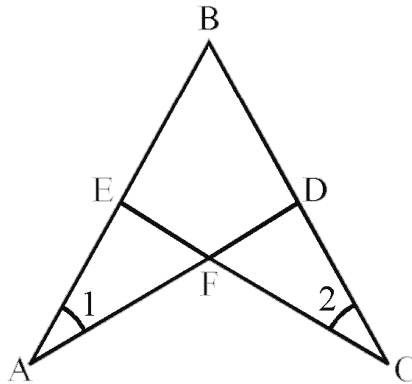


Exercise 10.1

- Q.1** In the given figure
 $\angle 1 \cong \angle 2$ and $\overline{AB} \cong \overline{CB}$
Prove that
 $\triangle ABD \cong \triangle CBE$



Proof

Statements	Reasons
In $\triangle ABD \leftrightarrow \triangle CBE$	
$\overline{AB} \cong \overline{CB}$	Given
$\angle BAD \cong \angle BCE$	Given $\angle 1 \cong \angle 2$
$\angle ABD \cong \angle CBE$	Common
$\triangle ABD \cong \triangle CBE$	S.A.A \cong S.A.A

- Q.2** From a point on the bisector of an angle, perpendiculars are drawn to the arms of the angle. Prove that these perpendiculars are equal in measure.

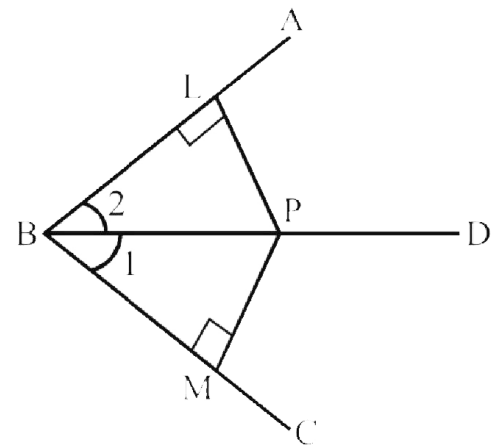
Given

\overline{BD} is bisector of $\angle ABC$. P is point on \overline{BD} and \overline{PL} and \overline{PM} are perpendicular to \overline{AB} and \overline{CB} respectively

To prove

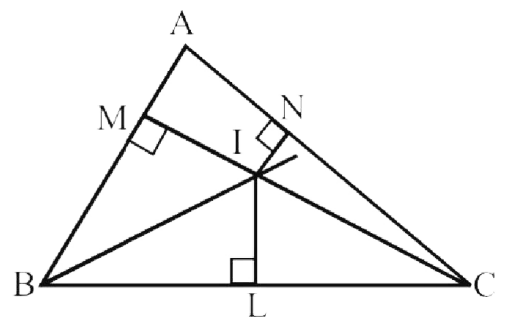
$\overline{PL} \cong \overline{PM}$

Proof



Statements	Reasons
In $\triangle BLP \leftrightarrow \triangle BMP$	
$\overline{BP} \cong \overline{BP}$	Common
$\angle BLP \cong \angle BMP$	Each right angle (given)
$\angle LBP \cong \angle MBP$	Given \overline{BD} is bisector of angle B
$\therefore \triangle BLP \cong \triangle BMP$	S.A.A \cong S.A.A
So $\overline{PL} \cong \overline{PM}$	Corresponding sides of congruent triangles

Q.3 In a triangle ABC, the bisectors of $\angle B$ and $\angle C$ meet in point I prove that I is equidistant from the three sides by ΔABC



Given

In ΔABC , the bisector of $\angle B$ and $\angle C$ meet at I and \overline{IL} , \overline{IM} , and \overline{IN} are perpendiculars to the three sides of ΔABC .

To prove

$$\overline{IL} \cong \overline{IM} \cong \overline{IN}$$

Proof

Statements	Reasons
In $\Delta ILB \leftrightarrow \Delta IMB$	
$\overline{BI} \cong \overline{BI}$	Common
$\angle IBL \cong \angle IBM$	Given BI is bisector of $\angle B$
$\angle ILB \cong \angle IMB$	Given each angle is right angles
$\Delta ILB \cong \Delta IMB$	SAA \cong S.A.A
$\therefore \overline{IL} \cong \overline{IM}$ _____ (i)	Corresponding sides of $\cong \Delta$'s
Similarly	
$\Delta IAC \cong \Delta INC$	
So $\overline{IL} \cong \overline{IN}$ _____ (ii)	
from (i) and (ii)	Corresponding sides of $\cong \Delta$ s
$\overline{IL} \cong \overline{IM} \cong \overline{IN}$	
\therefore I is equidistant from the three sides of ΔABC .	

Theorem 10.1.2

If two angles of a triangles are congruent then the sides opposite to them are also congruent

Given

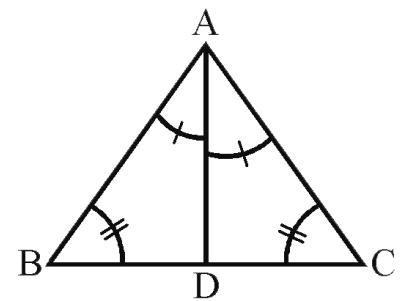
In ΔABC , $\angle B \cong \angle C$

To prove

$$\overline{AB} \cong \overline{AC}$$

Construction

Draw the bisector of $\angle A$, meeting \overline{BC} at point D



Proof

Statements	Reasons
In $\Delta ABD \leftrightarrow \Delta ACD$	
$\overline{AD} \cong \overline{AD}$	Common
$\angle B \cong \angle C$	Given
$\angle BAD \cong \angle CAD$	Construction
$\Delta ABD \cong \Delta ACD$	S.A.A \cong S.A.A
Hence $\overline{AB} \cong \overline{AC}$	(Corresponding sides of congruent triangles)

Example 1

If one angle of a right triangle is of 30° , the hypotenuse is twice as long as the side opposite to the angle.

Given

In $\triangle ABC$, $m\angle B = 90^\circ$ and $m\angle C = 30^\circ$

To prove

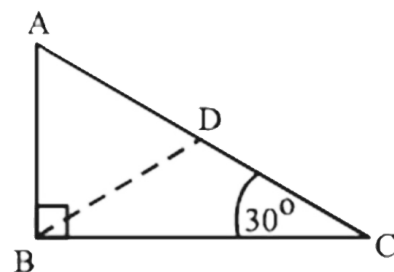
$$m\overline{AC} = 2m\overline{AB}$$

Constructions

At, B construct $\angle CBD$ of 30°

Let \overline{BD} cut \overline{AC} at the point D.

Proof



Statements	Reasons
In $\triangle ABD$, $m\angle A = 60^\circ$	$m\angle ABC = 90^\circ$, $m\angle C = 30^\circ$
$m\angle ABD = m\angle ABC$, $m\angle CBD = 60^\circ$	$m\angle ABC = 90^\circ$, $m\angle CBD = 30^\circ$
$\therefore m\angle ADB = 60^\circ$	Sum of measures of \angle s of a \triangle is 180°
$\therefore \triangle ABD$ is equilateral	Each of its angles is equal to 60°
$\therefore \overline{AB} \cong \overline{BD} \cong \overline{AD}$	Sides of equilateral \triangle
In $\triangle BCD$, $\overline{BD} \cong \overline{CD}$	$\angle C = \angle CBD$ (each of 30°),
Thus $m\overline{AC} = m\overline{AD} + m\overline{CD}$	$\overline{AD} \cong \overline{AB}$ and $\overline{CD} \cong \overline{BD} \cong \overline{AB}$
$= m\overline{AB} + m\overline{AB}$	
$= 2(m\overline{AB})$	

Example 2

If the bisector of an angle of a triangle bisects the side opposite to it, the triangle is isosceles.

Given

In $\triangle ABC$, \overline{AD} bisect $\angle A$ and $\overline{BD} \cong \overline{CD}$

To prove

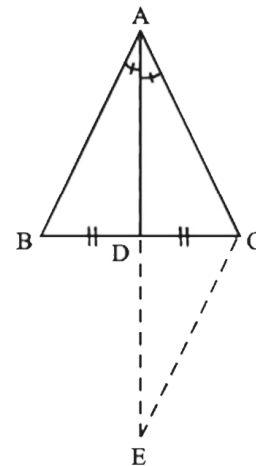
$$\overline{AB} \cong \overline{AC}$$

Construction

Produce \overline{AD} to E , and take $\overline{ED} \cong \overline{AD}$

Joint C to E

Proof



Statements	Reasons
In $\triangle ADB \leftrightarrow \triangle EDC$	Construction
$\overline{AD} \cong \overline{ED}$	Vertical angles
$\angle ADB \cong \angle EDC$	Given
$\overline{BD} \cong \overline{CD}$	S.A.S. Postulate
$\therefore \triangle ADB \cong \triangle EDC$	Corresponding sides
$\therefore \overline{AB} \cong \overline{EC} \dots (i)$	Corresponding angles
and $\angle BAD \cong \angle E$	Given
But $\angle BAD \cong \angle CAD$	Each $\cong \angle BAD$
$\therefore \angle E \cong \angle CAD$	$\angle E \cong \angle CAD$ (proved)
In $\triangle ACE$, $\overline{AC} \cong \overline{EC} \dots (ii)$	From (i) and (ii)
Hence $\overline{AB} \cong \overline{AC}$	

Exercise 10.2

Q.1 Prove that any two medians of an equilateral triangle are equal in measure.

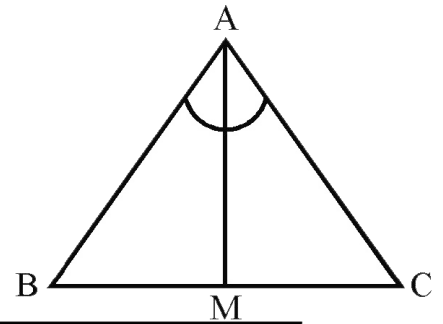
Given

In $\triangle ABC$, $\overline{AB} \cong \overline{AC}$ and M is midpoint of \overline{BC}

To prove

\overline{AM} bisects $\angle A$ and \overline{AM} is perpendicular to \overline{BC}

Proof



Statements	Reasons
In $\triangle ABM \leftrightarrow \triangle ACM$	
$\overline{AB} \cong \overline{AC}$	Given
$\overline{BM} \cong \overline{CM}$	Given M is midpoint of BC
$\overline{AM} \cong \overline{AM}$	Common
$\therefore \triangle ABM \cong \triangle ACM$	S.S.S \cong S.S.S
So $\angle BAM \cong \angle CAM$	Corresponding angles of congruent triangle
$m\angle AMB + m\angle AMC = 180^\circ$	
$\therefore m\angle AMB = m\angle AMC$	
i.e. \overline{AM} is perpendicular to \overline{BC}	

Q.2 Prove that a point which is equidistant from the end points of a line segment, is on the right bisector of line segment

Given

\overline{AB} is line segment. The point C is such that $\overline{CA} \cong \overline{CB}$

To prove

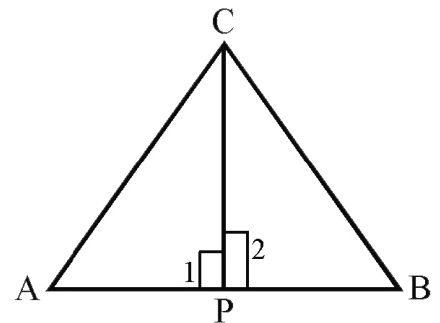
Point C lies on the right bisector of \overline{AB}

Construction

(i) Take P as midpoint of \overline{AB} i.e. $\overline{AP} \cong \overline{BP}$

(ii) Joint point C to A, P, B

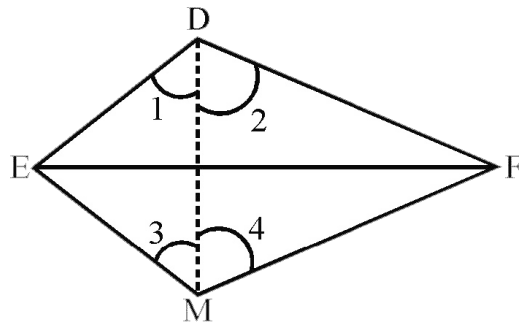
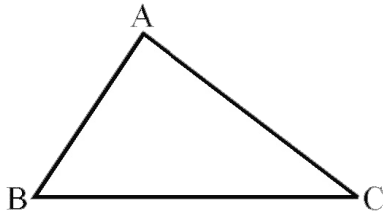
Proof:



Statements	Reasons
In $\triangle ABC$	
$\overline{CA} \cong \overline{CB}$	Given
$\angle A \cong \angle B$	Corresponding angles of congruent triangles
$\triangle CBP \leftrightarrow \triangle CAP$	
$\overline{CB} \cong \overline{CA}$	
$\triangle CAP \cong \triangle CBP$	S.A.S \cong S.A.S
$\therefore \angle 1 \cong \angle 2$	
$m\angle 1 + m\angle 2 = 180^\circ$	Adjacent angles on one side of a line
Thus $m\angle 1 = m\angle 2 = 90$	
Hence \overline{CP} is right bisector of \overline{AB} and point C lies on \overline{CB}	

Theorem 10.1.3

In a correspondence of two triangles if three sides of one triangle are congruent to the corresponding three sides of the other. Then the two triangles are congruent (S.S.S \cong S.S.S)

**Given:**In $\triangle ABC \leftrightarrow \triangle DEF$ $\overline{AB} \cong \overline{DE}, \overline{BC} \cong \overline{EF}$ and $\overline{CA} \cong \overline{FD}$ **To prove** $\triangle ABC \cong \triangle DEF$ **Construction**

Suppose that in $\triangle DEF$ the side \overline{EF} is not smaller than any of the remaining two sides. On \overline{EF} construct a $\triangle MEF$ in which, $\angle FEM \cong \angle B$ and $\overline{ME} \cong \overline{AB}$. Join D and M. as shown in the above figures we label some of the angles as 1, 2, 3, and 4

Proof:

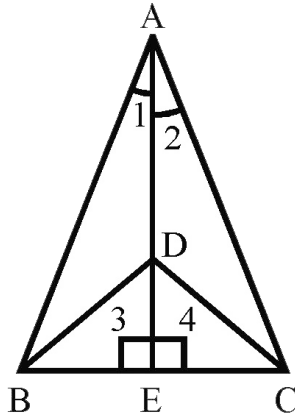
Statements	Reasons
In $\triangle ABC \leftrightarrow \triangle MEF$	
$\overline{BC} \cong \overline{EF}$	Given
$\angle B \cong \angle FEM$	Construction
$\overline{AB} \cong \overline{ME}$	Construction
$\therefore \triangle ABC \cong \triangle MEF$	S.A.S Postulate
and $\overline{CA} \cong \overline{FM}$ _____ (i)	(Corresponding sides of congruent triangles)
also $\overline{CA} \cong \overline{FD}$ _____ (ii)	Given
$\therefore \overline{FM} \cong \overline{FD}$	{ From (i) and (ii) }
In $\triangle FDM$	
$\angle 2 \cong \angle 4$ _____ (iii)	$\overline{FM} \cong \overline{FD}$ (proved)
Similarly $\angle 1 \cong \angle 3$ _____ (iv)	{ from (iii) and iv }
$\therefore m\angle 2 + m\angle 1 = m\angle 4 + m\angle 3$	
$\therefore m\angle EDF = m\angle EMF$	
Now in $\triangle DEF \leftrightarrow \triangle MEF$	
$\overline{FD} \cong \overline{FM}$	Proved
and $m\angle EDF \cong \angle EMF$	Proved
$\overline{DE} \cong \overline{ME}$	Each one $\cong \overline{AB}$
$\therefore \triangle DEF \cong \triangle MEF$	S.A.S postulates
also $\triangle ABC \cong \triangle MEF$	Proved
Hence $\triangle ABC \cong \triangle DEF$	Each $\triangle \cong \triangle MEF$ (proved)

Example 1

If two isosceles triangles are formed on the same side of their common base, the line through their vertices would be the right bisector of their common base.

Given

$\triangle ABC$ and $\triangle DBC$ formed on the same side of \overline{BC} such that $\overline{BA} \cong \overline{AC}$, $\overline{DB} \cong \overline{DC}$, \overline{AD} meets \overline{BC} at E .

**To prove**

$\overline{BE} \cong \overline{CE}$, $\overline{AE} \perp \overline{BC}$

Proof

Statements	Reasons
In $\triangle ADB \leftrightarrow \triangle ADC$	
$\overline{AB} \cong \overline{AC}$	Given
$\overline{DB} \cong \overline{DC}$	Given
$\overline{AD} \cong \overline{AD}$	Common
$\therefore \triangle ADB \cong \triangle ADC$	S.S.S \cong S.S.S
$\therefore \angle 1 \cong \angle 2$	Corresponding angles of $\cong \triangle s$
In $\triangle ABE \leftrightarrow \triangle ACE$	
$\overline{AB} \cong \overline{AC}$	Given
$\angle 1 \cong \angle 2$	Proved
$\triangle ABE \cong \triangle ACE$	S.A.S postulate
$\overline{AE} \cong \overline{AE}$	Common
$\therefore \overline{BE} \cong \overline{CE}$	Corresponding sides of $\cong \triangle s$
$\angle 3 \cong \angle 4$	Corresponding angles of $\cong \triangle s$
$m\angle 3 + m\angle 4 = 180^\circ$	Supplementary angles postulate
$m\angle 3 = m\angle 4 = 90^\circ$	From I and II
Hence $\overline{AE} \perp \overline{BC}$	

Exercise 10.3

Q.1 In the figure, $\overline{AB} \cong \overline{DC}$, $\overline{AD} \cong \overline{BC}$ prove that $\angle A = \angle C$, $\angle ABC \cong \angle ADC$

Given

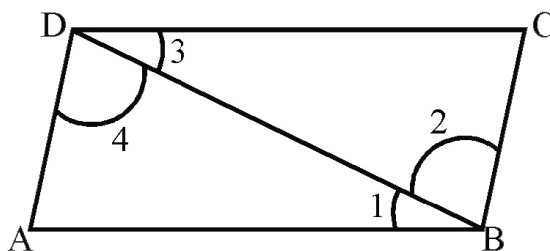
In the figure $\overline{AB} \cong \overline{DC}$, $\overline{AD} \cong \overline{BC}$

To prove

$\angle A \cong \angle C$

$\angle ABC \cong \angle ADC$

Proof



Statements	Reasons
In $\triangle ABD \leftrightarrow \triangle CDB$	
$\overline{AB} \cong \overline{DC}$	Given
$\overline{AD} \cong \overline{BC}$	Given
$\overline{BD} \cong \overline{BD}$	Common
$\triangle ABD \cong \triangle CDB$	S.S.S \cong S.S.S
\therefore Hence $\angle A \cong \angle C$	Corresponding angles of congruent triangles
$\angle 1 \cong \angle 3$	Corresponding angles of congruent triangles
$\angle 2 \cong \angle 4$	Corresponding angles of congruent triangles
$m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	
or $m\angle ABC = m\angle ADC$	
$\angle ABC \cong \angle ADC$	

Q.2 In the figure $\overline{LN} \cong \overline{MP}$, $\overline{MN} \cong \overline{LP}$ prove that $\angle N \cong \angle P$, $\angle NML \cong \angle PLM$

Given

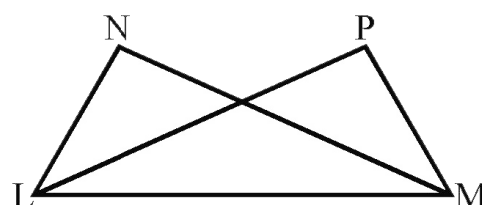
In the figure

$\overline{LN} \cong \overline{MP}$ and $\overline{LP} \cong \overline{MN}$

To prove

$\angle N \cong \angle P$ and $\angle NML \cong \angle PLM$

Proof



Statements	Reasons
$\triangle LMN \leftrightarrow \triangle MLP$	
$\overline{LN} \cong \overline{MP}$	Given
$\overline{LP} \cong \overline{MN}$	Given
$\overline{LM} \cong \overline{ML}$	Common
$\triangle LMN \cong \triangle MLP$	S.S.S \cong S.S.S
$\angle N \cong \angle P$	Corresponding angles of congruent triangles
$\angle NML \cong \angle PLM$	Corresponding angles of congruent triangles

Q.3 Prove that median bisecting the base of an isosceles triangle bisects the vertex angle and it is perpendicular to the base

Given

$\triangle ABC$

(i) $\overline{AB} \cong \overline{AC}$

(ii) Point P is mid point of \overline{BC} i.e. $\overline{BP} = \overline{CP}$

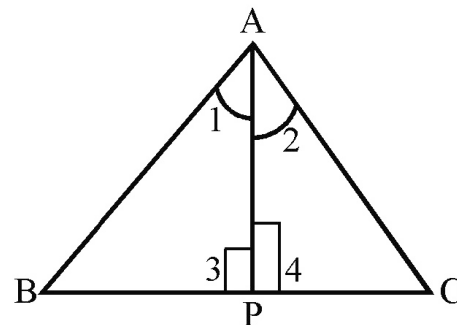
P is joined to A, i.e. \overline{AP} is median

To prove

$\angle 1 \cong \angle 2$

$\overline{AP} \perp \overline{BC}$

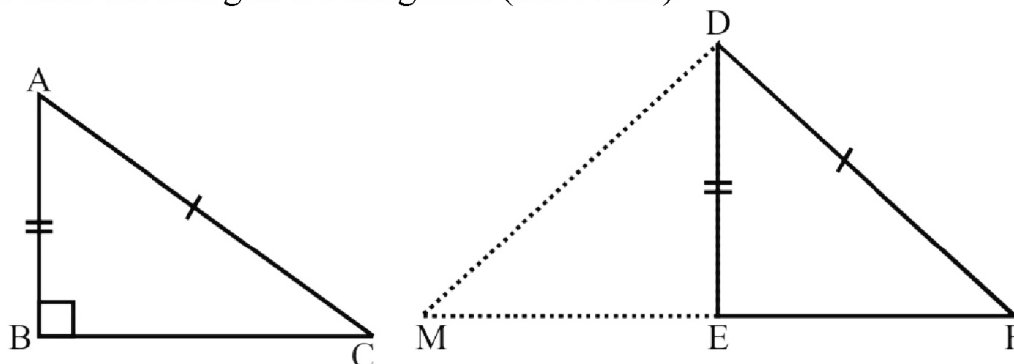
Proof



Statements	Reasons
$\triangle ABP \leftrightarrow \triangle ACP$	
$\overline{AB} \cong \overline{AC}$	Given
$\overline{BP} \cong \overline{CP}$	Given
$\overline{AP} \cong \overline{AP}$	Common
$\triangle ABP \cong \triangle ACP$	S.S.S \cong S.S.S
$\angle 1 \cong \angle 2$	Corresponding angles of congruent triangles
$\angle 3 \cong \angle 4$ _____ (i)	
$m\angle 3 + m\angle 4 = 180^\circ$ _____ (ii)	Corresponding angles of congruent triangles
Thus $m\angle 3 = m\angle 4 = 90$	
$\therefore \overline{AP} \perp \overline{BC}$	From equation (i) and (ii)

Theorem 10.1.4

If in the corresponding of the two right angled triangles, the hypotenuse and one side of one triangle are congruent to the hypotenuse and the corresponding side of the other then the triangles are congruent (H.S \cong H.S)



Given

$\triangle ABC \leftrightarrow \triangle DEF$

$\angle B \cong \angle E$ (right angles)

$\overline{CA} \cong \overline{FD}, \overline{AB} \cong \overline{DE}$

To Prove

$\triangle ABC \cong \triangle DEF$

Construction

Prove \overline{FE} to a point M such that $\overline{EM} \cong \overline{BC}$ and join the point D and M

Proof

Statements	Reasons
$m\angle DEF + \angle DEM = 180^\circ$ _____ (i)	Supplementary angles
Now $m\angle DEF = 90^\circ$ _____ (ii)	Given
$\therefore m\angle DEM = 90^\circ$	{ from (i) and (ii) }
In $\triangle ABC \leftrightarrow \triangle DEM$	
$\overline{BC} \cong \overline{EM}$	Construction
$\angle ABC \cong \angle DEM$	(Each angle equal to 90°)
$\overline{AB} \cong \overline{DE}$	Given
$\triangle ABC \cong \triangle DEM$	SAS postulate
ad $\angle C = \angle M$	Corresponding angles of congruent triangles
$\overline{CA} \cong \overline{MD}$	Corresponding sides of congruent triangles
But $\overline{CA} \cong \overline{FD}$	Given
$\overline{MD} \cong \overline{FD}$	Each is congruent to \overline{CA}
In $\triangle DMF$	
$\angle F \cong \angle M$	$\overline{MD} \cong \overline{FD}$ (proved)
But $\angle C \cong \angle M$	(Proved)
$\angle C \cong \angle F$	Each is congruent to $\angle M$
	Given
$\angle ABC \cong \angle DEF$	Given
$\overline{AB} \cong \overline{DE}$	(Proved)
Hence $\triangle ABC \cong \triangle DEF$	(S.A.A \cong S.A.A)

Example

If perpendiculars from two vertices of a triangle to the opposite sides are congruent, then the triangle is isosceles.

Given

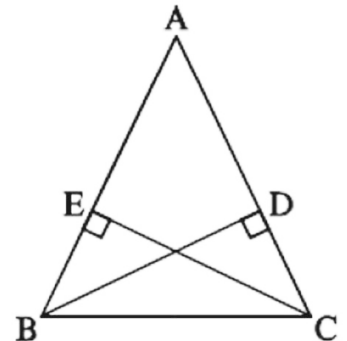
In $\triangle ABC$, $\overline{BD} \perp \overline{AC}$, $\overline{CE} \perp \overline{AB}$

Such that $\overline{BD} \cong \overline{CE}$

To prove

$\overline{AB} \cong \overline{AC}$

Proof



Statements	Reasons
In $\triangle BCD \leftrightarrow \triangle CEB$	
$\angle BDC \cong \angle CEB$	$\overline{BD} \perp \overline{AC}$, $\overline{CE} \perp \overline{AB}$ given \Rightarrow each angle = 90°
$\overline{BC} \cong \overline{CB}$	Common hypotenuse
$\overline{BD} \cong \overline{CE}$	Given
$\triangle BCD \cong \triangle CEB$	H.S \cong H.S
$\angle BCA \cong \angle CBE$	Corresponding angles \triangle s
Thus $\angle BCA \cong \angle CBA$	
Hence $\overline{AB} \cong \overline{AC}$	In $\triangle ABC$, $\angle BCA \cong \angle CBA$

Exercise 10.4

Q.1 In $\triangle PAB$ of figure $\overline{PQ} \perp \overline{AB}$ and $\overline{PA} \cong \overline{PB}$ prove that $\overline{AQ} \cong \overline{BQ}$ and $\angle APQ \cong \angle BPQ$

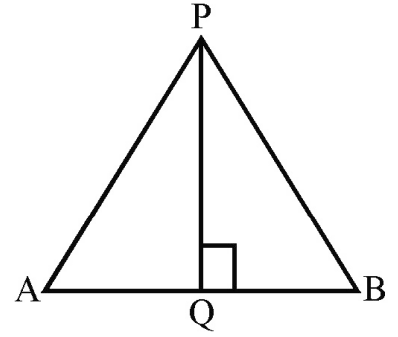
Given:

In $\triangle PAB$

$\overline{PQ} \perp \overline{AB}$ and $\overline{PA} \cong \overline{PB}$

To prove

$\overline{AQ} \cong \overline{BQ}$ and $\angle APQ \cong \angle BPQ$



Proof

Statements	Reasons
In $\triangle APQ \leftrightarrow \triangle BPQ$	
$\overline{PA} \cong \overline{PB}$	Given
$\angle AQP \cong \angle BQP$	Given $\overline{PQ} \perp \overline{AB}$
$\overline{PQ} \cong \overline{PQ}$	Common
$\therefore \triangle APQ \cong \triangle BPQ$	H.S \cong H.S
So $\overline{AQ} \cong \overline{BQ}$	Corresponding sides of congruent triangles
and $\angle APQ \cong \angle BPQ$	Corresponding angles of congruent triangles

Q.2 In the figure $m\angle C \cong m\angle D = 90^\circ$ and $\overline{BC} \cong \overline{AD}$ prove that $\overline{AC} \cong \overline{BD}$ and $\angle BAC \cong \angle ABD$

Given

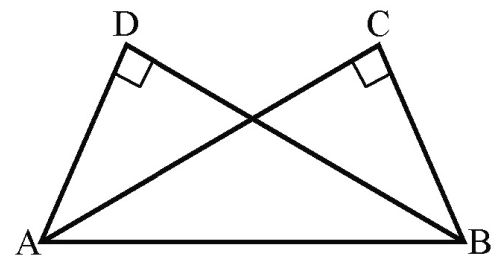
In the figure given $m\angle C = m\angle D = 90^\circ$

$\overline{BC} \cong \overline{AD}$

To Prove

$\overline{AC} \cong \overline{BD}$

$\angle BAC \cong \angle ABD$



Proof

Statements	Reasons
In $\triangle ABD \leftrightarrow \triangle BAC$	
$\overline{AD} \cong \overline{BC}$	Given
$\angle D \cong \angle C$	Each 90°
$\overline{AB} \cong \overline{BA}$	Common
Thus $\triangle ABD \cong \triangle BAC$	H-S \cong H-S
$\therefore \overline{AC} \cong \overline{BD}$	Corresponding sides of congruent triangles
$\therefore \angle BAC \cong \angle ABD$	Corresponding angles of congruent triangles

Q.3 In the figure, $m\angle B = m\angle D = 90^\circ$ and $\overline{AD} \cong \overline{BC}$ prove that ABCD is a rectangle

Given

In the figure

$m\angle B = m\angle D = 90^\circ$ and $\overline{AD} \cong \overline{BC}$

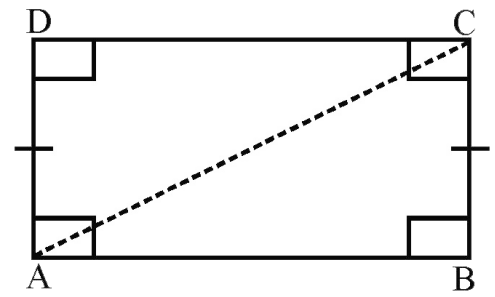
To prove

ABCD is a rectangle

Construction

Join A to C

Proof



Statements	Reasons
In $\triangle ABC \leftrightarrow \triangle CDA$	
$\angle B \cong \angle D$	Given each angle = 90°
$\overline{AC} \cong \overline{CA}$	Common
$\overline{BC} \cong \overline{DA}$	Given
$\therefore \triangle ABC \cong \triangle CDA$	H-S \cong H-S
$\overline{AB} \cong \overline{CD}$	Corresponding sides of congruent triangles
and $\angle ACB \cong \angle CAD$	Corresponding angles of congruent triangles
Hence ABCD is a rectangle	

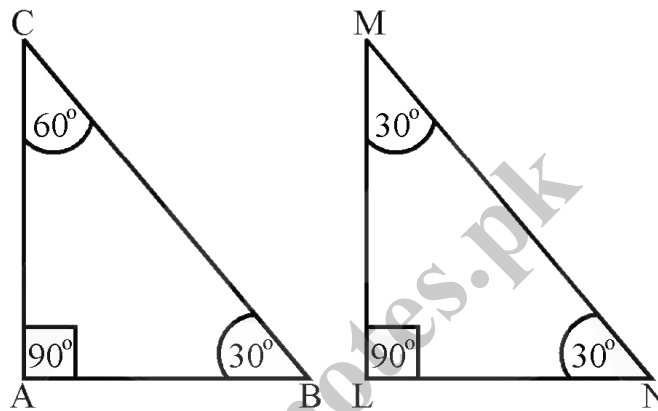
easynotes.pk

Review Exercise 10

Q.1 Which of the following are true and which are false.

- (i) A ray has two end points. (False)
- (ii) In a triangle there can be only are right angle. (True)
- (iii) Three points are said to be collinear if they lie on same line. (True)
- (iv) Two parallel lines intersect at a point. (False)
- (v) Two line can intersect only one point. (True)
- (vi) A triangle of congruent sides has non-congruent angles. (False)

Q.2 In $\triangle ABC \cong \triangle LMN$, then

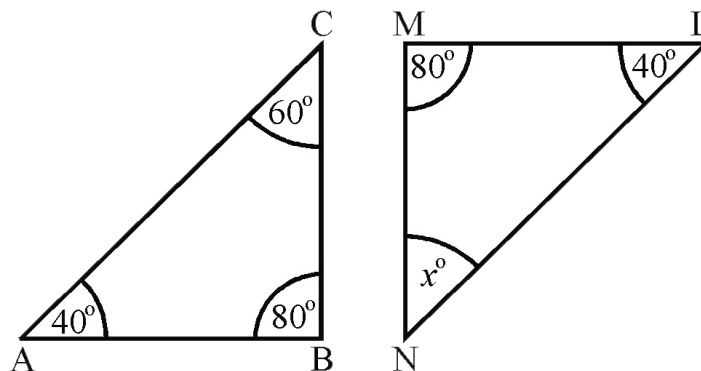


(i) $m\angle M \cong m\angle B = 30^\circ$

(ii) $m\angle N \cong m\angle C = 60^\circ$

(iii) $m\angle A \cong m\angle L = 90^\circ$

Q.3 If $\triangle ABC \cong \triangle LMN$ then find the value of x



$$m\angle N = m\angle C = 60^\circ$$

$$m\angle N = x = 60^\circ$$

Sum of three angle in a triangle is 180

So $x + 80 + 40 = 180$

$$x + 120 = 180$$

$$x = 180 - 120$$

$$x = 60^\circ$$

Q.4 Find the value of unknowns for the given congruent triangles.

It is an isosceles triangle

$$m \overline{AB} = m \overline{AC}$$

and $m \angle B = m \angle C$

when we draw a perpendicular from point A to BC it

Bisect

So $\overline{BD} \cong \overline{DC}$

$$5m - 3 = 2m + 6$$

$$5m - 2m = 6 + 3$$

$$3m = 9$$

$$m = \frac{9}{3}$$

$$m = 3$$

opposite angle are congruent

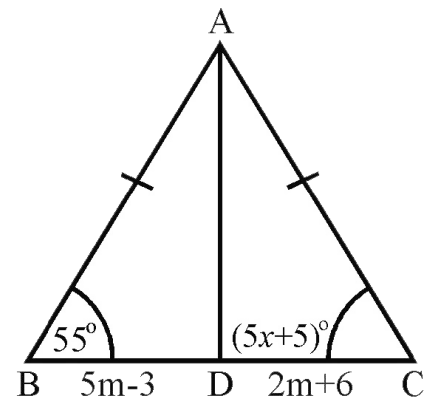
$$\therefore \angle B = \angle C$$

$$55 = 5x + 5$$

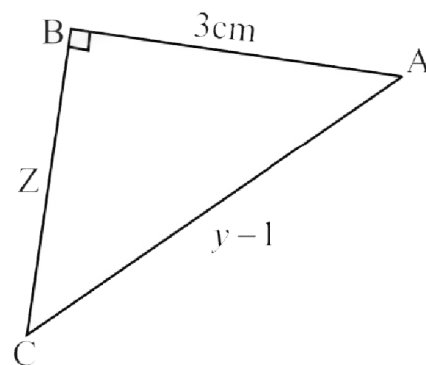
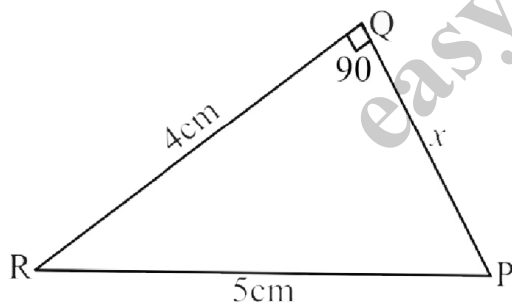
$$55 - 5 = 5x$$

$$\frac{50}{5} = x$$

$$x = 10$$



Q.5 If $\Delta PQR = \Delta ABC$, the find the unknowns



By using definition of congruent triangles.

$$\overline{RP} = \overline{AC}$$

$$5 = y - 1$$

$$5 + 1 = y$$

$$y = 6cm$$

$$\overline{AB} = \overline{QP}$$

$$3cm = x$$

Or

$$x = 3cm$$

$$\overline{BC} = \overline{QR}$$

$$Z = 4cm$$