

Exercise 1.1

Q.1 Find the order of the following matrices.

$$A = \begin{bmatrix} 2 & 3 \\ -5 & 6 \end{bmatrix}$$

It has 2 rows & 2 columns that's why its order is 2 - by -2

$$B = \begin{bmatrix} 2 & 0 \\ 3 & 5 \end{bmatrix}$$

It has 2 rows & 2 columns. So, its order is 2- by -2

$$C = [2 \quad 4]$$

It has 1 row and 2 columns. So, its order is 1 –by -2

$$D = \begin{bmatrix} 4 \\ 0 \\ 6 \end{bmatrix}$$

It has 3 rows and 1 column. So, its order is 3 – by -1

$$E = \begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix}$$

It has 3 rows and 2 columns. So, its order is 3 – by –2

$$F = [2]$$

It has 1 row & 1 column. So, its order is 1- by -1

$$G = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 2 & 3 \\ 2 & 4 & 5 \end{bmatrix}$$

It has 3 rows and 3 columns. So, its order is 3 –by -3

$$H = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 0 & 6 \end{bmatrix}$$

It has 2 rows & 3 columns. So, its order is 2- by -3

Q.2 Which one of the following matrices are equal?

1) $A = [3]$, 2) $B = [3 \ 5]$,

3) $C = [5-2]$ 4) $D = [5 \ 3]$

5) $E = \begin{bmatrix} 4 & 0 \\ 6 & 2 \end{bmatrix}$ 6) $F = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$

7) $G = \begin{bmatrix} 3-1 \\ 3+3 \end{bmatrix}$ 8) $H = \begin{bmatrix} 4 & 0 \\ 6 & 2 \end{bmatrix}$

9) $I = [3 \ 3+2]$ 10) $J = \begin{bmatrix} 2+2 & 2-2 \\ 2+4 & 2+0 \end{bmatrix}$

Solution:

Order of $A = [3]$ is equal to Order of $C = [5-2]$

Order of $B = [3 \ 5]$ is equal to Order of $I = [3 \ 3+2]$

Order of $C = [5-2]$ is equal to Order of $A = [3]$

$D = [5 \ 3]$ has no equal matrix.

$E = \begin{bmatrix} 4 & 0 \\ 6 & 2 \end{bmatrix}$ has equal matrices.

Order of $\Rightarrow H = \begin{bmatrix} 4 & 0 \\ 6 & 2 \end{bmatrix}$ is equal to Order of $J = \begin{bmatrix} 2+2 & 2-2 \\ 2+4 & 2+0 \end{bmatrix}$

Order of $F = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$ is equal to Order of $G = \begin{bmatrix} 3-1 \\ 3+3 \end{bmatrix}$

Q.3 Find the values of a, b, c & d.

$$\begin{bmatrix} a+c & a+2b \\ c-1 & 4d-6 \end{bmatrix} = \begin{bmatrix} 0 & -7 \\ 3 & +2d \end{bmatrix}$$

Solution:

As Matrices are equal so their corresponding entries are same.

$$a+c=0 \rightarrow (1)$$

$$a+2b=-7 \rightarrow (2)$$

$$c-1=3 \rightarrow (3)$$

$$4d-6=+2d \rightarrow (4)$$

Solving 3rd equation

$$c-1=3$$

$$c=3+1$$

$$c=4$$

Solving 2nd equation

$$a+2b=-7$$

$$-4+2b=-7$$

$$2b=-7+4$$

$$2b=-3$$

$$b = \frac{-3}{2}$$

Solving 1st equation

$$a+c=0$$

$$a+4=0$$

$$a=-4$$

Solving 4th equation

$$4d-6=2d$$

$$-6=2d-4d$$

$$-6=-2d$$

$$d = \frac{-6}{-2}$$

$$d=3$$

Exercise 1.2

Q.1 Identify the following matrices.

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

It's all members are 0. So, it's a null matrix.

$$B = [2 \quad 3 \quad 4]$$

It has only 1 row. So, it's a row matrix.

$$C = \begin{bmatrix} 4 \\ 0 \\ 6 \end{bmatrix}$$

It has only 1 column. So, it's a column matrix.

$$D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

It is an identity matrix because its diagonal entries are 1 and non-diagonal entries are zero.

$$E = [0]$$

It has only 0. So, it's a null matrix.

$$F = \begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix}$$

It has only 1 column. So, it's a column matrix.

Q.2 Identify the following matrices.

(1)
$$\begin{bmatrix} -8 & 2 & 7 \\ 12 & 0 & 4 \end{bmatrix}$$

Its number of rows & columns are not equal. So, it's a rectangular matrix.

$$(2) \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$$

It has only one column. So, it's a column matrix.

$$(3) \begin{bmatrix} 6 & -4 \\ 3 & -2 \end{bmatrix}$$

The number of rows & columns are equal. So, it's a square matrix.

$$(4) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Identity matrix – Because Diagonal entries are 1 and non-diagonal entries are 0.

$$(5) \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

Number of rows & columns are not equal. So, it's a rectangular matrix.

$$(6) [3 \ 10 \ -1]$$

It's a row matrix because it has only 1 row.

$$(7) \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Column matrix because it has only one column.

$$(8) \begin{bmatrix} 1 & 2 & 3 \\ -1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Square matrix because number of rows & columns are equal.

$$(9) \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Null matrix because all elements are 0.

Q.3 Identify the matrices.

$$(1) A = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

Scalar- matrix because its non-diagonal entries are 0 & diagonal entries are same.

$$(2) \quad B = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}$$

Diagonal matrix because its non-diagonal entries are 0.

$$(3) \quad C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Unit matrix because diagonal-entries are 1.

$$(4) \quad D = \begin{bmatrix} 3 & 0 \\ 0 & 0 \end{bmatrix}$$

Diagonal matrix because non-diagonal are 0.

$$(5) \quad E = \begin{bmatrix} 5-3 & 0 \\ 0 & 1+1 \end{bmatrix}$$

Scalar- because diagonal are same.

Q.4 Find the negative of matrices.

$$(1) \quad A = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$
$$-A = - \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

$$(2) \quad B = \begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}$$
$$-B = - \begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}$$
$$-B = \begin{bmatrix} -3 & +1 \\ -2 & -1 \end{bmatrix}$$

$$(3) \quad C = \begin{bmatrix} 2 & 6 \\ 3 & 2 \end{bmatrix}$$
$$-C = - \begin{bmatrix} 2 & 6 \\ 3 & 2 \end{bmatrix}$$

$$(4) \quad D = \begin{bmatrix} -2 & -6 \\ -3 & -2 \end{bmatrix}$$
$$-D = - \begin{bmatrix} -3 & 2 \\ -4 & 5 \end{bmatrix}$$
$$-D = \begin{bmatrix} +3 & -2 \\ +4 & -5 \end{bmatrix}$$

$$(5) \quad E = \begin{bmatrix} 1 & -5 \\ 2 & 3 \end{bmatrix}$$
$$-E = - \begin{bmatrix} 1 & -5 \\ 2 & 3 \end{bmatrix}$$
$$-E = \begin{bmatrix} -1 & +5 \\ -2 & -3 \end{bmatrix}$$

Q.5 Find the transpose.

$$(1) \quad A = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}$$

$$A^t = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}^t$$

$$A^t = \begin{bmatrix} 0 & 1 & -2 \end{bmatrix}$$

(2) $B = \begin{bmatrix} 5 & 1 & -6 \end{bmatrix}$

$$B^t = \begin{bmatrix} 5 & 1 & -6 \end{bmatrix}^t$$

$$B^t = \begin{bmatrix} 5 \\ 1 \\ -6 \end{bmatrix}$$

(3) $C = \begin{bmatrix} 1 & 2 \\ 2 & -1 \\ 3 & 0 \end{bmatrix}$

$$C^t = \begin{bmatrix} 1 & 2 \\ 2 & -1 \\ 3 & 0 \end{bmatrix}^t$$

$$C^t = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 0 \end{bmatrix}$$

(4) $D = \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix}$

$$D^t = \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix}^t$$

$$D^t = \begin{bmatrix} 2 & 0 \\ 3 & 5 \end{bmatrix}$$

(5) $E = \begin{bmatrix} 2 & 3 \\ -4 & 5 \end{bmatrix}$

$$E^t = \begin{bmatrix} 2 & 3 \\ -4 & 5 \end{bmatrix}^t$$

$$E^t = \begin{bmatrix} 2 & -4 \\ 3 & 5 \end{bmatrix}$$

(6) $F = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$$F^t = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}^t$$

$$F^t = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

Q.6 Verify if $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$

(i) $(A^t)^t = A$

Solution: $(A^t)^t = A$

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$A^t = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}^t$$

$$A^t = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$(A^t)^t = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}^t$$

$$(A^t)^t = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$(A^t)^t = A$$

Hence Proved.

(ii) $(B^t)^t = B$

Solution: $(B^t)^t = B$

$$B = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$$

$$B^t = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}^t$$

$$B^t = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$$

$$(B^t)^t = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}^t$$

$$(B^t)^t = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$$

$$(B^t)^t = B$$

Hence proved

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Exercise 1.3

Q.1 Which of the following are conformable for addition?

$$A = \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \\ 2 & -1 \\ 1 & -2 \end{bmatrix}, \quad D = \begin{bmatrix} 2+1 \\ 3 \end{bmatrix}$$

$$E = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}, \quad F = \begin{bmatrix} 3 & 2 \\ 1+1 & -4 \\ 3+2 & 2+1 \end{bmatrix}$$

Solution:

In the above matrices following matrices are suitable for addition.

- (i) A and E are conformable for addition because their order is same and both are square matrix.
- (ii) B and D are conformable for addition because the order is same i.e. they have two rows and 1 Columns and both are rectangular matrices.
- (iii) C and F are conformable for addition because their order is same i.e. they have three 3 rows and 2 columns and they are a rectangular matrix.

Q.2 Find the additive inverse of the following matrices:

(1) $A = \begin{bmatrix} 2 & 4 \\ -2 & 1 \end{bmatrix}$

Solution:

Additive inverse of a matrix is negative matrix.

$$A = \begin{bmatrix} 2 & 4 \\ -2 & 1 \end{bmatrix} \text{ is}$$

$$-A = -\begin{bmatrix} 2 & 4 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} (-1) \times 2 & (-1)4 \\ (-1)(-2) & (-1)1 \end{bmatrix}$$

$$-A = \begin{bmatrix} -2 & -4 \\ 2 & -1 \end{bmatrix}$$

(2) $B = \begin{bmatrix} 1 & 0 & -1 \\ 2 & -1 & 3 \\ 3 & -2 & 1 \end{bmatrix}$

Solution: $B = \begin{bmatrix} 1 & 0 & -1 \\ 2 & -1 & 3 \\ 3 & -2 & 1 \end{bmatrix}$

Its additive inverse is

$$-B = -\begin{bmatrix} +1 & 0 & -1 \\ +2 & -1 & 3 \\ +3 & -2 & 1 \end{bmatrix}$$

$$-B = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 1 & -3 \\ -3 & 2 & -1 \end{bmatrix}$$

$$(3) \quad C = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$$

$$\text{Solution: } C = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$$

$$-C = -\begin{bmatrix} 4 \\ -2 \end{bmatrix} = \begin{bmatrix} -1 \times 4 \\ -1 \times -2 \end{bmatrix}$$

The additive inverse is

$$-C = \begin{bmatrix} -4 \\ 2 \end{bmatrix}$$

$$(4) \quad D = \begin{bmatrix} 1 & 0 \\ -3 & -2 \\ 2 & 1 \end{bmatrix}$$

$$\text{Solution: } D = \begin{bmatrix} 1 & 0 \\ -3 & -2 \\ 2 & 1 \end{bmatrix}$$

The additive inverse is

$$-D = -\begin{bmatrix} 1 & 0 \\ -3 & -2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} -1 \times 1 & -1 \times 0 \\ -1 \times -3 & -1 \times -2 \\ -1 \times 2 & -1 \times 1 \end{bmatrix}$$

$$-D = \begin{bmatrix} -1 & 0 \\ 3 & 2 \\ -2 & -1 \end{bmatrix}$$

$$(5) \quad E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\text{Solution: } E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

The additive inverse of the given matrix is:

$$-E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 \times 1 & -1 \times 0 \\ -1 \times 0 & -1 \times 1 \end{bmatrix}$$

$$-E = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$(6) \quad F = \begin{bmatrix} \sqrt{3} & 1 \\ -1 & \sqrt{2} \end{bmatrix}$$

$$\text{Solution: } F = \begin{bmatrix} \sqrt{3} & 1 \\ -1 & \sqrt{2} \end{bmatrix}$$

Its additive inverse is

$$-F = -\begin{bmatrix} \sqrt{3} & 1 \\ -1 & \sqrt{2} \end{bmatrix}$$

$$= \begin{bmatrix} -1 \times \sqrt{3} & -1 \times 1 \\ -1 \times -1 & -1 \times \sqrt{2} \end{bmatrix}$$

$$-F = \begin{bmatrix} -\sqrt{3} & -1 \\ 1 & -\sqrt{2} \end{bmatrix}$$

$$\text{Q.3} \quad \text{If } A = \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 \\ -1 \end{bmatrix},$$

$$C = [1 \quad -1 \quad 2], D = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \end{bmatrix},$$

then find.

$$(i) \quad A + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\text{Solution: } A + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\text{As } A = \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix}$$

$$\text{So, } A + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

The order of matrix A and the given matrix order is same. So, they can be added easily.

$$= \begin{bmatrix} -1+1 & 2+1 \\ 2+1 & 1+1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 3 \\ 3 & 2 \end{bmatrix}$$

$$(ii) \quad B + \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

$$\text{Solution: } B + \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

$$\text{As } B = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\begin{aligned} \text{So, } B + \begin{bmatrix} -2 \\ 3 \end{bmatrix} \\ = \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \begin{bmatrix} -2 \\ 3 \end{bmatrix} \end{aligned}$$

The order of both above matrices are same, so, they can be easily added.

$$\begin{aligned} &= \begin{bmatrix} 1+(-2) \\ -1+3 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ -1 & +3 \end{bmatrix} \\ &= \begin{bmatrix} -1 \\ 2 \end{bmatrix} \end{aligned}$$

(iii) $C + [-2 \ 1 \ 3]$

Solution: $C + [-2 \ 1 \ 3]$

As $C = [1 \ -1 \ 2]$

So, $C + [-2 \ 1 \ 3]$

$$= [1 \ -1 \ 2] + [-2 \ 1 \ 3]$$

Their orders are same so they can added

$$= [1+(-2) \ -1+(1) \ 2+3]$$

$$= [1-2 \ -1+1 \ 5]$$

$$= [-1 \ 0 \ 5]$$

(iv) $D + \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$

Solution: $D + \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$

As $D = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \end{bmatrix}$

So, $D + \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

Their orders are same. So, they can be added.

$$= \begin{bmatrix} 1+0 & 2+1 & 3+0 \\ -1+2 & 0+0 & 2+1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 & 3 \\ 1 & 0 & 3 \end{bmatrix}$$

(v) $2A$

Solution: $2A$

As $A = \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix}$

So, $2A$

$$= (2) \times \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 2(-1) & 2 \times 2 \\ 2 \times 2 & 2 \times 1 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 4 \\ 4 & 2 \end{bmatrix}$$

(vi) $(-1)B$

Solution: $(-1)B$

As $B = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

So, $(-1)B$

$$= (-1) \times \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} (-1) \times 1 \\ (-1) \times (-1) \end{bmatrix}$$

$$= \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

(vii) $(-2)C$

Solution: $(-2)C$

As $C = [1 \ -1 \ 2]$

So, $(-2)C$

$$= (-2) \times [1 \ -1 \ 2]$$

$$= [(-2)(1) \ (-2)(-1) \ (-2)(2)]$$

$$= [-2 \ 2 \ -4]$$

(viii) $3D$

Solution: $3D$

As $D = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \end{bmatrix}$

So, $3D$

$$= (3) \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \times 1 & 3 \times 2 & 3 \times 3 \\ 3 \times -1 & 3 \times 0 & 3 \times 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 6 & 9 \\ -3 & 0 & 6 \end{bmatrix}$$

(ix) $3C$

Solution: $3C$

As $C = \begin{bmatrix} 1 & -1 & 2 \end{bmatrix}$

So, $3C$

$$\begin{aligned} &= (3) \times \begin{bmatrix} 1 & -1 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 3 \times 1 & 3 \times -1 & 3 \times 2 \end{bmatrix} \\ &= \begin{bmatrix} 3 & -3 & 6 \end{bmatrix} \end{aligned}$$

Q.4 Perform the indicated operations and simplify the following:

(i) $\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix} \right) + \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

Solution: $\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix} \right) + \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

$$= \begin{bmatrix} 1+0 & 0+2 \\ 0+3 & 1+0 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1 & 2+1 \\ 3+1 & 1+0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$$

(ii) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \left(\begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \right)$

Solution:

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \left(\begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0-1 & 2-1 \\ 3-1 & 0-0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} -1 & 1 \\ 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1-1 & 0+1 \\ 0+2 & 1+0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix}$$

(iii) $\begin{bmatrix} 2 & 3 & 1 \end{bmatrix} + \left(\begin{bmatrix} 1 & 0 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 2 & 2 \end{bmatrix} \right)$

Solution:

$$= \begin{bmatrix} 2 & 3 & 1 \end{bmatrix} + \begin{bmatrix} 1-2 & 0-2 & 2-2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 & 1 \end{bmatrix} + \begin{bmatrix} -1 & -2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2-1 & 3-2 & 1-0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$$

(iv) $\begin{bmatrix} 1 & 2 & 3 \\ -1 & -1 & -1 \\ 0 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$

Solution:

$$= \begin{bmatrix} 1 & 2 & 3 \\ -1 & -1 & -1 \\ 0 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1 & 2+1 & 3+1 \\ -1+2 & -1+2 & -1+2 \\ 3+0 & 1+2 & 2-1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 & 4 \\ 1 & 1 & 1 \\ 3 & 4 & 5 \end{bmatrix}$$

(v) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 0 \\ 0 & 2 & -1 \end{bmatrix}$

Solution:

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 0 \\ 0 & 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1 & 2+1 & 3+1 \\ -1+2 & -1+2 & -1+2 \\ 3+0 & 1+2 & 2-1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 & 1 \\ 0 & 2 & 1 \\ 3 & 3 & 1 \end{bmatrix}$$

$$(vi) \left(\left[\begin{array}{cc} 1 & 2 \\ 0 & 1 \end{array} \right] + \left[\begin{array}{cc} 2 & 1 \\ 1 & 0 \end{array} \right] \right) + \left[\begin{array}{cc} 1 & 1 \\ 1 & 1 \end{array} \right]$$

Solution:

$$= \left[\begin{array}{cc} 1+2 & 2+1 \\ 0+1 & 1+0 \end{array} \right] + \left[\begin{array}{cc} 1 & 1 \\ 1 & 1 \end{array} \right]$$

$$= \left[\begin{array}{cc} 3 & 3 \\ 1 & 1 \end{array} \right] + \left[\begin{array}{cc} 1 & 1 \\ 1 & 1 \end{array} \right]$$

$$= \left[\begin{array}{cc} 3+1 & 3+1 \\ 1+1 & 1+1 \end{array} \right]$$

$$= \left[\begin{array}{cc} 4 & 4 \\ 2 & 2 \end{array} \right]$$

Q.5 For the matrices $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix}$,

$$B = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix} \text{ and}$$

$$C = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix}, \text{ verify the}$$

following rules:

(i) $A + C = C + A$

Solutions:

L.H.S = A+C

R.H.S = C+A

LHS= A+C

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1-1 & 2+0 & 3+0 \\ 2+0 & 3-2 & 1+3 \\ 1+1 & -1+1 & 0+2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 2 & 3 \\ 2 & 1 & 4 \\ 2 & 0 & 2 \end{bmatrix}$$

RHS= C+A

$$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1+1 & 0+2 & 0+3 \\ 0+2 & -2+3 & 3+1 \\ 1+1 & 1-1 & 2-0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 2 & 3 \\ 2 & 1 & 4 \\ 2 & 0 & 2 \end{bmatrix}$$

$$A+C = C+A$$

Hence proved

$$\text{L.H.S} = \text{R.H.S}$$

(ii) $A + B = B + A$

Solution: $A + B = B + A$

L.H.S = A+B

R.H.S = B+A

LHS= A+B

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1 & 2-1 & +3+1 \\ 2+2 & 3-2 & 1+2 \\ 1+3 & -1+1 & 0+3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & 4 \\ 4 & 1 & 3 \\ 4 & 0 & 3 \end{bmatrix}$$

RHS= B+A

$$= \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1 & -1+2 & 1+3 \\ 2+2 & -2+3 & 2+1 \\ 3+3 & 1-1 & 3-0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & 4 \\ 4 & 1 & 3 \\ 6 & 0 & 3 \end{bmatrix}$$

$$A + B = B + A$$

Hence proved

$$\text{L.H.S} = \text{R.H.S}$$

(iii) $B + C = C + B$

Solution: $B + C = C + B$

L.H.S = B+C

R.H.S = C+B

L.H.S = B+C

$$= \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1-1 & -1+0 & 1+0 \\ 2+0 & -2-2 & 2+3 \\ 3+1 & 1+1 & 3+2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 1 \\ 2 & -4 & 5 \\ 4 & 2 & 5 \end{bmatrix}$$

R.H.S = C+B

$$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1+1 & 0-1 & 0+1 \\ 0+2 & -2-2 & 3+2 \\ 1+3 & 1+1 & 2+3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 1 \\ 2 & -4 & 5 \\ 4 & 2 & 5 \end{bmatrix}$$

L.H.S = R.H.S

B+C=C+B

Hence proved

(iv) $A + (B + A) = 2A + B$

Solution: $A + (B + A) = 2A + B$

L.H.S = A + (B+A)

R.H.S = 2A+B

L.H.S = A + (B+A)

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \left(\begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 1 & 4 \\ 4 & 1 & 3 \\ 4 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 3 & 7 \\ 6 & 4 & 4 \\ 5 & -1 & 3 \end{bmatrix}$$

R.H.S = 2A+B

$$= 2 \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 4 & 6 \\ 4 & 6 & 2 \\ 2 & -2 & 0 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 3 & 7 \\ 6 & 4 & 4 \\ 5 & -1 & 3 \end{bmatrix}$$

L.H.S = R.H.S

A + (B+A) = 2A+B

Hence proved

(v) $(C - B) + A = C + (A + B)$

Solution: $(C - B) + A = C + (A + B)$

L.H.S = (C-B) + A

R.H.S = C + (A-B)

L.H.S = (C-B) + A

$$= \left(\begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix} \right) + \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 1 & -1 \\ -2 & 0 & 1 \\ -2 & 0 & -1 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 3 & 2 \\ 0 & 3 & 2 \\ -1 & -1 & -1 \end{bmatrix}$$

RHS = C + (A-B)

$$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix} + \left(\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} - \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix} \right)$$

$$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 3 & 2 \\ 0 & 5 & -1 \\ -2 & -2 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 3 & 2 \\ 0 & 3 & 2 \\ -1 & -1 & -1 \end{bmatrix}$$

L.H.S = R.H.S

(C-B)+A=C+(A-B)

Hence proved

(vi) $2A + B = A + (A + B)$

Solution: $2A + B = A + (A + B)$

L.H.S = 2A+B

R.H.S = A + (A+B)

LHS = 2A+B

$$= 2 \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 4 & 6 \\ 4 & 6 & 2 \\ 2 & -2 & 0 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 3 & 7 \\ 6 & 4 & 4 \\ 5 & -1 & 3 \end{bmatrix}$$

$$\text{RHS} = A + (A+B)$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \left(\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 1 & 4 \\ 4 & 1 & 3 \\ 4 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 3 & 7 \\ 6 & 4 & 4 \\ 5 & -1 & 3 \end{bmatrix}$$

$$\text{L.H.S} = \text{R.H.S}$$

$$2A+B=A+(A+B)$$

Hence proved

$$\text{(vii)} \quad (C-B)-A=(C-A)-B$$

$$\text{Solution: } (C-B)-A=(C-A)-B$$

$$\text{L.H.S} = (C-B)-A$$

$$\text{R.H.S} = (C-A)-B$$

$$\text{LHS} = (C-B)-A$$

$$= \left(\begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix} \right) - \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 1 & -1 \\ -2 & 0 & 1 \\ -2 & 0 & -1 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & -1 & -4 \\ -4 & -3 & 0 \\ -3 & 1 & -1 \end{bmatrix}$$

$$\text{RHS} = (C-A)-B$$

$$= \left(\begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} \right) - \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -2 & -3 \\ -2 & -5 & 2 \\ 0 & 2 & 2 \end{bmatrix} - \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & -1 & -4 \\ -4 & -3 & 0 \\ -3 & 1 & -1 \end{bmatrix}$$

$$\text{L.H.S} = \text{R.H.S}$$

$$(C-B)-A=(C-A)-B$$

Hence proved

$$\text{(viii)} \quad (A+B)+C=A+(B+C)$$

$$\text{Solution: } (A+B)+C=A+(B+C)$$

$$\text{L.H.S} = (A+B)+C$$

$$\text{R.H.S} = A+(B+C)$$

$$\text{LHS} = (A+B)+C$$

$$= \left(\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix} \right) + \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & 4 \\ 4 & 1 & 3 \\ 4 & 0 & 3 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 4 \\ 4 & -1 & 6 \\ 5 & 1 & 5 \end{bmatrix}$$

$$\text{R.H.S} = A+(B+C)$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & -1 & 0 \end{bmatrix} + \left(\begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 2 & 1 & 4 \\ 4 & 1 & 3 \\ 4 & 0 & 3 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 4 \\ 4 & -1 & 6 \\ 5 & 1 & 5 \end{bmatrix}$$

L.H.S = R.H.S

$(A+B)+C=A+(B+C)$

Hence proved

(ix) $A+(B-C)=(A-C)+B$

Solution: $A+(B-C)=(A-C)+B$

L.H.S = $A+(B-C)$

R.H.S = $(A-C)+B$

L.H.S = $A+(B-C)$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \left(\begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix} - \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 2 & -1 & 1 \\ 2 & 0 & -1 \\ 2 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 1 & 4 \\ 4 & 3 & 0 \\ 3 & -1 & 1 \end{bmatrix}$$

RHS = $(A-C)+B$

$$= \left(\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} - \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix} \right) + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1 & 2-0 & 3-1 \\ 2-0 & 3+2 & 1-3 \\ 1-1 & -1-1 & 0-2 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 & 3 \\ 2 & 5 & -2 \\ 0 & -2 & -2 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 1 & 4 \\ 4 & 3 & 0 \\ 3 & -1 & 1 \end{bmatrix}$$

L.H.S = R.H.S

$A+(B-C)=(A-C)+B$

Hence proved

(x) $2A+2B=2(A+B)$

Solution: $2A+2B=2(A+B)$

L.H.S = $2A+2B$

R.H.S = $2(A+B)$

L.H.S = $2A+2B$

$$= 2 \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + 2 \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 4 & 6 \\ 4 & 6 & 2 \\ 2 & -2 & 0 \end{bmatrix} + \begin{bmatrix} 2 & -2 & 2 \\ 4 & -4 & 4 \\ 6 & 2 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 2 & 8 \\ 8 & 2 & 6 \\ 8 & 0 & 6 \end{bmatrix}$$

RHS = $2(A+B)$

$$= 2 \left(\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 1+1 & 2-1 & 3+1 \\ 2+2 & 3-2 & 1+2 \\ 1+3 & -1+1 & 0+3 \end{bmatrix}$$

$$= 2 \begin{bmatrix} 2 & 1 & 4 \\ 4 & 1 & 3 \\ 4 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 2 & 8 \\ 8 & 2 & 6 \\ 8 & 0 & 6 \end{bmatrix}$$

L.H.S = R.H.S

$2A+2B=2(A+B)$

Hence proved

Q.6 If $A = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 7 \\ -3 & 8 \end{bmatrix}$

find:

(i) $3A-2B$

Solution: $3A-2B$

$$3A-2B = 3 \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix} - 2 \begin{bmatrix} 0 & 7 \\ -3 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -6 \\ 9 & 12 \end{bmatrix} - \begin{bmatrix} 0 & 14 \\ -6 & 16 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -20 \\ 15 & -4 \end{bmatrix}$$

(ii) $2A^t - 3B^t$

Solution: $2A^t - 3B^t$

When we take transpose of any matrix we change rows into columns or columns into rows.

$$A^t = \begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix}$$

$$B^t = \begin{bmatrix} 0 & -3 \\ 7 & 8 \end{bmatrix}$$

$$2A^t - 3B^t = 2 \begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix} - 3 \begin{bmatrix} 0 & -3 \\ 7 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 6 \\ -4 & 8 \end{bmatrix} - \begin{bmatrix} 0 & -9 \\ 21 & 24 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 15 \\ -25 & -16 \end{bmatrix}$$

Q.7 If

$$2 \begin{bmatrix} 2 & 4 \\ -3 & a \end{bmatrix} + 3 \begin{bmatrix} 1 & b \\ 8 & -4 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 18 & 1 \end{bmatrix}$$

Solution:

$$2 \begin{bmatrix} 2 & 4 \\ -3 & a \end{bmatrix} + 3 \begin{bmatrix} 1 & b \\ 8 & -4 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 18 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 8 \\ -6 & 2a \end{bmatrix} + \begin{bmatrix} 3 & 3b \\ 24 & -12 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 18 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 8+3b \\ 18 & 2a+(-12) \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 18 & 1 \end{bmatrix}$$

$$8 + 3b = 10 \quad \text{(i)}$$

$$2a - 12 = 1 \quad \text{(ii)}$$

By solving equation (ii) we get the value of a

$$2a - 12 = 1$$

$$2a = 1 + 12$$

$$2a = 13$$

$$a = \frac{13}{2}$$

By solving equation (i) we get the value of b

$$8 + 3b = 10$$

$$3b = 10 - 8$$

$$3b = 2$$

$$b = \frac{2}{3}$$

Q.8 If $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ **and** $B = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$

Then verify that

(i) $(A+B)^t = A^t + B^t$

Solution: $(A+B)^t = A^t + B^t$

L.H.S = $(A+B)^t$

R.H.S = $A^t + B^t$

To solve L.H.S

L.H.S = $(A+B)^t$

$$= (A+B)^t = \left(\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} \right)^t$$

$$= \begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix}^t$$

R.H.S = $(A+B)^t = \begin{bmatrix} 2 & 2 \\ 3 & 1 \end{bmatrix}$

To solve R.H.S

R.H.S = $A^t + B^t$

$$A^t = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$B^t = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$$

$$\text{RHS} = A^t + B^t = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 \\ 3 & 1 \end{bmatrix}$$

L.H.S = R.H.S $\Rightarrow (A+B)^t = A^t + B^t$

Hence Proved

(ii) $(A-B)^t = A^t - B^t$

Solution: $(A-B)^t = A^t - B^t$

L.H.S = $(A-B)^t$

R.H.S = $A^t - B^t$

L.H.S = $(A-B)^t$

$$(A - B)^t = \left(\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} \right)^t$$

$$= \begin{bmatrix} 0 & 1 \\ -2 & 1 \end{bmatrix}^t$$

$$(A - B)^t = \begin{bmatrix} 0 & -2 \\ 1 & 1 \end{bmatrix}$$

$$\text{R.H.S} = A^t - B^t$$

$$= \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}^t - \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}^t$$

$$= \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1-1 & 0-2 \\ 2-1 & 1-0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -2 \\ 1 & 1 \end{bmatrix}$$

$$\text{L.H.S} = \text{R.H.S}$$

$$(A-B)^t = A^t - B^t$$

Hence proved

(iii) $A + A^t$ is a symmetric

Solution:

$A + A^t$ is a symmetric

To show that $A + A^t$ is symmetric, we will show that

$$(A + A^t)^t = (A + A^t)$$

$$A + A^t = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}^t$$

$$= \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1 & 2+0 \\ 0+2 & 1+1 \end{bmatrix}$$

$$A + A^t = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

$$(A + A^t)^t = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}^t$$

$$= \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

$$(A + A^t)^t = (A + A^t)$$

Hence Proved

$A + A^t$ symmetric

(iv) $A - A^t$ is a skew symmetric

Solution: $A - A^t$

To show that $A - A^t$ is skew symmetric we will show that

$$(A - A^t)^t = -(A - A^t)$$

$$A - A^t = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}^t$$

$$= \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1-1 & 2-0 \\ 0-2 & 1-1 \end{bmatrix}$$

$$A - A^t = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

$$(A - A^t)^t = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}^t$$

$$= \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$$

$$= - \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

$$(A - A^t)^t = -(A - A^t)$$

Hence proved

$A - A^t$ is a skew symmetric

(v) $B + B^t$ is a symmetric

Solution: $B + B^t$

To show that $B + B^t$ is symmetric we will show that

$$(B + B^t)^t = (B + B^t)$$

$$B + B^t = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}^t$$

$$= \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1 & 1+2 \\ 2+1 & 0+0 \end{bmatrix}$$

$$B + B^t = \begin{bmatrix} 2 & 3 \\ 3 & 0 \end{bmatrix}$$

$$(B + B^t)^t = \begin{bmatrix} 2 & 3 \\ 3 & 0 \end{bmatrix}^t$$

$$= \begin{bmatrix} 2 & 3 \\ 3 & 0 \end{bmatrix}$$

$$(B + B^t)^t = (B + B^t)$$

Hence proved

$B + B^t$ is a symmetric

(vi) $B - B^t$ is a skew symmetric

Solution: $B - B^t$

To show that $B - B^t$ is skew symmetric, we will show that

$$(B - B^t)^t = -(B - B^t)$$

$$B - B^t = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}^t$$

$$= \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1-1 & 1-2 \\ 2-1 & 0-0 \end{bmatrix}$$

$$B - B^t = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$(B - B^t)^t = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}^t$$

$$= \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$= - \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$(B - B^t)^t = -(B - B^t)$$

Hence proved

$B - B^t$ is a skew symmetric.

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Exercise 1.4

Q.1 Which of the following product of matrices is conformable for multiplication?

(i) $\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

Yes, these matrices can be multiplied because number of columns of 1st matrix is equal to number of rows of 2nd matrix.

(ii) $\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} -2 & -1 \\ 1 & 3 \end{bmatrix}$

Yes, these matrices can be multiplied because number of columns of 1st matrix is equal to number of rows of 2nd matrix.

(iii) $\begin{bmatrix} 1 \\ -1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}$

No, these matrices cannot be multiplied because number of columns of 1st matrix is not equal to the number of rows of 2nd matrix.

(iv) $\begin{bmatrix} 1 & 2 \\ 0 & -1 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \end{bmatrix}$

Yes, these matrices can be multiplied because number of columns of 1st matrix is equal to number of rows of 2nd matrix.

(v) $\begin{bmatrix} 3 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ -2 & 3 \end{bmatrix}$

Yes, these matrices can be multiplied because number of columns of 1st matrix is equal to number of rows of 2nd matrix.

Q.2 If $A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 6 \\ 5 \end{bmatrix}$ find

(i) AB

Solution: $AB = \begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 6 \\ 5 \end{bmatrix}$

$$AB = \begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 6 \\ 5 \end{bmatrix}$$

$$= \begin{bmatrix} (3 \times 6) + (0 \times 5) \\ (-1 \times 6) + (2 \times 5) \end{bmatrix}$$

$$= \begin{bmatrix} 18 + 0 \\ -6 + 10 \end{bmatrix} = \begin{bmatrix} 18 \\ 4 \end{bmatrix}$$

(ii) BA (if possible)

Solution:

BA is not possible because number of columns of B not equal to number of rows of A .

Q.3 Find the following products

(i) $[1 \ 2] \begin{bmatrix} 4 \\ 0 \end{bmatrix}$

Solution: $[1 \ 2] \begin{bmatrix} 4 \\ 0 \end{bmatrix}$

$$= [(1 \times 4) + (2 \times 0)]$$

$$= [4 + 0]$$

$$= [4]$$

(ii) $[1 \ 2] \begin{bmatrix} 5 \\ -4 \end{bmatrix}$

Solution: $[1 \ 2] \begin{bmatrix} 4 \\ -0 \end{bmatrix}$

$$= [(1 \times 5) + (2 \times -4)]$$

$$= [5 + (-8)]$$

$$= [5 - 8]$$

$$= [-3]$$

(iii) $[-3 \ 0] \begin{bmatrix} 4 \\ 0 \end{bmatrix}$

Solution: $[-3 \ 0] \begin{bmatrix} 4 \\ 0 \end{bmatrix}$

$$= [(-3 \times 4) + (0 \times 0)]$$

$$= [-12 + 0]$$

$$= [-12]$$

(iv) $[6 \ 0] \begin{bmatrix} 4 \\ -0 \end{bmatrix}$

Solution: $[6 \ 0] \begin{bmatrix} 4 \\ -0 \end{bmatrix}$

$$[6 \ 0] \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

$$= [6 \times 4 + (-0)(0)]$$

$$= [24 - 0]$$

$$= [24]$$

(v) $\begin{bmatrix} 1 & 2 \\ -3 & 0 \\ 6 & -1 \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 0 & -4 \end{bmatrix}$

Solution: $\begin{bmatrix} 1 & 2 \\ -3 & 0 \\ 6 & -1 \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 0 & -4 \end{bmatrix}$

$$= \begin{bmatrix} 1 \times 4 + 2 \times 0 & 1 \times 5 + 2 \times (-4) \\ -3 \times 4 + 0 \times 0 & -3(5) + 0 \times (-4) \\ 6(4) + (-1)(0) & 6(5) + (-1)(-4) \end{bmatrix}$$

$$= \begin{bmatrix} 4 + 0 & 5 - 8 \\ -12 + 0 & -15 - 0 \\ 24 - 0 & 30 + 4 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -3 \\ -12 & -15 \\ 24 & 34 \end{bmatrix}$$

Q.4 Multiply the following matrices.

(a) $\begin{bmatrix} 2 & 3 \\ 1 & 1 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 3 & 0 \end{bmatrix}$

Solution: $\begin{bmatrix} 2 & 3 \\ 1 & 1 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 3 & 0 \end{bmatrix}$

$$\begin{bmatrix} 2 \times 2 + (3 \times 3) & (2 \times -1) + (3 \times 0) \\ (1 \times 2) + (1 \times 3) & (1 \times -1) + (1 \times 0) \\ (0 \times 2) + (-2 \times 3) & (0 \times -1) + (-2 \times 0) \end{bmatrix}$$

$$= \begin{bmatrix} 4 + 9 & -2 + 0 \\ 2 + 3 & -1 + 0 \\ 0 + -6 & 0 + 0 \end{bmatrix}$$

$$= \begin{bmatrix} 13 & -2 \\ 5 & -1 \\ 0-6 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 13 & -2 \\ 5 & -1 \\ -6 & 0 \end{bmatrix}$$

$$(b) \quad \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ -1 & 1 \end{bmatrix}$$

$$\text{Solution: } \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (1 \times 1) + (2 \times 3) + (3 \times -1) & (1 \times 2) + (2 \times 4) + (3 \times 1) \\ (4 \times 1) + (5 \times 3) + (6 \times -1) & (4 \times 2) + (5 \times 4) + (6 \times 1) \end{bmatrix}$$

$$= \begin{bmatrix} 1+6+(-3) & 2+8+3 \\ 4+15+(-6) & 8+20+6 \end{bmatrix}$$

$$= \begin{bmatrix} 7-3 & 13 \\ 19-6 & 34 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 13 \\ 13 & 34 \end{bmatrix}$$

$$(c) \quad \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

$$\text{Solution: } \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} (1 \times 1) + (2 \times 4) & (1 \times 2) + (2 \times 5) & (1 \times 3) + (2 \times 6) \\ (3 \times 1) + (4 \times 4) & (3 \times 2) + (4 \times 5) & (3 \times 3) + (4 \times 6) \\ (-1 \times 1) + (1 \times 4) & (-1 \times 2) + (1 \times 5) & (-1 \times 3) + (1 \times 6) \end{bmatrix}$$

$$= \begin{bmatrix} 1+8 & 2+10 & 3+12 \\ 3+16 & 6+20 & 9+24 \\ -1+4 & -2+5 & -3+6 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 12 & 15 \\ 19 & 26 & 33 \\ 3 & 3 & 3 \end{bmatrix}$$

$$(d) \quad \begin{bmatrix} 8 & 5 \\ 6 & 4 \end{bmatrix} \begin{bmatrix} 2 & -\frac{5}{2} \\ -4 & 4 \end{bmatrix}$$

$$\text{Solution: } \begin{bmatrix} 8 & 5 \\ 6 & 4 \end{bmatrix} \begin{bmatrix} 2 & -\frac{5}{2} \\ -4 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} (8 \times 2) + (5 \times -4) & \left(8 \times -\frac{5}{2}\right) + (5 \times 4) \\ (6 \times 2) + (4 \times -4) & \left(6 \times -\frac{5}{2}\right) + (4 \times 4) \end{bmatrix}$$

$$= \begin{bmatrix} 16 + (-20) & \frac{-40}{2} + 20 \\ 12 + (-16) & \frac{-30}{2} + 16 \end{bmatrix}$$

$$= \begin{bmatrix} 16-20 & -20+20 \\ 12-16 & -15+16 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & 0 \\ -4 & 1 \end{bmatrix}$$

$$(e) \quad \begin{bmatrix} -1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\text{Solution: } \begin{bmatrix} -1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} (-1 \times 0) + (2 \times 0) & (-1 \times 0) + (2 \times 0) \\ (1 \times 0) + (3 \times 0) & (1 \times 0) + (3 \times 0) \end{bmatrix}$$

$$= \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\text{Q.5 Let } A = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}$$

$$\text{and } C = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \text{ verify whether}$$

$$(i) \quad AB = BA$$

$$\text{Solution: } AB = BA$$

$$\text{L.H.S} = AB$$

$$\text{R.H.S} = BA$$

$$\text{L.H.S} = AB$$

$$\begin{aligned}
&= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix} \\
&= \begin{bmatrix} (-1 \times 1) + (3 \times -3) & (-1 \times 2) + (3 \times -5) \\ (2 \times 1) + (0 \times -3) & (2 \times 2) + (0 \times -5) \end{bmatrix} \\
&= \begin{bmatrix} -1 + (-9) & -2 + (-15) \\ 2 + 0 & 4 + 0 \end{bmatrix} \\
&= \begin{bmatrix} -1 - 9 & -2 - 15 \\ 2 & 4 \end{bmatrix} \\
&= \begin{bmatrix} -10 & -17 \\ 2 & 4 \end{bmatrix}
\end{aligned}$$

$$\begin{aligned}
\text{R.H.S} = \text{BA} &= \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix} \times \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \\
&= \begin{bmatrix} 1 \times (-1) + 2 \times 2 & 1 \times 3 + 2 \times 0 \\ -3 \times (-1) + (-5) \times 2 & -3 \times 3 + (-5) \times (0) \end{bmatrix} \\
&= \begin{bmatrix} -1 + 4 & 3 + 0 \\ 3 - 10 & -9 - 0 \end{bmatrix} \\
&= \begin{bmatrix} 3 & 3 \\ -7 & -9 \end{bmatrix}
\end{aligned}$$

Since L.H.S \neq R.H.S
L.H.S \neq R.H.S
L.H.S \neq R.H.S

(ii) $A(BC) = (AB)C$

Solution: $A(BC) = (AB)C$

L.H.S = A(BC)

R.H.S = (AB)C

L.H.S

L.H.S = A(BC)

$$\begin{aligned}
&= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \times \left(\begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix} \times \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \right) \\
&= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \times \begin{bmatrix} 2+2 & 1+6 \\ -6+(-5) & -3+(-15) \end{bmatrix} \\
&= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \times \begin{bmatrix} 4 & 7 \\ -6-5 & -3-15 \end{bmatrix} \\
&= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \times \begin{bmatrix} 4 & 7 \\ -11 & -18 \end{bmatrix} \\
&= \begin{bmatrix} (-1 \times 4) + (3 \times -11) & (-1 \times 7) + (3 \times -18) \\ (2 \times 4) + (0 \times -11) & (2 \times 7) + (0 \times -18) \end{bmatrix}
\end{aligned}$$

$$\begin{aligned}
&= \begin{bmatrix} -4 + (-33) & -7 + (-54) \\ 8 + 0 & 14 + 0 \end{bmatrix} \\
&= \begin{bmatrix} -4 - 33 & -7 - 54 \\ 8 & 14 \end{bmatrix} \\
&= \begin{bmatrix} -37 & -61 \\ 8 & 14 \end{bmatrix}
\end{aligned}$$

R.H.S = (AB)C

$$\begin{aligned}
&= \left(\begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix} \right) \times \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \\
&= \begin{bmatrix} (-1 \times 1) + (3 \times -3) & (-1 \times 2) + (3 \times -5) \\ (2 \times 1) + (0 \times -3) & (2 \times 2) + (0 \times -5) \end{bmatrix} \\
&= \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \\
&= \begin{bmatrix} -1 + (-9) & -2 + (-15) \\ 2 + 0 & -4 + 0 \end{bmatrix} \times \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \\
&= \begin{bmatrix} -1 - 9 & -2 - 15 \\ 2 & 4 \end{bmatrix} \times \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \\
&= \begin{bmatrix} -10 & -17 \\ 2 & 4 \end{bmatrix} \times \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \\
&= \begin{bmatrix} (-10 \times 2) + (-17 \times 1) & (-10 \times 1) + (-17 \times 3) \\ (2 \times 2) + (4 \times 1) & (2 \times 1) + (4 \times 3) \end{bmatrix} \\
&= \begin{bmatrix} -20 + (-17) & -10 + (-51) \\ 4 + 4 & 2 + 12 \end{bmatrix} \\
&= \begin{bmatrix} -20 - 17 & -10 - 51 \\ 8 & 14 \end{bmatrix} \\
&= \begin{bmatrix} -37 & -61 \\ 8 & 14 \end{bmatrix}
\end{aligned}$$

Since

L.H.S = R.H.S $\Rightarrow A(BC) = (AB)C$

Hence proved

(iii) $A(B+C) = AB + AC$

Solution: $A(B+C) = AB + AC$

L.H.S = A(B+C)

R.H.S = AB + AC

L.H.S

L.H.S = A(B+C)

$$= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \left(\begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \right)$$

$$= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \times \begin{bmatrix} 1+2 & 2+1 \\ -3+1 & -5+3 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \times \begin{bmatrix} 3 & 3 \\ -2 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} (-1 \times 3) + (3 \times -2) & (-1 \times 3) + (3 \times -2) \\ (2 \times 3) + (0 \times -2) & (2 \times 3) + (0 \times -2) \end{bmatrix}$$

$$= \begin{bmatrix} -3 + (-6) & -3 + (-6) \\ 6 + 0 & 6 + 0 \end{bmatrix}$$

$$= \begin{bmatrix} -3 - 6 & -3 - 6 \\ 6 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} -9 & -9 \\ 6 & 6 \end{bmatrix}$$

R.H.S = AB + AC

$$= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix} + \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \times \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} (-1 \times 1) + (3 \times -3) & (-1 \times 2) + (3 \times -5) \\ (2 \times 1) + (0 \times -3) & (2 \times 2) + (0 \times -5) \end{bmatrix}$$

$$+ \begin{bmatrix} (-1 \times 2) + (3 \times 1) & (-1 \times 1) + (3 \times 3) \\ (2 \times 2) + (0 \times 1) & (2 \times 1) + (0 \times 3) \end{bmatrix}$$

$$= \begin{bmatrix} -1 + (-3) & -2 + (-15) \\ 2 + 0 & 4 + 0 \end{bmatrix} + \begin{bmatrix} -2 + 3 & -1 + 9 \\ 4 + 0 & 2 + 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1 - 3 & -2 - 15 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 8 \\ 4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -10 & -17 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 8 \\ 4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -10 + 1 & -17 + 8 \\ 2 + 4 & 4 + 2 \end{bmatrix}$$

$$= \begin{bmatrix} -9 & -9 \\ 6 & 6 \end{bmatrix}$$

Since LHS = RHS

A(B + C) = AB + AC

Hence proved

(iv) A(B - C) = AB - AC

Solution: A(B - C) = AB - AC

L.H.S = A(B - C)

R.H.S = AB - AC

L.H.S = A(B - C)

$$= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \left(\begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \right)$$

$$= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \times \begin{bmatrix} 1 - 2 & 2 - 1 \\ -3 - 1 & -5 - 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \times \begin{bmatrix} -1 & 1 \\ -4 & -8 \end{bmatrix}$$

$$= \begin{bmatrix} (-1 \times -1) + (3 \times -4) & (-1 \times 1) + (3 \times -8) \\ (2 \times -1) + (0 \times -4) & (2 \times 1) + (0 \times -8) \end{bmatrix}$$

$$= \begin{bmatrix} +1 + (-12) & -1 + (-24) \\ -2 + 0 & 2 + 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 - 12 & -1 - 24 \\ -2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -11 & -25 \\ -2 & 2 \end{bmatrix}$$

R.H.S = AB - AC

$$= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix} - \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \times \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} (-1 \times 1) + (3 \times -3) & (-1 \times 2) + (3 \times -5) \\ (2 \times 1) + (0 \times -3) & (2 \times 2) + (0 \times -5) \end{bmatrix}$$

$$- \begin{bmatrix} (-1 \times 2) + (3 \times 1) & (-1 \times 1) + (3 \times 3) \\ (2 \times 2) + (0 \times 1) & (2 \times 1) + (0 \times 3) \end{bmatrix}$$

$$= \begin{bmatrix} (-1 \times 1) - (3 \times 3) & (-1 \times 2) + (3 \times -5) \\ (2 \times 1) + (0 \times -3) & (2 \times 2) + (0 \times -5) \end{bmatrix}$$

$$- \begin{bmatrix} (-1 \times 2) + (3 \times 1) & (-1 \times 1) + (3 \times 3) \\ (2 \times 2) + (0 \times 1) & (2 \times 1) + (0 \times 3) \end{bmatrix}$$

$$= \begin{bmatrix} -1 - 9 & -2 - 15 \\ 2 + 0 & 4 + 0 \end{bmatrix} - \begin{bmatrix} -2 + 3 & -1 + 9 \\ 4 + 0 & 2 + 0 \end{bmatrix}$$

$$= \begin{bmatrix} -10 & -17 \\ 2 & 4 \end{bmatrix} - \begin{bmatrix} 1 & 8 \\ 4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -10 - 1 & -17 - 8 \\ 2 - 4 & 4 - 2 \end{bmatrix}$$

$$= \begin{bmatrix} -11 & -25 \\ -2 & 2 \end{bmatrix}$$

Since L.H.S = R.H.S

A(B - C) = AB - AC, Hence **proved**.

Q.6 For the matrices $A = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}$,

$B = \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}$ and $C = \begin{bmatrix} -2 & 6 \\ 3 & -9 \end{bmatrix}$

Verify that

(i) $(AB)^t = B^t A^t$

Solution: $(AB)^t = B^t A^t$

L.H.S = $(AB)^t$

R.H.S = $B^t A^t$

$$(AB) = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} (-1 \times 1) + (3 \times -3) & (-1 \times 2) + (3 \times -5) \\ (+2 \times 1) + (0 \times -3) & (2 \times 2) + (0 \times -5) \end{bmatrix}$$

$$= \begin{bmatrix} -1 + (-9) & -2 + (-15) \\ 2 + 0 & 4 + 0 \end{bmatrix}$$

$$= \begin{bmatrix} -10 & -17 \\ 2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -10 & -17 \\ 2 & 4 \end{bmatrix}$$

L.H.S = $(AB)^t$

$$= \begin{bmatrix} -10 & -17 \\ 2 & 4 \end{bmatrix}^t$$

$$= \begin{bmatrix} -10 & 2 \\ -17 & 4 \end{bmatrix}$$

$$B^t = \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}^t$$

$$B^t = \begin{bmatrix} 1 & -3 \\ 2 & -5 \end{bmatrix}$$

$$A^t = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}^t$$

$$A^t = \begin{bmatrix} -1 & 2 \\ 3 & 0 \end{bmatrix}$$

L.H.S = $B^t A^t$

$$= \begin{bmatrix} 1 & -3 \\ 2 & -5 \end{bmatrix} \times \begin{bmatrix} -1 & 2 \\ 3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} (1 \times -1) + (-3 \times 3) & (1 \times 2) + (-3 \times 0) \\ (2 \times -1) + (-5 \times 3) & (2 \times 2) + (-5 \times 0) \end{bmatrix}$$

$$= \begin{bmatrix} -1 + (-9) & 2 + 0 \\ -2 + (-15) & 4 + 0 \end{bmatrix}$$

$$= \begin{bmatrix} -10 & 2 \\ -17 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -10 & 2 \\ -17 & 4 \end{bmatrix}$$

Since L.H.S = R.H.S

$(AB)^t = B^t A^t$

Hence proved

L.H.S = R.H.S

(ii) $(BC)^t = C^t B^t$

Solution: $(BC)^t = C^t B^t$

L.H.S = $(BC)^t$

R.H.S = $C^t B^t$

To solve L.H.S

$$BC = \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix} \times \begin{bmatrix} -2 & 6 \\ 3 & -9 \end{bmatrix}$$

$$= \begin{bmatrix} (1 \times -2) + (2 \times 3) & (1 \times 6) + (2 \times -9) \\ (-3 \times -2) + (-5 \times 3) & (-3 \times 6) + (-5 \times -9) \end{bmatrix}$$

$$= \begin{bmatrix} -2 + 6 & 6 + (-18) \\ 6 + (-15) & -18 + 45 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 6 - 18 \\ 6 - 15 & 27 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -12 \\ -9 & 27 \end{bmatrix}$$

Taking transpose of BC:-

$$(BC)^t = \begin{bmatrix} 4 & -12 \\ -9 & 27 \end{bmatrix}^t$$

$$L.H.S = (BC)^t = \begin{bmatrix} 4 & -9 \\ -12 & 27 \end{bmatrix}$$

To solve R.H.S =

Taking transpose of matrix C

$$C^t = \begin{bmatrix} -2 & 3 \\ 6 & -9 \end{bmatrix}$$

Taking transpose of matrix B

$$B^t = \begin{bmatrix} 1 & -3 \\ 2 & -5 \end{bmatrix}$$

Now, multiplying matrices, $B^t C^t$

$$R.H.S = C^t B^t = \begin{bmatrix} -2 & 3 \\ 6 & -9 \end{bmatrix} \times \begin{bmatrix} 1 & -3 \\ 2 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} (-2 \times 1) + (3 \times 2) & (-2 \times -3) + (3 \times -5) \\ (6 \times 1) + (-9 \times 2) & (6 \times -3) + (-9 \times -5) \end{bmatrix}$$

$$\begin{aligned} &= \begin{bmatrix} -2+6 & 6+(-15) \\ 6+(-18) & -18+45 \end{bmatrix} \\ &= \begin{bmatrix} 4 & 6-15 \\ 6-18 & 27 \end{bmatrix} \\ &= \begin{bmatrix} 4 & -9 \\ -12 & 27 \end{bmatrix} \end{aligned}$$

Hence proved
L.H.S = R.H.S

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Exercise 1.5

Q.1 Find the determinant of following matrices.

(i) $A = \begin{bmatrix} -1 & 1 \\ 2 & 0 \end{bmatrix}$

Solution:

$$A = \begin{bmatrix} -1 & 1 \\ 2 & 0 \end{bmatrix}$$

To write the determinant form

$$|A| = \begin{vmatrix} -1 & 1 \\ 2 & 0 \end{vmatrix}$$

$$= (-1)(0) - (2)(1)$$

$$= 0 - 2$$

$$= -2$$

(ii) $B = \begin{bmatrix} 1 & 3 \\ 2 & -2 \end{bmatrix}$

Solution:

$$B = \begin{bmatrix} 1 & 3 \\ 2 & -2 \end{bmatrix}$$

To write in determinant form

$$|B| = \begin{vmatrix} 1 & 3 \\ 2 & -2 \end{vmatrix}$$

$$= (1)(-2) - (2)(3)$$

$$= -2 - 6$$

$$= -8$$

(iii) $C = \begin{bmatrix} 3 & 2 \\ 3 & 2 \end{bmatrix}$

Solution:

$$C = \begin{bmatrix} 3 & 2 \\ 3 & 2 \end{bmatrix}$$

To write in determinant form

$$\begin{aligned} |C| &= \begin{vmatrix} 3 & 2 \\ 3 & 2 \end{vmatrix} \\ &= (3)(2) - (3)(2) \\ &= 6 - 6 \\ &= 0 \end{aligned}$$

(iv) $D = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$

Solution:

$$D = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$$

To write in determinant form

$$|D| = \begin{vmatrix} 3 & 2 \\ 1 & 4 \end{vmatrix}$$

$$= (3)(4) - (2)(1)$$

$$= 12 - 2$$

$$= 10$$

Q.2 Find which of the following matrices are singular or non-singular?

(i) $A = \begin{bmatrix} 3 & 6 \\ 2 & 4 \end{bmatrix}$

Solution:

$$A = \begin{bmatrix} 3 & 6 \\ 2 & 4 \end{bmatrix}$$

To write in determinant form

$$|A| = \begin{vmatrix} 3 & 6 \\ 2 & 4 \end{vmatrix}$$

$$|A| = (3)(4) - (2)(6)$$

$$|A| = 12 - 12$$

$$|A| = 0$$

It is a singular matrix.

(ii) $B = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$

Solution:

$$B = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$$

To write in determinant form

$$|B| = \begin{vmatrix} 4 & 1 \\ 3 & 2 \end{vmatrix}$$

$$|B| = (4)(2) - (3)(1)$$

$$|B| = 8 - 3$$

$$|B| = 5$$

It is non-singular matrix.

$$(iii) \quad C = \begin{bmatrix} 7 & -9 \\ 3 & 5 \end{bmatrix}$$

Solution:

$$C = \begin{bmatrix} 7 & -9 \\ 3 & 5 \end{bmatrix}$$

To write in determinant form

$$|C| = \begin{vmatrix} 7 & -9 \\ 3 & 5 \end{vmatrix}$$

$$|C| = (7)(5) - (3)(-9)$$

$$|C| = 35 + 27$$

$$|C| = 62$$

It is not equal to zero so

It is non-singular matrix.

$$(iv) \quad D = \begin{bmatrix} 5 & -10 \\ -2 & 4 \end{bmatrix}$$

Solution:

$$D = \begin{bmatrix} 5 & -10 \\ -2 & 4 \end{bmatrix}$$

To write in determinant form

$$|D| = \begin{vmatrix} 5 & -10 \\ -2 & 4 \end{vmatrix}$$

$$|D| = (5)(4) - (-2)(-10)$$

$$|D| = 20 - 20$$

$$|D| = 0$$

It is singular matrix.

Q.3 Find the multiplicative inverse of each

$$(i) \quad A = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}$$

Solution:

$$A = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}$$

To write in determinant form

$$|A| = \begin{vmatrix} -1 & 3 \\ 2 & 0 \end{vmatrix}$$

$$|A| = (-1)(0) - (2)(3)$$

$$|A| = 0 - 6$$

$$|A| = -6 \neq 0 \text{ (Non-Singular)}$$

A^{-1} exists

To write in Adj A

$$Adj A = \begin{bmatrix} 0 & -3 \\ -2 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times Adj A$$

Putting the values

$$A^{-1} = \frac{1}{-6} \times \begin{bmatrix} 0 & -3 \\ -2 & -1 \end{bmatrix} = \begin{bmatrix} 0 \times \frac{1}{-6} & -3 \times \frac{1}{-6} \\ -2 \times \frac{1}{-6} & -1 \times \frac{1}{-6} \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 0 & \frac{+3}{-6} \\ \frac{+2}{-6} & \frac{+1}{-6} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{6} \end{bmatrix}$$

$$(ii) \quad B = \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}$$

Solution:

$$B = \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}$$

To write in determinant form

$$|B| = \begin{vmatrix} 1 & 2 \\ -3 & -5 \end{vmatrix}$$

$$|B| = (-1)(-5) - (-3)(2)$$

$$|B| = -5 + 6$$

$$|B| = 1 \neq 0 \text{ (Non-Singular)}$$

B^{-1} exists

$$\text{Adj}B = \begin{bmatrix} -5 & -2 \\ 3 & 1 \end{bmatrix}$$

$$B^{-1} = \frac{1}{|B|} \times \text{Adj}B$$

Putting the values

$$B^{-1} = \frac{1}{1} \times \begin{bmatrix} -5 & -2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \times -5 & \frac{1}{1} \times -2 \\ \frac{1}{1} \times 3 & \frac{1}{1} \times 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-5}{1} & \frac{-2}{1} \\ \frac{3}{1} & \frac{1}{1} \end{bmatrix}$$

$$= \begin{bmatrix} -5 & -2 \\ 3 & 1 \end{bmatrix}$$

$$\text{(iii)} \quad C = \begin{bmatrix} -2 & 6 \\ 3 & -9 \end{bmatrix}$$

Solution:

To write in determinant form

$$|C| = \begin{vmatrix} -2 & 6 \\ 3 & -9 \end{vmatrix}$$

$$|C| = (-2)(-9) - (3)(6)$$

$$|C| = 18 - 18$$

$$|C| = 0 \text{ Singular}$$

C^{-1} Does not exist.

$$\text{(iv)} \quad D = \begin{bmatrix} \frac{1}{2} & \frac{3}{4} \\ 1 & 2 \end{bmatrix}$$

Solution:

To write in determinant form

$$D = \begin{vmatrix} \frac{1}{2} & \frac{3}{4} \\ 1 & 2 \end{vmatrix}$$

$$|D| = \begin{vmatrix} \frac{1}{2} & \frac{3}{4} \\ 1 & 2 \end{vmatrix} = \frac{1}{2} \times 2 - \frac{3}{4} \times 1$$

$$= 1 - \frac{3}{4}$$

$$= \frac{4-3}{4}$$

$$|D| = \frac{1}{4} \neq 0 \text{ (Non Singular)}$$

D^{-1} exists

$$\text{Adj}D = \begin{bmatrix} 2 & \frac{-3}{4} \\ -1 & \frac{1}{2} \end{bmatrix}$$

$$D^{-1} = \frac{1}{|D|} \times \text{Adj}D$$

By putting the values

$$= \frac{1}{\frac{1}{4}} \begin{bmatrix} 2 & \frac{-3}{4} \\ -1 & \frac{1}{2} \end{bmatrix}$$

$$= 1 \div \frac{1}{4} \begin{bmatrix} 2 & \frac{-3}{4} \\ -1 & \frac{1}{2} \end{bmatrix}$$

$$= 1 \times \frac{4}{1} \begin{bmatrix} 2 & \frac{-3}{4} \\ -1 & \frac{1}{2} \end{bmatrix}$$

$$= 4 \begin{bmatrix} 2 & \frac{-3}{4} \\ -1 & \frac{1}{2} \end{bmatrix}$$

$$= \begin{bmatrix} 8 & -3 \\ -4 & 2 \end{bmatrix}$$

Q.4 If $A = \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix}$,

then

Then verify that

(i) $A(\text{Adj}A) = (\text{Adj}A)A = (\det A)I$

Solution: $A(\text{Adj}A) = (\text{Adj}A)A = (\det A)I$

$$A = \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix}$$

$$\text{Adj}A = \begin{bmatrix} 6 & -2 \\ -4 & 1 \end{bmatrix}$$

$$\det A = \begin{vmatrix} 1 & 2 \\ 4 & 6 \end{vmatrix}$$

$$= 1 \times 6 - 2 \times 4$$

$$= 6 - 8$$

$$= -2$$

$$A(\text{Adj}A) = \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix} \begin{bmatrix} 6 & -2 \\ -4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 6-8 & (-2)+2 \\ 24-4 & -8+6 \end{bmatrix}$$

$$A(\text{Adj}A) = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix} \text{----- (i)}$$

$$(\text{Adj}A)A = \begin{bmatrix} 6 & -2 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix}$$

$$(\text{Adj}A)A = \begin{bmatrix} (6) \times 1 + (-2) \times 4 & (6) \times 2 + (-2) \times 6 \\ (-4) \times 1 + (1) \times 4 & (-4) \times 2 + (1) \times 6 \end{bmatrix}$$

$$= \begin{bmatrix} 6-8 & 12-12 \\ -4+4 & -8+6 \end{bmatrix}$$

$$(\text{Adj}A)A = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix} \text{----- (ii)}$$

$$(\det A)I = -2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -2 \times 1 & 0 \times 2 \\ -2 \times 0 & 1 \times -2 \end{bmatrix}$$

$$(\det A)I = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix} \text{----- (iii)}$$

Hence proved

From eq (i), (ii) and (iii)

$$A(\text{Adj}A) = (\text{Adj}A)A = (\det A)I$$

(ii) $BB^{-1} = I = B^{-1}B$

Solution: $BB^{-1} = I = B^{-1}B$

To write in determinant form

$$|B| = \begin{vmatrix} 3 & -1 \\ 2 & -2 \end{vmatrix}$$

$$= -6 - (-2)$$

$$= -6 + 2$$

$$= -4 \neq 0 \text{ (None singular)}$$

$$= B^{-1} \text{ exists.}$$

To write in AdjB

$$\text{Adj}B = \begin{bmatrix} -2 & 1 \\ -2 & 3 \end{bmatrix}$$

$$B^{-1} = \frac{1}{|B|} \text{Adj}B$$

$$B^{-1} = \frac{1}{-4} \begin{bmatrix} -2 & 1 \\ -2 & 3 \end{bmatrix}$$

New

$$BB^{-1} = \begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix} \times \frac{1}{-4} \begin{bmatrix} -2 & 1 \\ -2 & 3 \end{bmatrix}$$

$$= \frac{1}{-4} \begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} -2 & 1 \\ -2 & 3 \end{bmatrix}$$

$$= -\frac{1}{4} \begin{bmatrix} -6+2 & 3-3 \\ -4+4 & 2-6 \end{bmatrix}$$

$$= \frac{1}{-4} \begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{4}{-4} & 0 \\ 0 & \frac{4}{-4} \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$BB^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

and

$$B^{-1}B = \frac{1}{-4} \begin{bmatrix} -2 & 1 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix}$$

$$= \frac{1}{-4} \begin{bmatrix} -6+2 & 2-2 \\ -6+6 & 2-6 \end{bmatrix}$$

$$= \frac{1}{-4} \begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-4}{-4} & 0 \\ 0 & \frac{-4}{-4} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$B^{-1}B = I$$

From (i) and (ii)

$$BB^{-1} = I = B^{-1}B$$

Hence proved

Q.5 Determine whether the given matrices are multiplicative inverses of each other.

(i) $\begin{bmatrix} 3 & 5 \\ 4 & 7 \end{bmatrix}$ and $\begin{bmatrix} 7 & -5 \\ -4 & 3 \end{bmatrix}$

Solution: $\begin{bmatrix} 3 & 5 \\ 4 & 7 \end{bmatrix}$ and $\begin{bmatrix} 7 & -5 \\ -4 & 3 \end{bmatrix}$

$$\begin{bmatrix} 3 & 5 \\ 4 & 7 \end{bmatrix} \begin{bmatrix} 7 & -5 \\ -4 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 21 + (-20) & -15 + 15 \\ 28 + (-28) & -20 + 21 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

The given matrices are multiplicative inverse of each other.

(ii) $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ and $\begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}$

Solution: $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ and $\begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}$

$$\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -3 + 4 & 2 + (-2) \\ -6 + 6 & 4 + (-3) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Given matrices are multiplicative inverse of each other

Q.6

(i) $(AB)^{-1} = B^{-1}A^{-1}$

Solution: $(AB)^{-1} = B^{-1}A^{-1}$

$$A = \begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix}, B = \begin{bmatrix} -4 & -2 \\ 1 & -1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} -4 & -2 \\ 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 \times (-4) + 0(1) & 4 \times (-2) + 0(-1) \\ -1 \times (-4) + 2(1) & -1 \times (-2) + 2(-1) \end{bmatrix}$$

$$= \begin{bmatrix} -16 + 0 & -8 + 0 \\ 4 + 2 & 2 + (-2) \end{bmatrix}$$

$$= \begin{bmatrix} -16 & -8 \\ 6 & 0 \end{bmatrix}$$

To write in determinant form

$$|AB| = \begin{vmatrix} -16 & -8 \\ 6 & 0 \end{vmatrix}$$

$$|AB| = 0 - (-48)$$

$$|AB| = 48$$

To write in Adj (AB)

$$\text{Adj}(AB) = \begin{bmatrix} 0 & 8 \\ -6 & -16 \end{bmatrix}$$

$$(AB)^{-1} = \frac{1}{|AB|} \times \text{Adj}AB$$

$$= \frac{1}{48} \times \begin{bmatrix} 0 & 8 \\ -6 & -16 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{0}{48} & \frac{8}{48} \\ \frac{-6}{48} & \frac{-16}{48} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & \frac{1}{6} \\ -\frac{1}{8} & -\frac{1}{3} \end{bmatrix}$$

To solve R.H. S

To write in determinant form

$$|B| = \begin{vmatrix} -4 & -2 \\ 1 & -1 \end{vmatrix}$$

$$|B| = 4 - (-2)$$

$$|B| = 4 + 2$$

$$|B| = 6$$

To write in Adj B

$$AdjB = \begin{bmatrix} -1 & 2 \\ -1 & -4 \end{bmatrix}$$

$$B^{-1} = \frac{1}{|B|} \times AdjB$$

By putting value

$$B^{-1} = \frac{1}{6} \times \begin{bmatrix} -1 & 2 \\ -1 & -4 \end{bmatrix}$$

To write in determinant form

$$|A| = \begin{vmatrix} 4 & 0 \\ -1 & 2 \end{vmatrix}$$

$$|A| = 8 - (-0)$$

$$|A| = 8$$

To write in Adj A

$$AdjA = \begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times AdjA$$

$$= \frac{1}{8} \times \begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$$

To solve R.H.S

$$B^{-1}A^{-1} = \frac{1}{6} \begin{bmatrix} -1 & 2 \\ -1 & -4 \end{bmatrix} \times \frac{1}{8} \begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$$

$$= \frac{1}{6} \times \frac{1}{8} \begin{bmatrix} -1 & 2 \\ -1 & -4 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$$

$$= \frac{1}{48} \begin{bmatrix} -2+2 & 0+8 \\ -2-4 & 0-16 \end{bmatrix}$$

$$= \frac{1}{48} \begin{bmatrix} 0 & 8 \\ -6 & -16 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{0}{48} & \frac{8}{48} \\ \frac{-6}{48} & \frac{-16}{48} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & \frac{1}{6} \\ -\frac{1}{8} & -\frac{1}{3} \end{bmatrix}$$

$$= \begin{bmatrix} 0 \times \frac{1}{48} & 8 \times \frac{1}{48} \\ -6 \times \frac{1}{48} & -16 \times \frac{1}{48} \end{bmatrix}$$
$$= \begin{bmatrix} 0 & \frac{1}{6} \\ -\frac{1}{8} & -\frac{1}{3} \end{bmatrix}$$

Hence proved

L.H.S = R.H.S

Exercise 1.6

Q.1 Use of matrices, if possible to solve the following systems of linear equations.

- (i) The matrices inversion method
 (ii) The Cramer's rule

(i) $2x - 2y = 4$
 $3x + 2y = 6$

By matrices inversion method

$$\begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

Let $A = \begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$

$$A = \begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & -2 \\ 3 & 2 \end{vmatrix}$$

$$|A| = (2)(2) - (-2)(3)$$

$$|A| = 4 + 6$$

$$|A| = 10$$

Then, solution is possible because A is non-singular matrix.

$$AdjA = \begin{bmatrix} 2 & 2 \\ -3 & 2 \end{bmatrix}$$

As we know that

$$AX = B$$

$$X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{|A|} \times AdjA \times B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{10} \begin{bmatrix} 2 & 2 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

$$= \frac{1}{10} \begin{bmatrix} 2 \times 4 + 2 \times 6 \\ -3 \times 4 + 2 \times 6 \end{bmatrix}$$

$$= \frac{1}{10} \begin{bmatrix} 8 + 12 \\ -12 + 12 \end{bmatrix}$$

$$= \frac{1}{10} \begin{bmatrix} 20 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{20}{10} \\ \frac{0}{10} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$x = 2, y = 0$$

Solution Set = $\{(2, 0)\}$

By Cramer's rule

$$A = \begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}, B = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & -2 \\ 3 & 2 \end{vmatrix}$$

$$= (2)(2) - (-2)(3)$$

$$= 4 - (-6)$$

$$= 4 + 6$$

$$= 10$$

$$|A_x| = \begin{vmatrix} 4 & -2 \\ 6 & 2 \end{vmatrix}$$

$$= (4)(2) - (-2)(6)$$

$$= 8 + 12$$

$$= 20$$

$$|A_y| = \begin{vmatrix} 2 & 4 \\ 3 & 6 \end{vmatrix}$$

$$= (2)(6) - (4)(3)$$

$$= 12 - 12$$

$$= 0$$

$$x = \frac{|A_x|}{|A|}$$

$$x = \frac{20}{10}$$

$$x = 2$$

$$y = \frac{|A_y|}{|A|}$$

$$y = \frac{0}{10}$$

$$y = 0$$

$$\text{Solution Set} = \{(2, 0)\}$$

$$\text{(ii)} \quad 2x + y = 3$$

$$6x + 5y = 1$$

Matrices inversion method

$$\begin{bmatrix} 2 & 1 \\ 6 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\text{Let } A = \begin{bmatrix} 2 & 1 \\ 6 & 5 \end{bmatrix} X = \begin{bmatrix} x \\ y \end{bmatrix} B = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 1 \\ 6 & 5 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & 1 \\ 6 & 5 \end{vmatrix}$$

$$= (2)(5) - (1)(6)$$

$$= 10 - 6$$

$$= 4$$

Solution is possible because A is non-singular matrix.

$$\text{Adj}A = \begin{bmatrix} 5 & -1 \\ -6 & 2 \end{bmatrix}$$

$$AX = B$$

$$X = A^{-1}B$$

$$X = \frac{1}{|A|} \times \text{Adj}A \times B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 5 & -1 \\ -6 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 5 \times 3 + (-1 \times 1) \\ -6 \times 3 + 2 \times 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 15 + (-1) \\ -18 + 2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 14 \\ -16 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{14}{4} \\ \frac{-16}{4} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{7}{2} \\ -4 \end{bmatrix}$$

$$x = \frac{7}{2}, y = -4$$

$$\text{Solution Set} = \left\{ \left(\frac{7}{2}, -4 \right) \right\}$$

By Cramer's Rule

$$A = \begin{bmatrix} 2 & 1 \\ 6 & 5 \end{bmatrix} B = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & 1 \\ 6 & 5 \end{vmatrix}$$

$$= (2)(5) - (1)(6)$$

$$= 10 - 6$$

$$= 4$$

Solution is possible because A is non-singular matrix.

$$|A_x| = \begin{vmatrix} 3 & 1 \\ 1 & 5 \end{vmatrix}$$

$$= (3)(5) - (1)(1)$$

$$= 15 - 1$$

$$= 14$$

$$|A_y| = \begin{vmatrix} 2 & 3 \\ 6 & 1 \end{vmatrix}$$

$$= (2)(1) - (3)(6)$$

$$= 2 - 18$$

$$= -16$$

$$x = \frac{|A_x|}{|A|}$$

$$x = \frac{14}{4}$$

$$x = \frac{7}{2}$$

$$y = \frac{|A_y|}{|A|}$$

$$y = \frac{-16}{4}$$

$$y = -4$$

$$\text{Solution Set} = \left\{ \left(\frac{7}{2}, -4 \right) \right\}$$

$$(iii) \quad 4x + 2y = 8$$

$$3x - y = -1$$

By Matrices Inversion Method

$$\begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ -1 \end{bmatrix}$$

$$\text{Let } A = \begin{bmatrix} 3 & -2 \\ 3 & 2 \end{bmatrix} X = \begin{bmatrix} x \\ y \end{bmatrix} B = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 4 & 2 \\ 3 & -1 \end{vmatrix}$$

$$= (4)(-1) - (2)(3)$$

$$= -4 - 6$$

$$= -10$$

Solution is possible because A is non singular matrix.

$$AdjA = \begin{bmatrix} -1 & -2 \\ -3 & 4 \end{bmatrix}$$

As we know that

$$AX = B$$

$$X = A^{-1}B$$

$$X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{|A|} \times AdjA \times B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-10} \begin{bmatrix} -1 & -2 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} 8 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-10} \begin{bmatrix} -8 + 2 \\ -24 + (-4) \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-10} \begin{bmatrix} -6 \\ -28 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{-6}{-10} \\ \frac{-28}{-10} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{3}{5} \\ \frac{14}{5} \end{bmatrix}$$

$$x = \frac{3}{5}, y = \frac{14}{5}$$

$$\text{Solution Set} = \left\{ \left(\frac{3}{5}, \frac{14}{5} \right) \right\}$$

By Cramer's rule

$$A = \begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix} B = \begin{bmatrix} 8 \\ -1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 4 & 2 \\ 3 & -1 \end{vmatrix}$$

$$= (4)(-1) - (2)(3)$$

$$= -4 - 6$$

$$= -10$$

Solution is possible because A is non singular matrix.

$$|A_x| = \begin{vmatrix} 8 & 2 \\ -1 & -1 \end{vmatrix}$$

$$= (8)(-1) - (2)(-1)$$

$$= -8 - (-2)$$

$$= -6$$

$$x = \frac{|A_x|}{|A|}$$

$$x = \frac{6}{10}$$

$$x = \frac{3}{5}$$

$$|A_y| = \begin{vmatrix} 4 & 8 \\ 3 & -1 \end{vmatrix}$$

$$= (4)(-1) - (8)(3)$$

$$= -4 - 24$$

$$= -28$$

$$y = \frac{|A_y|}{|A|}$$

$$y = \frac{28}{10}$$

$$y = \frac{14}{5}$$

$$\text{Solution Set} = \left\{ \left(\frac{3}{5}, \frac{14}{5} \right) \right\}$$

$$(iv) \quad 3x - 2y = -6$$

$$5x - 2y = -10$$

By Matrices Inversion Method

$$\begin{bmatrix} 3 & -2 \\ 5 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -6 \\ -10 \end{bmatrix}$$

$$\text{Let } A = \begin{bmatrix} 3 & -2 \\ 3 & 2 \end{bmatrix} X = \begin{bmatrix} x \\ y \end{bmatrix} B = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & -2 \\ 5 & -2 \end{vmatrix}$$

$$= (3)(-2) - (-2)(5)$$

$$= -6 - (-10)$$

$$= -6 + 10$$

$$= 4$$

Solution is possible because A is non singular matrix.

$$AdjA = \begin{bmatrix} -2 & 2 \\ -5 & 3 \end{bmatrix}$$

$$X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{|A|} \times AdjA \times B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4} \begin{bmatrix} -2 & 2 \\ -5 & 3 \end{bmatrix} \begin{bmatrix} -6 \\ -10 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4} \begin{bmatrix} -2 \times -6 + 2 \times -10 \\ -5 \times -6 + 3 \times -10 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 12 + (-20) \\ 30 + (-30) \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 12 - 20 \\ 30 - 30 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4} \begin{bmatrix} -8 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{-8}{4} \\ \frac{0}{4} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$$

$$x = -2, y = 0$$

$$\text{Solution Set} = \{(-2, 0)\}$$

By Cramer's rule

$$A = \begin{bmatrix} 3 & -2 \\ 5 & -2 \end{bmatrix}, B = \begin{bmatrix} -6 \\ -10 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & -2 \\ 5 & -2 \end{vmatrix}$$

$$= (3)(-2) - (-2)(5)$$

$$= -6 - (-10)$$

$$= -6 + 10$$

$$= 4$$

Solution is possible because A is non singular matrix.

$$|A_x| = \begin{vmatrix} -6 & -2 \\ -10 & -2 \end{vmatrix}$$

$$= (-6)(-2) - (-2)(-10)$$

$$= +12 - (+20)$$

$$= 12 - 20$$

$$= -8$$

$$|A_y| = \begin{vmatrix} 3 & -6 \\ 5 & -10 \end{vmatrix}$$

$$= (3)(-10) - (-6)(5)$$

$$= -30 - (-30)$$

$$= -30 + 30$$

$$= 0$$

$$x = \frac{|A_x|}{|A|}$$

$$x = \frac{-8}{4}$$

$$x = -2$$

$$y = \frac{|A_y|}{|A|}$$

$$= \frac{0}{4}$$

$$y = 0$$

$$\text{Solution Set} = \{(-2, 0)\}$$

$$(v) \quad 3x - 2y = 4$$

$$-6x + 4y = 7$$

By Matrices Inversion Method

$$\begin{bmatrix} 3 & -2 \\ -6 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & -2 \\ 3 & 2 \end{bmatrix} X = \begin{bmatrix} x \\ y \end{bmatrix} B = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & -2 \\ -6 & 4 \end{vmatrix}$$

$$= (3)(4) - (-2)(-6)$$

$$= 12 - (+12)$$

$$= 12 - 12$$

$$= 0$$

Solution is not possible because A is singular matrix.

$$\begin{aligned} \text{(vi)} \quad 4x + y &= 9 \\ -3x - y &= -5 \end{aligned}$$

By Matrices Inversion Method

$$\begin{bmatrix} 4 & 1 \\ -3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ -5 \end{bmatrix}$$

$$\text{Let } A = \begin{bmatrix} 3 & -2 \\ 3 & 2 \end{bmatrix} X = \begin{bmatrix} x \\ y \end{bmatrix} B = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 4 & 1 \\ -3 & -1 \end{vmatrix}$$

$$= (4)(-1) - (-1)(-3)$$

$$= -4 + 3$$

$$= -1$$

Solution is possible because $|A|$ is non singular

$$\text{Adj}A = \begin{bmatrix} -1 & -1 \\ 3 & 4 \end{bmatrix}$$

As we know that

$$X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{|A|} \times \text{Adj}A \times B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-1} \begin{bmatrix} -1 & -1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 9 \\ -5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-1} \begin{bmatrix} -9 + 5 \\ 27 + (-20) \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-1} \begin{bmatrix} -4 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -4 \\ -1 \\ 7 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -7 \end{bmatrix}$$

$$x = 4, y = -7$$

$$\text{Solution Set} = \{(4, -7)\}$$

By Cramer's rule

$$A = \begin{bmatrix} 4 & 1 \\ -3 & -1 \end{bmatrix}, B = \begin{bmatrix} 9 \\ -5 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 4 & 1 \\ -3 & -1 \end{vmatrix}$$

$$= (4)(-1) - (1)(-3)$$

$$= -4 - (-3)$$

$$= -4 + 3$$

$$= -1$$

$$|A_x| = \begin{vmatrix} 9 & 1 \\ -5 & -1 \end{vmatrix}$$

$$= (9)(-1) - (1)(-3)$$

$$= -9 - (-5)$$

$$= -9 + 5$$

$$= -4$$

$$x = \frac{|A_x|}{|A|}$$

$$= \frac{-4}{-1}$$

$$x = 4$$

$$|A_y| = \begin{vmatrix} 4 & 9 \\ -3 & -5 \end{vmatrix}$$

$$= (4)(-5) - (9)(-3)$$

$$= -20 - (-27)$$

$$= -20 + 27$$

$$= 7$$

$$y = \frac{|A_y|}{|A|}$$

$$= \frac{7}{-1}$$

$$y = -7$$

$$\text{Solution Set} = \{(4, -7)\}$$

$$\text{(vii)} \quad 2x - 2y = 4$$

$$-5x - 2y = -10$$

By Matrices Inversion Method

$$\begin{bmatrix} 2 & -2 \\ -5 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -10 \end{bmatrix}$$

$$\text{Let } A = \begin{bmatrix} 3 & -2 \\ 3 & 2 \end{bmatrix} X = \begin{bmatrix} x \\ y \end{bmatrix} B = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & -2 \\ -5 & -2 \end{vmatrix}$$

$$= (2)(-2) - (-2)(-5)$$

$$= -4 - (+10)$$

$$= -4 - 10$$

$$= -14$$

Solution is possible

$$AdjA = \begin{bmatrix} -2 & 2 \\ 5 & 2 \end{bmatrix}$$

As we know that

$$X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{|A|} \times AdjA \times B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-14} \begin{bmatrix} -2 & 2 \\ 5 & 2 \end{bmatrix} \begin{bmatrix} 4 \\ -10 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-14} \begin{bmatrix} -8 + (-20) \\ 20 + (-20) \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-14} \begin{bmatrix} -8 - 20 \\ 20 - 20 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-14} \begin{bmatrix} -28 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{-28}{-14} \\ \frac{0}{14} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$x = 2, y = 0$$

Solution Set = $\{(2, 0)\}$

By Cramer's rule

$$A = \begin{bmatrix} 2 & -2 \\ -5 & -2 \end{bmatrix} B = \begin{bmatrix} 4 \\ -10 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & -2 \\ -5 & -2 \end{vmatrix}$$

$$= (2)(-2) - (-2)(-5)$$

$$= -4 - (+10)$$

$$= -4 - 10$$

$$= -14$$

Set is possible

$$|A_x| = \begin{vmatrix} 4 & -2 \\ -10 & -2 \end{vmatrix}$$

$$= (4)(-2) - (-2)(-10)$$

$$= -8 - (+20)$$

$$= -8 - 20$$

$$= -28$$

$$|A_y| = \begin{vmatrix} 2 & 4 \\ -5 & -10 \end{vmatrix}$$

$$= (2)(-10) - (4)(-5)$$

$$= -20 - (-20)$$

$$= -20 + 20$$

$$= 0$$

$$x = \frac{|A_x|}{|A|}$$

$$x = \frac{-28}{-14}$$

$$x = 2$$

Solution Set = $\{(2, 0)\}$

(viii) $3x - 4y = 4$

$$x + 2y = 8$$

By Matrices Inversion Method

$$\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$$

$$\text{Let } A = \begin{bmatrix} 3 & -2 \\ 3 & 2 \end{bmatrix} X = \begin{bmatrix} x \\ y \end{bmatrix} B = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & -4 \\ 1 & 2 \end{vmatrix}$$

$$= (3)(2) - (-4)(1)$$

$$= 6 - (-4)$$

$$|A| = 6 + 4$$

$$= 10$$

Solution is possible because A is non singular matrix.

$$AdjA = \begin{bmatrix} 2 & 4 \\ -1 & 3 \end{bmatrix}$$

As we know that

$$AX = B$$

$$X = A^{-1}B$$

$$X = \frac{1}{|A|} \times AdjA \times B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{10} \begin{bmatrix} 2 & 4 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 8 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{10} \begin{bmatrix} 2 \times 4 + 4 \times 8 \\ -1 \times 4 + 3 \times 8 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{10} \begin{bmatrix} 8 + 32 \\ -4 + 24 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{10} \begin{bmatrix} 40 \\ 20 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{40}{10} \\ \frac{20}{10} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

$$x = 4, y = 2$$

Solution Set = $\{(4, 2)\}$

By Cramer's rule

$$A = \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} B = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & -4 \\ 1 & 2 \end{vmatrix}$$

$$= (3)(2) - (-4)(1)$$

$$= 6 - (-4)$$

$$= 6 + 4$$

$$= 10$$

Solution is possible

$$|A_x| = \begin{vmatrix} 4 & -4 \\ 8 & 2 \end{vmatrix}$$

$$|A_x| = \begin{vmatrix} 4 & -4 \\ 8 & 2 \end{vmatrix}$$

$$= (4)(2) - (-4)(8)$$

$$= 8 - (-32)$$

$$= 8 + 32$$

$$= 40$$

$$|A_y| = \begin{vmatrix} 3 & 4 \\ 1 & 8 \end{vmatrix}$$

$$|A_y| = \begin{vmatrix} 3 & 4 \\ 1 & 8 \end{vmatrix}$$

$$= (3)(8) - (4)(1)$$

$$= 24 - 4$$

$$= 20$$

$$x = \frac{|A_x|}{|A|}$$

$$x = \frac{40}{10}$$

$$x = 4$$

$$y = \frac{|A_y|}{|A|}$$

$$y = \frac{20}{10}$$

$$y = 2$$

Solution Set = $\{(4, 2)\}$

Q.2 The length of a rectangle is 4 times it width. The perimeter of the rectangle is 150cm. Find the dimensions of the rectangle.

Solution:

Let width of rectangle = x

Length of rectangle = y

According to 1st condition

$$y = 4x$$

$$-4x + y = 0 \quad \rightarrow \dots(i)$$

According to 2nd condition

2(length + Width) = Perimeter

$$2(y + x) = 150$$

$$y + x = \frac{150}{2}$$

$$x + y = 75 \quad \rightarrow \dots(ii)$$

$$-4x + y = 0$$

$$x + y = 75$$

Changing into matrix form

$$\begin{bmatrix} -4 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 75 \end{bmatrix}$$

$$X = A^{-1}B$$

(By matrix inversion method)

$$\text{Let } A = \begin{bmatrix} -4 & 1 \\ 1 & 1 \end{bmatrix} X = \begin{bmatrix} x \\ y \end{bmatrix} B = \begin{bmatrix} 0 \\ 75 \end{bmatrix}$$

$$|A| = \begin{vmatrix} -4 & 1 \\ 1 & 1 \end{vmatrix}$$

$$= (-4)(1) - (1)(1)$$

$$= -4 - 1$$

$$= -5$$

$$\text{Adj}A = \begin{bmatrix} 1 & -1 \\ -1 & -4 \end{bmatrix}$$

As we know that

$$X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{|A|} \times \text{Adj}A \times B$$

$$= -\frac{1}{5} \begin{bmatrix} 1 & -1 \\ -1 & -4 \end{bmatrix} \begin{bmatrix} 0 \\ 75 \end{bmatrix}$$

$$= -\frac{1}{5} \begin{bmatrix} 0 - 75 \\ 0 - 300 \end{bmatrix}$$

$$= \frac{1}{5} \begin{bmatrix} -75 \\ -300 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{-75}{-5} \\ \frac{-300}{-5} \end{bmatrix}$$

$$= \begin{bmatrix} 15 \\ 60 \end{bmatrix}$$

$$x = 15, y = 60$$

Width of rectangle = $x = 15$ cm

Length of rectangle = $y = 60$ cm

By Cramer's rule

$$A = \begin{bmatrix} -4 & 1 \\ 1 & 1 \end{bmatrix} B = \begin{bmatrix} 0 \\ 75 \end{bmatrix}$$

$$|A| = \begin{vmatrix} -4 & 1 \\ 1 & 1 \end{vmatrix}$$

$$= (-4)(1) - (1)(1)$$

$$= -4 - 1$$

$$= -5$$

$$|A_x| = \begin{vmatrix} 0 & 1 \\ 75 & 1 \end{vmatrix}$$

$$= (0)(1) - (1)(75)$$

$$= 0 - 75$$

$$= -75$$

$$|A_y| = \begin{vmatrix} -4 & 0 \\ 1 & 75 \end{vmatrix}$$

$$= (-4)(75) - (0)(1)$$

$$= 0 - 300$$

$$= -300$$

$$x = \frac{|A_x|}{|A|}$$

$$= \frac{-75}{-5}$$

$$x = 15$$

$$y = \frac{|A_y|}{|A|}$$

$$= \frac{-300}{-5}$$

$$y = 60$$

Then

Width of rectangle = $x = 15$ cm

Length of rectangle = $y = 60$ cm

Q.3 Two sides of a rectangle differ by 3.5 cm. Find the dimension of the rectangle if its perimeter is 67 cm.

Solution:

Suppose Width of rectangle = x

Length of rectangle = y

According to 1st condition

$$y - x = 3.5$$

$$-x + y = 3.5 \quad \rightarrow \dots (i)$$

According to 2nd condition

$$2(L + B) = P$$

$$2(y + x) = 67$$

$$x + y = \frac{67}{2}$$

$$x + y = 33.5 \quad \rightarrow (ii)$$

Changing into matrix form

$$\begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3.5 \\ 33.5 \end{bmatrix}$$

(By matrix inversion method)

$$\text{Let } A = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix} X = \begin{bmatrix} x \\ y \end{bmatrix} B = \begin{bmatrix} 3.5 \\ 33.5 \end{bmatrix}$$

$$|A| = \begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix}$$

$$= (-1)(1) - (1)(1)$$

$$= -1 - 1$$

$$= -2$$

$$\text{Adj}A = \begin{bmatrix} 1 & -1 \\ -1 & -1 \end{bmatrix}$$

As we know that

$$X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{|A|} \times \text{Adj}A \times B$$

$$= \frac{1}{-2} \begin{bmatrix} 1 & -1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} 3.5 \\ +33.5 \end{bmatrix}$$

$$= \frac{1}{-2} \begin{bmatrix} 1 \times 3.5 & 1 \times 33.5 \\ -1 \times 3.5 & 1 \times 33.5 \end{bmatrix}$$

$$= \frac{1}{-2} \begin{bmatrix} 3.5(-33.5) \\ -3.5(-33.5) \end{bmatrix}$$

$$= \frac{1}{-2} \begin{bmatrix} -30 \\ -37 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{30}{2} \\ \frac{37}{2} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 15 \\ \frac{37}{2} \end{bmatrix}$$

$$x = 15, y = \frac{37}{2} = 18.5$$

By Cramer's rule

$$A = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix} B = \begin{bmatrix} 3.5 \\ 33.5 \end{bmatrix}$$

$$|A| = \begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix}$$

$$= (-1)(1) - (1)(1)$$

$$= -1 - 1$$

$$= -2$$

$$|A_x| = \begin{vmatrix} 3.5 & 1 \\ 33.5 & 1 \end{vmatrix}$$

$$= (3.5)(1) - (1)(33.5)$$

$$= 3.5 - 33.5$$

$$= -30$$

$$|A_y| = \begin{vmatrix} -1 & 3.5 \\ 1 & 33.5 \end{vmatrix}$$

$$= (-1)(33.5) - (3.5)(1)$$

$$= -33.5 - 3.5$$

$$= -37$$

$$x = \frac{|A_x|}{|A|}$$

$$x = \frac{-30}{-2}$$

$$x = 15$$

$$y = \frac{|A_y|}{|A|}$$

$$y = \frac{-37}{-2}$$

$$y = \frac{37}{2} = 18.5$$

Width of rectangle = $x = 15$ cm

Length of rectangle = $y = 18.5$ cm

Q.4 The third angle of an isosceles Δ is 16° less than the sum of two equal angles. Find three angles of the triangle.

Solution:

Let each equal angles are x and third angle is y

According to condition $y = 2x - 16$

$$2x - y = 16 \quad (i)$$

As we know that

$$x + x + y = 180$$

$$2x + y = 180 \quad (ii)$$

$$2x - y = 16$$

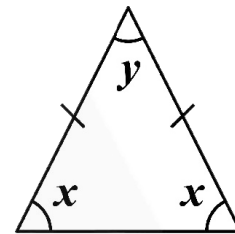
$$2x + y = 180$$

Changing into matrix form

$$\begin{bmatrix} 2 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 16 \\ 180 \end{bmatrix}$$

$$X = A^{-1}B$$

$$\text{Let } A = \begin{bmatrix} 2 & -1 \\ 2 & 1 \end{bmatrix} X = \begin{bmatrix} x \\ y \end{bmatrix} B = \begin{bmatrix} 16 \\ 180 \end{bmatrix}$$



$$X = A^{-1}B$$

Where

$$A = \begin{bmatrix} 2 & -1 \\ 2 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 16 \\ 180 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & -1 \\ 2 & 1 \end{vmatrix}$$

$$= 2 \times 1 - (-1) \times 2$$

$$= 2 + 2$$

$$= 4 \neq 0 \text{ (None singular)}$$

A^{-1} exist

$$\text{Adj}A = \begin{bmatrix} 1 & 1 \\ -2 & 2 \end{bmatrix}$$

$$X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 1 & 1 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} 16 \\ 180 \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} 1 \times 16 + 1 \times 180 \\ -2 \times 16 + 2 \times 180 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} 16+180 \\ -32+360 \end{bmatrix}$$

$$= \begin{bmatrix} 196 \\ 328 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{196}{4} \\ \frac{328}{4} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 49 \\ 82 \end{bmatrix}$$

$$x = 49$$

$$y = 82$$

Cramer Rule

$$A = \begin{bmatrix} 2 & -1 \\ 2 & 1 \end{bmatrix} B = \begin{bmatrix} 16 \\ 180 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & -1 \\ 2 & 1 \end{vmatrix}$$

$$= (2)(1) - (-1)(2)$$

$$= 2 - (-2)$$

$$= 2 + 2$$

$$= 4$$

$$|A_x| = \begin{vmatrix} 16 & -1 \\ 180 & 1 \end{vmatrix}$$

$$= (16)(1) - (-1)(180)$$

$$= 16 + 180$$

$$= 196$$

$$|A_y| = \begin{vmatrix} 2 & 16 \\ 2 & 180 \end{vmatrix}$$

$$= (2)(180) - (16)(2)$$

$$= 360 - 32$$

$$= 328$$

$$x = \frac{|A_x|}{|A|}$$

$$x = \frac{196}{4}$$

$$x = 49$$

$$y = \frac{|A_y|}{|A|}$$

$$y = \frac{328}{4}$$

$$y = 82$$

$$1^{\text{st}} \text{ angle} = x = 49^\circ \text{ Ans}$$

$$2^{\text{nd}} \text{ angle} = x = 49^\circ \text{ Ans}$$

$$3^{\text{rd}} \text{ angle} = y = 82^\circ \text{ Ans}$$

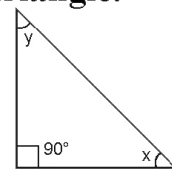
Q.5 One acute angle of a right triangle is 12° more than twice the other acute angle. Find the acute angles of the right triangle.

Solution:

Let one acute angle = x

And other acute angle = y

According to 1st condition



$$x = 2y + 12$$

$$x - 2y = 12 \quad \rightarrow \text{(i)}$$

As we know

$$x + y = 90 \quad \rightarrow \text{(ii)}$$

By matrices inversion method

Changing into matrix form

$$\begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ 90 \end{bmatrix}$$

$$X = A^{-1}B$$

$$\text{Let } A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix} X = \begin{bmatrix} x \\ y \end{bmatrix} B = \begin{bmatrix} 12 \\ 90 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & -2 \\ 1 & 1 \end{vmatrix}$$

$$= (1)(1) - (-2)(1)$$

$$= 1 - (-2)$$

$$= 3 \text{ (Non singular)}$$

$\therefore A^{-1}$ exists

$$\text{Adj}A = \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$$

As we know that

$$X = A^{-1}B \text{ or}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{|A|} \times \text{Adj}A \times B$$

$$= \frac{1}{3} \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 12 \\ 90 \end{bmatrix}$$

$$= \frac{1}{3} \begin{bmatrix} 12+180 \\ -12+90 \end{bmatrix}$$

$$= \frac{1}{3} \begin{bmatrix} 192 \\ 78 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{192}{3} \\ \frac{78}{3} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 64 \\ 26 \end{bmatrix}$$

$$x = 64, y = 26$$

Then

$$1^{\text{st}} \text{ angle} = x = 64^\circ$$

$$2^{\text{nd}} \text{ angle} = y = 26^\circ$$

By Cramer's rule

$$A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix} B = \begin{bmatrix} 12 \\ 90 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & -2 \\ 1 & 1 \end{vmatrix}$$

$$= (1)(1) - (-2)(1)$$

$$= 1 - (-2)$$

$$= 1 + 2$$

$$= 3$$

$$|A_x| = \begin{vmatrix} 12 & -2 \\ 90 & 1 \end{vmatrix}$$

$$= (12)(1) - (-2)(90)$$

$$= 12 + 180$$

$$= 192$$

$$|A_y| = \begin{vmatrix} 1 & 12 \\ 1 & 90 \end{vmatrix}$$

$$= (90) - (12)$$

$$= 90 - 12$$

$$= 78$$

$$x = \frac{|A_x|}{|A|}$$

$$x = \frac{192}{3}$$

$$x = 64$$

$$y = \frac{|A_y|}{|A|}$$

$$y = \frac{78}{3}$$

$$y = 26$$

$$1^{\text{st}} \text{ angle} = x = 64^\circ$$

$$2^{\text{nd}} \text{ angle} = y = 26^\circ$$

Q.6 Two cars that are 600 km apart are moving towards each other. Their speeds differ by 6 km per hour and the cars are 123 km apart after $4\frac{1}{2}$

hours. Find the speed of each car.

Solution:

Suppose speed of 1st car = x

Suppose speed of 2nd car = y

According to 1st condition

$$x - y = 6 \quad \rightarrow (i)$$

According to 2nd condition

$$\text{Total distance} = 600 \text{ km}$$

$$\text{Left distance} = 123 \text{ km}$$

Covered distance = total distance - left distance

$$\text{Covered distance} = 600 - 123$$

$$= 477 \text{ km}$$

$$\text{Total time} = 4\frac{1}{2} \text{ hours} = \text{or } \frac{9}{2} \text{ hours}$$

$$\text{Total Speed} = \frac{\text{Total Distance Covered}}{\text{Total Time Taken}}$$

$$x + y = \frac{477}{\frac{9}{2}} = 477 \div \frac{9}{2} = 477 \times \frac{2}{9}$$

$$x + y = \frac{53 \cancel{477} \times 2}{\cancel{9}}$$

$$x + y = 106 \quad \rightarrow (ii)$$

$$x - y = 6$$

$$x + y = 106$$

By matrices inversion method

Changing into matrix form

$$\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 106 \end{bmatrix}$$

$X = A^{-1}B$, where

$$\text{Let } A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 6 \\ 106 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix}$$

$$= (1)(1) - (-1)(1)$$

$$= 1 - (-1)$$

$$= 1 + 1$$

$$= 2$$

$$\text{Adj}A = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

$$X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{|A|} \times \text{Adj}A \times B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 6 \\ 106 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} 6 + 106 \\ -6 + 106 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} 112 \\ 100 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{112}{2} \\ \frac{100}{2} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 56 \\ 50 \end{bmatrix}$$

$$x = 56, y = 50$$

Speed of 1st car = $x = 56$ km/h

Speed of 2nd car = $y = 50$ km/h

By Cramer's rule

$$A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} B = \begin{bmatrix} 6 \\ 106 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix}$$

$$= (1)(1) - (-1)(1)$$

$$= 1 - (-1)$$

$$= 1 + 1$$

$$= 2$$

$$|A_x| = \begin{vmatrix} 6 & -1 \\ 106 & 1 \end{vmatrix}$$

$$= (6)(1) - (-1)(106)$$

$$= 6 - (-106)$$

$$= 6 + 106$$

$$= 112$$

$$|A_y| = \begin{vmatrix} 1 & 6 \\ 1 & 106 \end{vmatrix}$$

$$= (106)(1) - (6)(1)$$

$$= 106 - 6$$

$$= 100$$

$$x = \frac{|A_x|}{|A|}$$

$$x = \frac{112}{2}$$

$$x = 56$$

$$y = \frac{|A_y|}{|A|}$$

$$y = \frac{100}{2}$$

$$y = 50$$

Then

Speed of 1st car = $x = 56$ km/h

Speed of 2nd car = $y = 50$ km/h

Review Exercise 1

Q.1 Select the correct answer in each of the following.

(i) The order of matrix $\begin{bmatrix} 2 & 1 \end{bmatrix}$ is....

- (a) 2-by-1
(c) 1-by-1

- (b) 1-by-2
(d) 2-by-2

(ii) $\begin{bmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{2} \end{bmatrix}$ is called ...matrix.

- (a) Zero
(c) Scalar

- (b) Unit
(d) Singular

(iii) Which is order of a square matrix?

- (a) 2-by-2
(c) 2-by-1

- (b) 1-by-2
(d) 3-by-2

(iv) Order of transpose of $\begin{bmatrix} 2 & 1 \\ 0 & 1 \\ 3 & 2 \end{bmatrix}$ is...

- (a) 3-by-2
(c) 1-by-3

- (b) 2-by-3
(d) 3-by-1

(v) Adjoint of $\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$ is...

(a) $\begin{bmatrix} -1 & -2 \\ 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & 2 \\ 0 & -1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & -2 \\ 0 & -1 \end{bmatrix}$

(d) $\begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}$

(vi) Product of $\begin{bmatrix} x & y \end{bmatrix}$ $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$ is...

(a) $[2x + y]$
(c) $[2x - y]$

(b) $[x - 2y]$
(d) $[x + 2y]$

(vii) If $\begin{bmatrix} 2 & 6 \\ 3 & x \end{bmatrix} = 0$, then x is equal to...

- (a) 9
(c) 6

- (b) -6
(d) -9

(viii) If $X + \begin{bmatrix} -1 & -2 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then X is equal to...

(a) $\begin{bmatrix} 2 & 2 \\ 2 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 2 \\ 2 & 2 \end{bmatrix}$

$$(c) \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$(d) \begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix}$$

ANSWER KEY

i	ii	iii	iv	v	vi	vii	viii
b	c	a	b	a	c	a	d

Q.2 Complete the following:

(i) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ is called ... matrix.

(ii) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is called ... matrix.

(iii) Additive inverse of $\begin{bmatrix} 1 & -2 \\ 0 & -1 \end{bmatrix}$ is....

(iv) In matrix multiplication, in general, $AB \dots BA$.

(v) Matrix $A+B$ may be found if order of A and B is...

(vi) A matrix is called ... matrix if number of rows and columns are equal.

ANSWER KEY

i	ii	iii	iv	v	vi
Null	Unit	$\begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}$	\neq	Same	Square

Q.3 If $\begin{bmatrix} a+3 & 4 \\ 6 & b-1 \end{bmatrix} = \begin{bmatrix} -3 & 4 \\ 6 & 2 \end{bmatrix}$, then find a and b .

Solution: $\begin{bmatrix} a+3 & 4 \\ 6 & b-1 \end{bmatrix} = \begin{bmatrix} -3 & 4 \\ 6 & 2 \end{bmatrix}$
 $a+3 = -3$ $b-1 = 2$
 $a = -3-3$ $b = 2+1$
 $a = -6$ $b = 3$ **Ans**

Solution: (i)

$$\begin{aligned} 2A+3B &= 2 \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} + 3 \begin{bmatrix} 5 & 4 \\ -2 & -1 \end{bmatrix} \\ &= \begin{bmatrix} 4 & 6 \\ 2 & 0 \end{bmatrix} + \begin{bmatrix} 15 & -12 \\ -6 & -3 \end{bmatrix} \\ &= \begin{bmatrix} 4+15 & 6-12 \\ 2-6 & 0-3 \end{bmatrix} \\ &= \begin{bmatrix} 19 & -6 \\ -4 & -3 \end{bmatrix} \text{ **Ans** } \end{aligned}$$

Q.4 If $A = \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 5 & -4 \\ -2 & -1 \end{bmatrix}$, then find the following.

- (i) $2A+3B$
(ii) $-3A+2B$
(iii) $-3(A+2B)$
(iv) $\frac{2}{3}(2A-3B)$

Solution: (ii)

$$\begin{aligned} -3A+2B &= -3 \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} + 2 \begin{bmatrix} 5 & -4 \\ -2 & -1 \end{bmatrix} \\ &= \begin{bmatrix} -6 & -9 \\ -3 & 0 \end{bmatrix} + \begin{bmatrix} 10 & -8 \\ -4 & -2 \end{bmatrix} \\ &= \begin{bmatrix} -6+10 & -9-8 \\ -3-4 & 0-2 \end{bmatrix} \end{aligned}$$

$$= \begin{bmatrix} 4 & -17 \\ -7 & -2 \end{bmatrix} \text{ Ans}$$

Solution: (iii)

$$\begin{aligned} -3(A+2B) &= -3 \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} + 2 \begin{bmatrix} 5 & -4 \\ -2 & -1 \end{bmatrix} \\ &= -3 \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 10 & -8 \\ -4 & -2 \end{bmatrix} \\ &= -3 \begin{bmatrix} 2+10 & 3-8 \\ 1-4 & 0-2 \end{bmatrix} \\ &= \begin{bmatrix} 12 & -5 \\ -3 & -2 \end{bmatrix} \\ &= \begin{bmatrix} -36 & 15 \\ 9 & 6 \end{bmatrix} \text{ Ans} \end{aligned}$$

Solution: (iv) $\frac{2}{3}(2A-3B)$

$$\begin{aligned} &= \frac{2}{3} \left(2 \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} - 3 \begin{bmatrix} 5 & -4 \\ -2 & -1 \end{bmatrix} \right) \\ &= \frac{2}{3} \left(\begin{bmatrix} 4 & 6 \\ 2 & 0 \end{bmatrix} - \begin{bmatrix} 15 & -12 \\ -6 & -3 \end{bmatrix} \right) \\ &= \frac{2}{3} \begin{bmatrix} 4-15 & 6-(-12) \\ 2-(-6) & 0-(-3) \end{bmatrix} \\ &= \frac{2}{3} \begin{bmatrix} -11 & 6+12 \\ 2+6 & 0+3 \end{bmatrix} \\ &= \frac{2}{3} \begin{bmatrix} -11 & 18 \\ 8 & 3 \end{bmatrix} \\ &= \begin{bmatrix} -11 \times \frac{2}{3} & 18 \times \frac{2}{3} \\ 8 \times \frac{2}{3} & 3 \times \frac{2}{3} \end{bmatrix} \\ &= \begin{bmatrix} \frac{-22}{3} & 12 \\ \frac{16}{3} & 2 \end{bmatrix} \text{ Ans} \end{aligned}$$

Q.5 Find the value of X, if

$$\begin{bmatrix} 2 & 1 \\ 3 & -3 \end{bmatrix} + X = \begin{bmatrix} 4 & -2 \\ -1 & -2 \end{bmatrix}.$$

Solution: Given that

$$\begin{aligned} \begin{bmatrix} 2 & 1 \\ 3 & -3 \end{bmatrix} + X &= \begin{bmatrix} 4 & -2 \\ -1 & -2 \end{bmatrix} \\ X &= \begin{bmatrix} 4 & -2 \\ -1 & -2 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 3 & -3 \end{bmatrix} \\ &= \begin{bmatrix} 4-2 & -2-1 \\ -1-3 & -2-(-3) \end{bmatrix} \\ &= \begin{bmatrix} 2 & -3 \\ -4 & -2+3 \end{bmatrix} \\ X &= \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix} \text{ Ans} \end{aligned}$$

Q.6 If $A = \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 4 \\ 5 & -2 \end{bmatrix}$,

then prove that

- (i) $AB \neq BA$
(ii) $A(BC) = (AB)C$

Solution: Given that

$$A = \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix}, B = \begin{bmatrix} -3 & 4 \\ 5 & -2 \end{bmatrix}$$

(i) $AB \neq BA$

$$\begin{aligned} \text{L.H.S} = AB &= \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} -3 & 4 \\ 5 & -2 \end{bmatrix} \\ &= \begin{bmatrix} 0 \times (-3) + 1 \times 5 & 0 \times 4 + 1 \times (-2) \\ 2 \times (-3) + (-3) \times 5 & 2 \times 4 + (-3) \times (-2) \end{bmatrix} \\ &= \begin{bmatrix} 0+5 & 0-2 \\ -6-15 & 8+6 \end{bmatrix} \\ &= \begin{bmatrix} 5 & -2 \\ -21 & 14 \end{bmatrix} \rightarrow \text{(i)} \end{aligned}$$

$$\begin{aligned} \text{R.H.S} = BA &= \begin{bmatrix} -3 & 4 \\ 5 & -2 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix} \\ &= \begin{bmatrix} -3(0) + 4(2) & -3(1) + 4(-3) \\ 5(0) + (-2)(2) & 5(1) + (-2)(-3) \end{bmatrix} \\ &= \begin{bmatrix} 0+8 & -3-12 \\ 0-4 & 5+6 \end{bmatrix} \\ &= \begin{bmatrix} 8 & -15 \\ -4 & 11 \end{bmatrix} \rightarrow \text{(ii)} \end{aligned}$$

From (i) and (ii), we get

$$L.H.S \neq R.H.S$$

$$AB \neq BA$$

Hence proved

$$(ii) A(BC) = (AB)C$$

Solution:

We cannot solve because matrix C is not given.

$$\text{Q.7 If } A = \begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 4 \\ -3 & -5 \end{bmatrix},$$

then verify that

$$(i) (AB)^t = B^t A^t$$

$$(ii) (AB)^{-1} = B^{-1} A^{-1}$$

Solution: Given that

$$A = \begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 4 \\ -3 & -5 \end{bmatrix}$$

$$(i) (AB)^t = B^t A^t$$

$$\begin{aligned} AB &= \begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ -3 & -5 \end{bmatrix} \\ &= \begin{bmatrix} 3(2)+2(-3) & 3(4)+2(-5) \\ 1(2)+(-1)(-3) & 1(4)+(-1)(-5) \end{bmatrix} \\ &= \begin{bmatrix} 6-6 & 12-10 \\ 2+3 & 4+5 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 2 \\ 5 & 9 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \text{L.H.S} &= (AB)^t = \begin{bmatrix} 0 & 2 \\ 5 & 9 \end{bmatrix}^t \\ &= \begin{bmatrix} 0 & 5 \\ 2 & 9 \end{bmatrix} \rightarrow (i) \end{aligned}$$

$$\begin{aligned} A^t &= \begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}^t \\ &= \begin{bmatrix} 3 & 1 \\ 2 & -1 \end{bmatrix} \end{aligned}$$

$$B^t = \begin{bmatrix} 2 & 4 \\ -3 & -5 \end{bmatrix}^t = \begin{bmatrix} 2 & -3 \\ 4 & -5 \end{bmatrix}$$

$$\text{R.H.S} = B^t A^t = \begin{bmatrix} 2 & -3 \\ 4 & -5 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 2 & -1 \end{bmatrix}$$

$$\begin{aligned} &= \begin{bmatrix} 2 \times 3 + (-3) \times 2 & 2(1) + (-3)(-1) \\ 4 \times 3 + (-5) \times 2 & 4(1) + (-5)(-1) \end{bmatrix} \\ &= \begin{bmatrix} 6-6 & 2+3 \\ 12-10 & 4+5 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 5 \\ 2 & 9 \end{bmatrix} \rightarrow (ii) \end{aligned}$$

From equal (i) and (ii) we get

$$\text{L.H.S} = \text{R.H.S}$$

$$(AB)^t = B^t A^t$$

Hence proved

$$(ii) (AB)^{-1} = B^{-1} A^{-1}$$

$$\begin{aligned} |AB| &= \begin{vmatrix} 0 & 2 \\ 5 & 9 \end{vmatrix} \\ &= 0 \times 9 - 2 \times 5 \\ &= 0 - 10 \\ &= -10 \text{ (Non singular)} \end{aligned}$$

Inverse exists

$$\text{Adj}(AB) = \begin{bmatrix} 9 & -2 \\ -5 & 0 \end{bmatrix}$$

$$\text{L.H.S} = (AB)^{-1} = \frac{1}{|AB|} \text{Adj}(AB)$$

$$\begin{aligned} &= \frac{1}{-10} \begin{bmatrix} 9 & -2 \\ -5 & 0 \end{bmatrix} \\ &= \begin{bmatrix} \frac{9}{-10} & \frac{-2}{-10} \\ \frac{-5}{-10} & \frac{0}{-10} \end{bmatrix} \\ &= \begin{bmatrix} -\frac{9}{10} & \frac{1}{5} \\ \frac{1}{2} & 0 \end{bmatrix} \rightarrow (i) \end{aligned}$$

$$B = \begin{bmatrix} 2 & 4 \\ -3 & -5 \end{bmatrix}$$

$$|B| = \begin{vmatrix} 2 & 4 \\ -3 & -5 \end{vmatrix}$$

$$\begin{aligned} &= 2(-5) - 4 \times (-3) \\ &= -10 + 12 \\ &= 2 \text{ (non singular)} \end{aligned}$$

$\therefore B^{-1}$ exists

$$AdjB = \begin{bmatrix} -5 & -4 \\ 3 & 2 \end{bmatrix}$$

$$B^{-1} = \frac{1}{|B|} AdjB$$

$$= \frac{1}{2} \begin{bmatrix} -5 & -4 \\ 3 & 2 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & 2 \\ 1 & -1 \end{vmatrix}$$

$$= 3(-1) - 2 \times 1$$

$$= -3 - 2$$

$$= -5 \text{ (non singular)}$$

$\therefore A^{-1}$ exists

$$AdjA = \begin{bmatrix} -1 & -2 \\ -1 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times AdjA$$

$$= \frac{1}{-5} \begin{bmatrix} -1 & -2 \\ -1 & 3 \end{bmatrix}$$

$$\text{R.H.S} = B^{-1}A^{-1}$$

$$= \left(\frac{1}{2} \begin{bmatrix} -5 & -4 \\ 3 & 2 \end{bmatrix} \right) \times \left(\frac{1}{-5} \begin{bmatrix} -1 & -2 \\ -1 & 3 \end{bmatrix} \right)$$

$$= \frac{1}{2} \left(-\frac{1}{5} \right) \begin{bmatrix} -5 & -4 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} -1 & -2 \\ -1 & 3 \end{bmatrix}$$

$$= -\frac{1}{10} \begin{bmatrix} -5(-1) + (-4)(-1) & -5(-2) + (-4)(3) \\ 3(-1) + 2(-1) & 3(-2) + 2(3) \end{bmatrix}$$

$$= -\frac{1}{10} \begin{bmatrix} 5+4 & 10-12 \\ -3-2 & -6+6 \end{bmatrix}$$

$$= -\frac{1}{10} \begin{bmatrix} 9 & -2 \\ -5 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{9}{-10} & \frac{-2}{-10} \\ \frac{-5}{-10} & \frac{0}{-10} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-9}{10} & \frac{1}{5} \\ \frac{1}{2} & 0 \end{bmatrix}$$

\rightarrow (ii)

From equation (i) and (ii) we get

L.H.S = R.H.S

$$(AB)^{-1} = B^{-1}A^{-1}$$

Hence proved