^2 + (\overline{OC})^2 \longrightarrow \text{(ii)}$$

ΔCOD

$$(\overline{CD})^2 = (\overline{OD})^2 + (\overline{OC})^2 \longrightarrow \text{(iii)}$$

ΔDOA

$$(\overline{AD})^2 = (\overline{OA})^2 + (\overline{OD})^2 \longrightarrow \text{(iv)}$$

By adding (i) and (iii)

$$(\overline{AB})^2 + (\overline{CD})^2 = (\overline{OB})^2 + (\overline{OA})^2 + (\overline{OD})^2 + (\overline{OC})^2 \rightarrow \text{(v)}$$

By adding (ii) and (iv)

$$(\overline{AD})^2 + (\overline{BC})^2 = (\overline{OB})^2 + (\overline{OC})^2 + (\overline{OA})^2 + (\overline{OD})^2 \rightarrow \text{(vi)}$$

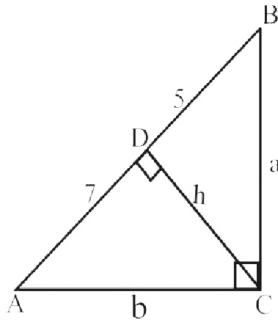
By comparing v and vi

$$(\overline{AB})^2 + (\overline{CD})^2 = (\overline{AD})^2 + (\overline{BC})^2$$

Hence proved

Q.6 the $\triangle ABC$ as shown in the figure $m\angle ACB = 90^\circ$ and $\overline{CD} \perp \overline{AB}$ find the length a , h and b if $m\overline{BD} = 5$ units and $m\overline{AD} = 7$ units

(i)



$\triangle ACB$

$$(7+5)^2 = (b)^2 + (a)^2$$

$$a^2 + b^2 = (12)^2$$

$$a^2 + b^2 = 144 \quad \text{_____ (i)}$$

$\triangle ADC$

$$(b)^2 = (7)^2 + (h)^2$$

$$b^2 - h^2 = 49 \quad \text{_____ (ii)}$$

$\triangle CDB$

$$a^2 = (5)^2 + (h)^2$$

$$a^2 - h^2 = 25 \quad \text{_____ (iii)}$$

Subtracting ii from iii

$$a^2 - \cancel{h^2} = 25$$

$$\pm b^2 \mp \cancel{h^2} = \pm 49$$

$$\underline{a^2 - b^2 = -24}$$

$$a^2 - b^2 = -24 \quad \text{_____ (iv)}$$

Adding equation I and IV

$$a^2 + \cancel{b^2} = 144$$

$$a^2 - \cancel{b^2} = -24$$

$$\underline{2a^2 = 120}$$

$$2a^2 = 120$$

$$a^2 = \frac{120}{2}$$

$$a^2 = 60$$

$$a^2 = 4 \times 15$$

Taking square root both side

Prime factor	
2	60
2	30
	15

$$\sqrt{a^2} = \sqrt{4 \times 15}$$

$$a = 2\sqrt{15}$$

Putting the value of a in equation

(i)

$$(2\sqrt{15})^2 + b^2 = 144$$

Prime factor

$$4 \times 15 + b^2 = 144$$

$$60 + b^2 = 144$$

$$b^2 = 144 - 60$$

$$b^2 = 84$$

$$b^2 = 4 \times 21$$

$$2 \times 2 \times 21$$

$$4 \times 21$$

Taking square root both side

$$b^2 = \sqrt{4 \times 21}$$

$$b = 2\sqrt{21}$$

Putting the value of b in equation

(ii)

$$(2\sqrt{21})^2 - h^2 = 49$$

$$4 \times 21 - 49 = h^2$$

$$h^2 = 84 - 49$$

$$h^2 = 35$$

Taking square root both side

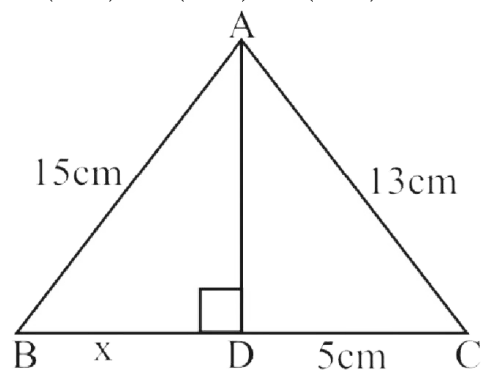
$$\sqrt{h^2} = \sqrt{35}$$

$$h = \sqrt{35}$$

(ii) Find the value of x in the shown figure

From $\triangle ADC$

$$(\overline{AC})^2 = (\overline{DC})^2 + (\overline{AD})^2$$



$$(13)^2 = (5)^2 + (\overline{AD})^2$$

$$169 = 25 + (\overline{AD})^2$$

$$169 - 25 = (\overline{AD})^2$$

$$(\overline{AD})^2 = 144$$

Taking square root both side

$$\sqrt{(\overline{AD})^2} = \sqrt{(144)}$$

$$\overline{AD} = 12$$

From ΔADB

$$(\overline{AB})^2 = (\overline{BD})^2 + (\overline{AD})^2$$

$$(15)^2 = x^2 + (12)^2$$

$$225 = x^2 + 144$$

$$225 - 144 = x^2$$

$$x^2 = 81$$

Taking square on both side

$$\sqrt{x^2} = \sqrt{81}$$

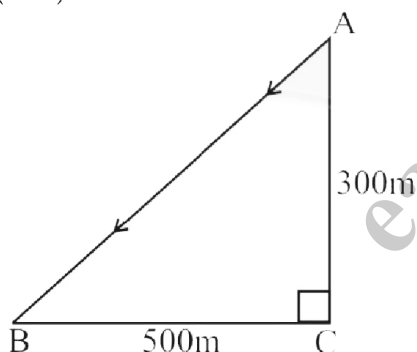
$$x = 9$$

Q.7 A plane is at a height of 300m and is 500m away from the airport as shown in the figure How much distance will it travel to land at the airport?

ΔABC is right angle triangle

$$(\overline{AB})^2 = (\overline{BC})^2 + (\overline{AC})^2$$

$$(\overline{AB})^2 = (500)^2 + (300)^2$$



Airport

$$(\overline{AB})^2 = 250000 + 90000$$

$$(\overline{AB})^2 = 340000$$

$$(\overline{AB})^2 = 10000 \times 34$$

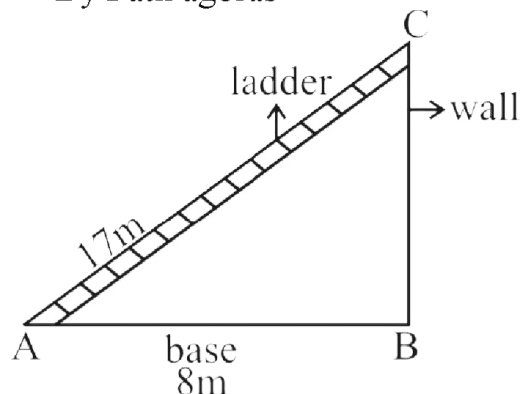
Taking square root on both side

$$\sqrt{(\overline{AB})^2} = \sqrt{10000 \times 34}$$

$$\overline{AB} = 100\sqrt{34}m$$

Q.8 A ladder 17m long rests against a vertical wall. The foot of the ladder is 8m away from the base of the wall. How high up the wall will the ladder reach?

By Path agoras



$$(\overline{AC})^2 = (\overline{AB})^2 + (\overline{BC})^2$$

$$(17)^2 = (8)^2 + (\overline{BC})^2$$

$$289 = 64 + (\overline{BC})^2$$

$$289 - 64 = (\overline{BC})^2$$

$$(\overline{BC})^2 = 225$$

Taking square root on both side

$$\sqrt{(\overline{BC})^2} = \sqrt{225}$$

$$\overline{BC} = 15m$$

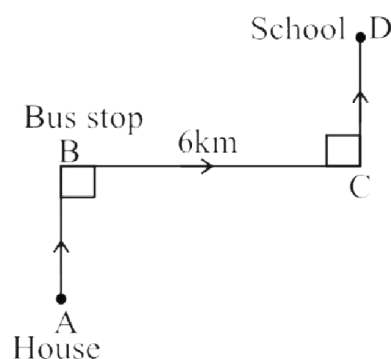
The height of wall = $\overline{BC} = 15m$

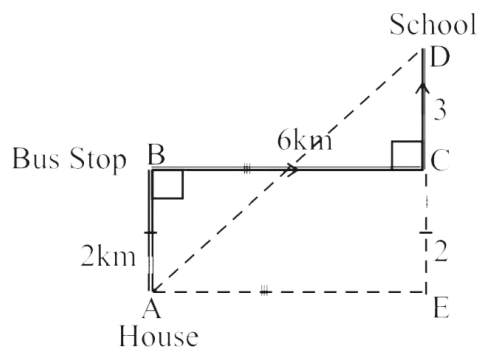
Q.9 A student travels to his school by the route as shown in the figure.

Find $m\overline{AD}$, the direct distance from his house to school.

Solution:

As we know that in rectangular opposite sides are equal so





$$\overline{AB} = \overline{CE} = 2km$$

$$\overline{BC} = \overline{AE} = 6km$$

$$\overline{DE} = \overline{DC} + \overline{CE}$$

∴ We get triangle

ΔADE which is right angled

triangle

$$(\overline{AD})^2 = (\overline{AE})^2 + (\overline{ED})^2$$

$$(\overline{AD})^2 = (6)^2 + (3 + 2)^2$$

$$(\overline{AD})^2 = 36 + (5)^2$$

$$(\overline{AD})^2 = 36 + 25$$

$$(\overline{AD})^2 = 61$$

Taking square root on both side

$$\sqrt{(\overline{AD})^2} = \sqrt{61}$$

$$\overline{AD} = \sqrt{61}km$$

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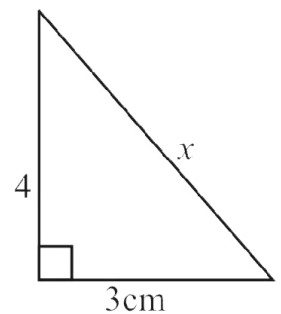
Review Exercise 15

Q.1 Which of the following are true and which are false

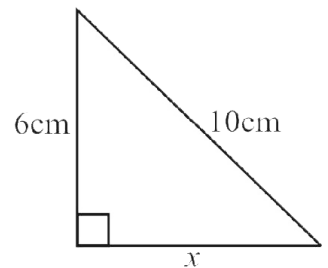
- (i) In a right angled triangle greater angle is of 90° (True)
(ii) In a right angled triangle right angle is of 60° (False)
(iii) In a right triangle hypotenuse is a side opposite to right angle (True)
(iv) If a,b,c are sides of right angled triangle with c as longer side then $c^2 = a^2 + b^2$ (True)
(v) If 3cm and 4cm are two sides of a right angled triangle, the hypotenuse is 5cm (True)
(vi) If hypotenuse of an isosceles right triangle is $\sqrt{2}$ cm then each of other side is of length 2cm (False)

Q.2 Find the unknown value in each of the following figures.

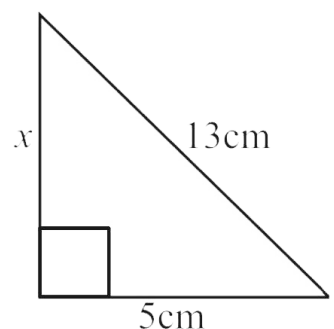
- (i) By Path agoras theorem
(Hypotenuse)² = (Base)² + (Perpendicular)²
 $(x)^2 = (3)^2 + (4)^2$
 $x^2 = 9 + 16$
 $x^2 = 25$
Taking square root on both side
 $\sqrt{x^2} = \sqrt{25}$
 $x = 5 \text{ cm}$



- (ii) By Pythagoras theorem
(Hypotenuse)² = (Base)² + (Perpendicular)²
 $(10)^2 = (x)^2 + (6)^2$
 $100 = x^2 + 36$
 $100 - 36 = x^2$
 $x^2 = 64$
Taking square root on both side
 $\sqrt{x^2} = \sqrt{64}$
 $x = 8 \text{ cm}$



- (iii) By Pythagoras theorem
(Hypotenuse)² = (Base)² + (Perpendicular)²
 $(13)^2 = (5)^2 + (x)^2$
 $169 = 25 + x^2$
 $169 - 25 = x^2$
 $x^2 = 144$



Taking square root on both side

$$\sqrt{x^2} = \sqrt{144}$$

$$x = 12 \text{ cm}$$

(iv) By Path agoras theorem

$$(\text{Hypotenuse})^2 = (\text{base})^2 + (\text{Perpendicular})^2$$

$$(\sqrt{2})^2 = (1)^2 + (x)^2$$

$$2 = 1 + x^2$$

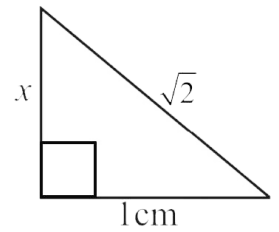
$$2 - 1 = x^2$$

$$x^2 = 1$$

Taking square root on both side

$$\sqrt{x^2} = \sqrt{1}$$

$$x = 1 \text{ cm}$$



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