CHAPTER NO. 7(OSCILLATIONS)

Question 7.1:- A 100.0 g body hung on a spring elongates the spring by 4.0 cm. When a certain object is hung on the spring and is set vibrating, its period is 0.568 s. What is the mass of the object pulling the spring?

Solution:- Mass of hanging object = M = 100.0 g = 0.1 kg

Elongation in the spring = x = 4.0 cm = 0.04 m

According to Hook's law F = k x

k = F/x = Mg/x = (0.1)(9.8)/(0.04) = 24.5 N m^{-1}

Time period of oscillation = T = 0.568 s

Mass of hanging object = m

Time period of oscillation of spring mass system is $T = 2\pi \sqrt{\frac{m}{k}}$

$T^2 = 4\pi^2 \frac{m}{k}$

$m = kT^2/4\pi^2$

$m = (24.5)(0.568)^2/(4)(3.14)^2$

m = 0.20 kg

Question 7.2:- A load of 15.0 g elongates a spring by 2.00 cm. If a body of mass 294 g is attached to the spring and is set into vibration with an amplitude of 10.0 cm, what will be its (i) period (ii) spring constant (iii) maximum speed of its vibration.

Solution:- Load = M = 15.0 g = 0.015 kg

Extension in the spring = x = 2.00 cm = 0.02 m

According to Hook's law F = k x

k = F/x = Mg/x = (0.015)(9.8)/(0.02) = 7.35 N m^{-1}

Amplitude of vibration = x_o = 10.0 cm = 0.10 m

Mass of vibrating object = m = 294 g = 0.294 kg

(i) Time period of oscillation of spring mass system is $T = 2\pi \sqrt{\frac{m}{k}}$

$T = 2(3.14) \sqrt{\frac{0.294}{7.35}} = (6.28)(0.2)$

T = 1.26 s

(ii) Spring constant = k = 7.35 N m^{-1}

(iii) Maximum speed of vibration = $v_o = x_o \sqrt{\frac{k}{m}}$

$v_o = (0.10) \sqrt{\frac{7.35}{0.294}} = (0.10)(5) = 0.5$ m s^{-1}

$v_o = 50$ cm s^{-1}

Question 7.3:- An 8.0 kg body executes SHM with amplitude 30 cm. The restoring force is 60 N when the displacement is 30 cm. Find (i) Period (ii) Acceleration, speed, kinetic energy and potential energy when the displacement is 12 cm.

Solution:- Mass of the object = m = 8.0 kg

Amplitude of vibration = x_o = 30 cm = 0.30 m

Restoring force = F = 60 N

Extension = x = 30 cm = 0.30 m

According to Hook's law F = k x

k = F/x = (60)/(0.30) = 200 N m^{-1}

(i) Time period of oscillation of spring mass system is $T = 2\pi\sqrt{\frac{m}{k}}$

$T = 2\pi\sqrt{\frac{8}{200}} = (6.28)(0.2) = 1.256$ s

T = 1.3 s

(ii) Instantaneous displacement of object = x = 12 cm = 0.12 m

Acceleration = a = F/x = kx/m = (200)(0.12)/8

a = 3.0 m s^{-2}

Speed = $v = \omega\sqrt{x_o^2 - x^2} = \sqrt{\frac{k}{m}}\sqrt{x_o^2 - x^2} = \sqrt{\frac{200}{8}}\sqrt{(0.30)^2 - (0.12)^2} = 1.37$ m s^{-1}

v = 1.4 m s^{-1}

Kinetic energy = K.E. = $\frac{1}{2}$ m v^2 = $\frac{1}{2}(8)(1.4)^2$

K.E. = 7.6 J

Potential energy = P.E. = $\frac{1}{2}$ k x^2 = $\frac{1}{2}(200)(0.12)^2$

P.E. = 1.44 J

Question 7.4:- A block of mass 4.0 kg is dropped from a height of 0.80 m on to a spring of spring constant k = 1960 N m^{-1}, find the maximum distance through which the spring will be compressed.

Solution:- Mass of the block = m = 4.0 kg

Height of block above the spring = h = 0.80 kg

Spring constant = k = 1960 N m^{-1}

The block at the top of the spring has gravitational potential energy. When block is dropped on the spring, its gravitational potential energy is converted into maximum elastic potential energy of the spring as the spring will be compressed.

$P.E._{grav} = P.E._{elas}$

$m\,g\,h = \frac{1}{2}k\,x_o^2$

$(4)\ (9.8)\ (0.80) = \frac{1}{2}(1960)\ (x_o^2)$

$x_o^2 = 0.032\ m^2$

<u>$x_o = 0.18$ m</u>

Question 7.5:- A simple pendulum is 50.0 cm long. What will be its frequency of vibration at a place where g = 9.8 m s^{-2}?

Solution:- Length of simple pendulum = l = 50.0 cm = 0.50 m

Acceleration due to gravity = g = 9.8 m s^{-2}

The frequency of oscillation of simple pendulum is $f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$

$f = \frac{1}{(2)(3.14)}\sqrt{\frac{9.8}{0.50}} = 4.43/6.28$

<u>f = 0.70 Hz</u>

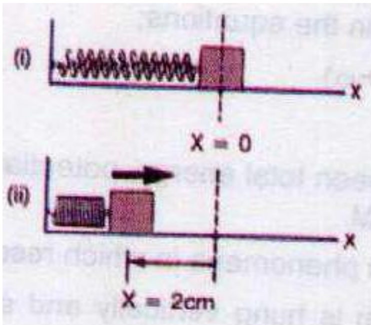
Alternately, the time period of oscillation of simple pendulum is given as $T = 2\pi\sqrt{\frac{l}{g}}$

$T = 2(3.14)\sqrt{\frac{0.50}{9.8}} = (6.28)\ (0.23) = 1.4$ s

f = 1/T = 1/1.4

<u>f = 0.70 Hz</u>

Question 7.6:- A block of mass 1.6 kg is attached to a spring with spring constant 1000 N m^{-1}, as shown in figure. The spring is compressed through a distance of 2.0 cm and the block is released from rest. Calculate the velocity of the block as it passes through the equilibrium position, x = 0, if the surface is frictionless.



Solution:- Mass of the block = m = 1.6 kg

Spring constant = k = 1000 N m^{-1}

Amplitude of vibration = x_o = 2.0 cm = 0.02 m

Velocity at mean position = $v_o = x_o\sqrt{\frac{k}{m}} = (0.02)\sqrt{\frac{1000}{1.6}}$

$v_o = (0.02)\ (25)$

<u>$v_o = 0.50$ m s^{-1}</u>

Question 7.7:- A car of mass 1300 kg is constructed using a frame supported by four springs. Each spring has a spring constant 20,000 N m^{-1}. If two people riding in the car have a

combined mass of 160 kg, find the frequency of vibration of car, when it is driven over a pot hole in the road. Assume the weight is evenly distributed.

Solution:- Mass of the car = M_{car} = 1300 kg

Mass of the riders = M_{riders} = 160 kg

Total mass = m = $M_{car} + M_{riders}$ = 1300 + 160 = 1460 kg

Spring constant of one spring = k_1 = 20000 N m^{-1}

Number of springs = n = 4

When n-identical springs are connected in parallel, net spring constant is k = n k_1 = (4)(20000) = 80000 N m^{-1}

Time period of oscillation of spring mass system is $T = 2\pi \sqrt{\frac{m}{k}}$

$T = 2(3.14) \sqrt{\frac{1460}{80000}} = (6.28)(0.14)$

T = 0.85 s

f = 1/T = 1/0.85 = 1.178 Hz

<u>**f = 1.18 Hz**</u>

Question 7.8:- Find the amplitude, frequency and period of an object vibrating at the end of a spring, if the equation for its position, as a function of time, is $x = 0.25 \cos (\frac{\pi}{8})t$. What is the displacement of the object after 2.0 s?

Solution:- The instantaneous displacement of harmonic oscillator is given as $x = 0.25 \cos (\frac{\pi}{8})t$

The standard representation of instantaneous displacement of harmonic oscillator (started from extreme position) is given as $x = x_o \cos \omega t$

<u>**Comparing both equations gives x_o = 0.25 m and $\omega = \frac{\pi}{8}$ rad s^{-1}**</u>

<u>**Amplitude = x_o = 0.25 m**</u>

Frequency = f = ω/2π = $\frac{1}{2\pi}(\frac{\pi}{8})$

<u>**f = 1/16 Hz**</u>

Time period = T = 1/f = 1/(1/16)

<u>**T = 16 s**</u>

Instantaneous displacement at t = 2 s is $x = 0.25 \cos (\frac{\pi}{8})(2) = 0.25 \cos (\frac{\pi}{4}) = (0.25)(0.707)$

<u>**x = 0.18 m**</u>