CHAPTER NO. 20(ATOMIC SPECTRA)

Question 20.1:- A hydrogen atom is in its ground state (n=1). Using Bohr's theory, calculate (a) the radius of the orbit, (b) the linear momentum of the electron, (c) the angular momentum of the electron, (d) the kinetic energy (e) the potential energy and (f) the total energy.

Solution:- Energy state of electron in hydrogen atom = n = 1

(a) Radius of n^{th} orbit in hydrogen atom is $r_n = \frac{n^2 h^2}{4\pi^2 k m e^2}$

$r_1 = \frac{(1)^2 (6.63 \, x \, 10^{-34})^2}{4\,(3.14)^2\,(9 \, x \, 10^9)\,(9.1 \, x \, 10^{-31})\,(1.6 \, x \, 10^{-19})^2}$

$r_1 = \frac{43.96 \, x \, 10^{-68}}{8268.81 \, x \, 10^{-60}} = 0.00529 \text{ x } 10^{-8}$ m

$r_1 = 0.529 \text{ x } 10^{-10}$ m

(b) Linear momentum of electron in 1^{st} hydrogen orbit = $P_1 = m \, v_1$

Second postulate of Bohr's theory of hydrogen atom is $m \, v_n \, r_n = \frac{n h}{2\pi}$

Put n = 1

$m \, v_1 \, r_1 = \frac{h}{2\pi}$

$P_1 = m \, v_1 = \frac{h}{2\pi r_1} = \frac{6.63 \, x \, 10^{-34}}{4\,(3.14)\,(0.529 \, x \, 10^{-10})}$

$P_1 = 1.99 \text{ x } 10^{-24}$ kg m s^{-1}

(c) Angular momentum of electron in 1^{st} hydrogen orbit = $L_1 = m \, v_1 \, r_1$

Second postulate of Bohr's theory of hydrogen atom is $m \, v_n \, r_n = \frac{n h}{2\pi}$

Put n = 1

$m \, v_1 \, r_1 = \frac{h}{2\pi}$

$L_1 = m \, v_1 \, r_1 = \frac{h}{2\pi} = \frac{6.63 \, x \, 10^{-34}}{4\,(3.14)}$

$P_1 = 1.05 \text{ x } 10^{-34}$ kg m^2 s^{-1}

(d) K.E. of electron in n^{th} hydrogen orbit = $K.E._n = \frac{k e^2}{2 r_n}$

Put n = 1

$K.E._1 = \frac{k e^2}{2 r_1} = \frac{(9 \, x \, 10^9)(1.6 \, x \, 10^{-19})^2}{2\,(0.529 \, x \, 10^{-10})}$

$K.E._1 = 21.78 \text{ x } 10^{-19}$ J

$K.E._1 = (21.78 \text{ x } 10^{-19})/(1.6 \text{ x } 10^{-19})$ eV

$K.E._1 = +13.6$ eV

(e) P.E. of electron in n^{th} hydrogen orbit = $P.E._n = -\frac{k e^2}{r_n}$

Put n = 1

$P.E._1 = -\frac{k\,e^2}{r_1} = \frac{(9\ x\ 10^9)(1.6\ x\ 10^{-19})^2}{(0.529\ x\ 10^{-10})}$

$P.E._1 = -\ 43.56 \times 10^{-19}$ J

$P.E._1 = -\ (43.56 \times 10^{-19})/(1.6 \times 10^{-19})$ eV

$P.E._1 = -\ 27.2$ eV

(f) The total energy of electron in n^{th} hydrogen orbit = $E_n = K.E._n + P.E._n$

Put n = 1

$E_1 = K.E._1 + P.E._1$

$E_1 = (+13.6\text{ eV}) + (-27.2\text{ eV})$

$E_1 = -13.6$ eV

Question 20.2:- What are the energies in eV of quanta of wavelength λ=400, 500 and 700 nm.

Solution:- (a) Wavelength = λ = 400 nm = 4×10^{-7} m

$E = \frac{h\,c}{\lambda} = \frac{(6.63\ x\ 10^{-34})\ (3\ x\ 10^8)}{4\ x\ 10^{-7}} = \frac{1.989\ x\ 10^{-25}}{4\ x\ 10^{-7}} = 4.97 \times 10^{-19}$ J

$E = \frac{4.97 \times 10^{-19}}{1.6 \times 10^{-19}}$ eV

E = 3.1 eV

(b) Wavelength = λ = 500 nm = 5×10^{-7} m

$E = \frac{h\,c}{\lambda} = \frac{(6.63\ x\ 10^{-34})\ (3\ x\ 10^8)}{5\ x\ 10^{-7}} = \frac{1.989\ x\ 10^{-25}}{5\ x\ 10^{-7}} = 3.98 \times 10^{-19}$ J

$E = \frac{3.98 \times 10^{-19}}{1.6 \times 10^{-19}}$ eV

E = 2.49 eV

(c) Wavelength = λ = 700 nm = 7×10^{-7} m

$E = \frac{h\,c}{\lambda} = \frac{(6.63\ x\ 10^{-34})\ (3\ x\ 10^8)}{7\ x\ 10^{-7}} = \frac{1.989\ x\ 10^{-25}}{7\ x\ 10^{-7}} = 2.84 \times 10^{-19}$ J

$E = \frac{2.84 \times 10^{-19}}{1.6 \times 10^{-19}}$ eV

E = 1.77 eV

Question 20.3:- An electron jumps from a level $E_i = -3.5 \times 10^{-19}$ J to $E_f = -1.20 \times 10^{-18}$ J. What is the wavelength of emitted light?

Solution:- Energy of higher energy level = $E_i = -3.5 \times 10^{-19}$ J

Energy of lower energy level = $E_f = -1.20 \times 10^{-18}$ J

Energy gap = $\Delta E = E_i - E_f = (-3.5 \times 10^{-19}) - (-1.20 \times 10^{-18})$

$\Delta E = 10^{-18}\ (-3.5 \times 10^{-1}) + (1.20 \times 10^{-18}) = 0.85 \times 10^{-18}$ J

$\Delta E = 8.5 \times 10^{-19}$ J

$\Delta E = \frac{h\,c}{\lambda}$

$\lambda = \frac{h\,c}{\Delta E} = \frac{(6.63 \times 10^{-34})\,(3 \times 10^{8})}{8.5 \times 10^{-19}} = \frac{1.989 \times 10^{-25}}{8.5 \times 10^{-19}}$

$\lambda = 0.234 \times 10^{-6}$ m $= 234 \times 10^{-9}$ m

$\lambda = 234$ nm

Question 20.4:- Find the wavelength of the spectral line corresponding to the transition in hydrogen from n = 6 state to n = 3 state?

Solution:- Higher energy state = n = 6

Lower energy state = p = 6

The result for hydrogen emission spectrum is $\frac{1}{\lambda} = R_H\,(\frac{1}{p^2} - \frac{1}{n^2})$

$\frac{1}{\lambda} = (1.0974 \times 10^7)\,(\frac{1}{3^2} - \frac{1}{6^2}) = (1.0974 \times 10^7)\,(\frac{1}{9} - \frac{1}{36}) = (1.0974 \times 10^7)\,(\frac{4-1}{36})$

$\frac{1}{\lambda} = (1.0974 \times 10^7)\,(\frac{1}{12})$

$\lambda = 12/(1.0974 \times 10^7)$

$\lambda = 10.94 \times 10^{-7}$ m $= 1094 \times 10^{-9}$ m

$\lambda = 1094$ nm

Question 20.5:- Compute the shortest wavelength radiation in Balmer series? What value of n must be used?

Solution:- The Rydberg's result for Balmer series is $\frac{1}{\lambda} = R_H\,(\frac{1}{2^2} - \frac{1}{n^2})$ where n = 3, 4, 5,……

Shortest wavelength will be emitted when energy gap is largest, it means we must choose **n = ∞** for emission of shortest wavelength.

$\frac{1}{\lambda_{min}} = R_H\,(\frac{1}{2^2} - \frac{1}{\infty^2}) = R_H\,(\frac{1}{2^2})$

$\lambda_{min} = 4/R_H = 4/(1.0974 \times 10^7)$

$\lambda_{min} = 3.645 \times 10^{-7}$ m $= 364.5 \times 10^{-9}$ m

$\lambda_{min} = 364.5$ nm

Question 20.6:- Calculate the longest wavelength of radiation for the Paschen series.

Solution:- The Rydberg's result for Paschen series is $\frac{1}{\lambda} = R_H\,(\frac{1}{3^2} - \frac{1}{n^2})$ where n = 4, 5, 6,……

Longest wavelength will be emitted when energy gap is minimum, it means we must choose **n = 4** for emission of longest wavelength.

$\frac{1}{\lambda_{max}} = R_H\,(\frac{1}{3^2} - \frac{1}{4^2}) = R_H\,(\frac{1}{9} - \frac{1}{16}) = (1.0974 \times 10^7)\,(\frac{16-9}{144})$

$\lambda_{max} = 144/7R_H = 144/(7)(1.0974 \times 10^7)$

$\lambda_{max} = 18.75 \times 10^{-7}$ m $= 1875 \times 10^{-9}$ m

$\lambda_{max} = 1875$ nm

Question 20.7:- Electrons in an X-ray tube are accelerated through a potential difference of 3000 V. If the electrons were slowed down in a target, what will be the minimum wavelength of X-ray produced?

Solution:- Accelerating voltage = V = 3000 V

The minimum wavelength of Bremsstrahlung radiations is $\lambda_{min} = \frac{h\,c}{V\,e}$

$$\lambda_{min} = \frac{h\,c}{V\,e} = \frac{(6.63\ x\ 10^{-34})\ (3\ x\ 10^{8})}{(3.0\ x\ 10^{3})(1.6\ x\ 10^{-19})} = \frac{1.989\ x\ 10^{-25}}{4.8\ x\ 10^{-16}}$$

$\lambda_{min} = 0.414 \text{ x } 10^{-9}$ m

$\lambda_{min} = 4.14 \text{ x } 10^{-10}$ m

Question 20.8:- The wavelength of K X-ray from copper is 1.377 x 10^{-10} m. What is the energy difference between two levels from which this transition results?

Solution:- Wavelength of emitted characteristic radiation = λ = 1.277 x 10^{-10} m

$$\Delta E = \frac{h\,c}{\lambda}$$

$$\Delta E = \frac{h\,c}{\lambda} = \frac{(6.63\ x\ 10^{-34})\ (3\ x\ 10^{8})}{1.377\ x\ 10^{-10}} = \frac{1.989\ x\ 10^{-25}}{1.377\ x\ 10^{-10}}$$

$\Delta E = 1.44 \text{ x } 10^{-15}$ J

$$\Delta E = \frac{1.44\ x\ 10^{-15}}{1.6\ x\ 10^{-19}} = 0.903 \text{ x } 10^{4} \text{ eV} = 9.03 \text{ x } 10^{3} \text{ eV}$$

ΔE = 9.03 keV

Question 20.9:- A tungsten target is struck by electrons that have been accelerated from the rest through 40 kV potential difference. Find the shortest wavelength of the bremsstrahlung radiation emitted.

Solution:- Accelerating voltage = V = 40 kV = 4.0 x 10^{4} V

The minimum wavelength of Bremsstrahlung radiations is $\lambda_{min} = \frac{h\,c}{V\,e}$

$$\lambda_{min} = \frac{h\,c}{V\,e} = \frac{(6.63\ x\ 10^{-34})\ (3\ x\ 10^{8})}{(4.0\ x\ 10^{4})(1.6\ x\ 10^{-19})} = \frac{1.989\ x\ 10^{-25}}{6.4\ x\ 10^{-15}}$$

$\lambda_{min} = 0.31 \text{ x } 10^{-10}$ m

Question 20.10:- The orbital electron of hydrogen atom moves with a speed of 5.456 x 10^{5} m s^{-1}. (a) find the value of quantum number n associated with this electron (b) calculate the radius of this orbit (c) find energy of electron in this orbit.

Solution:- Speed of electron = v_n = 5.456 x 10^{5} m s^{-1}

(a) Speed of electron in n^{th} orbit in hydrogen atom is $v_n = \frac{2\,\pi\,k\,e^2}{n\,h}$

$$n = \frac{2\,\pi\,k\,e^2}{v_n\,h} = \frac{(2)(3.14)(9\ x\ 10^{9})(1.6\ x\ 10^{-19})(1.6\ x\ 10^{-19})}{(5.456\ x\ 10^{5})(6.63\ x\ 10^{-34})}$$

$$n = \frac{(144.69\ x\ 10^{-29})}{(36.17\ x\ 10^{-29})}$$

$n = 4$

(b) Radius of n^{th} orbit in hydrogen atom is $r_n = n^2 r_1$ where $r_1 = 0.053$ nm

$r_4 = (4)^2 (0.053 \text{ nm})$

$r_4 = 0.846$ nm

(c) Energy of electron in n^{th} orbit in hydrogen atom is $E_n = -\frac{E_o}{n^2}$ where $E_o = 13.6$ eV

$E_4 = -13.6 \text{ eV}/(4)^2$

$E_4 = -0.85$ eV