CHAPTER NO. 6(FLUID DYNAMICS)

Question 6.1:- Certain globular protein particle has a density of 1246 kg m^{-3}. It falls through pure water (η = 8.0 x 10^{-4} N m^{-2} s) with a terminal speed of 3.0 cm h^{-1}. Find the radius of the particle.

Solution:- Density of particle = ρ = 1246 kg m^{-3}

Coefficient of viscosity of water = η = 8.0 x 10^{-4} N m^{-2} s

Terminal speed = v_t = 3.0 cm h^{-1} = 3.0 $\frac{0.01}{3600}$ m s^{-1} = 8.33 x 10^{-6} m s^{-1}

$r^2 = \frac{9\,\eta\,v_t}{2\,\rho\,g}$

$r = \sqrt{\frac{9\,\eta\,v_t}{2\,\rho\,g}} = \sqrt{\frac{9\,(8.0\,x\,10^{-4})\,(8.33\,x\,10^{-6})}{2\,(1246)\,(9.8)}}$

r = 1.6 x 10^{-6} m

Question 6.2:- Water flows through a hose, whose internal diameter is 1 cm at a speed of 1 m s^{-1}. What should be the diameter of the nozzle if the water is to emerge at 21 m s^{-1}?

Solution:- Diameter at first point = d_1 = 1 cm = 0.01 m

Speed at first point = v_1 = 1 m s^{-1}

Diameter at second point = d_2

Speed at second point = v_2 = 21 m s^{-1}

$A_2 v_2 = A_1 v_1$

$v_2\,(\pi d_2^2/4) = v_1\,(\pi d_1^2/4)$

$v_2 d_2^2 = v_1 d_1^2$

$(21)\, d_2^2 = (1)(0.01)^2$

$d_2^2 = 10^{-4}/21$

d_2^2 = 4.76 x 10^{-6} m^2

d_2 = 0.002 m

d_2 = 2 cm

Question 6.3:- The pipe near the lower end of a large water storage tank develops a small leak and a stream of water shoots from it. The top of water in the tank is 15 m above the point of leak.

a) With what speed the does the water rush from the hole?

b) If the hole has an area of 0.060 cm^2, how much water flows out in one second?

Solution:- Height of water above the hole = h = 15 m

(a) The speed of ejection of water = v = $\sqrt{2\,g\,h} = \sqrt{2\,(9.8)(15)}$

v = 17 m s^{-1}

(b) Area of hole = A = 0.060 cm^2 = 0.060 x 10^{-4} m^2

Volume flow in one second = A v = (0.060 x 10^{-4})(17)

Volume flow in one second = 1.02 x 10^{-4} m^3

Volume flow in one second = 102 x 10^{-6} m^3

Volume flow in one second = 102 cm^3

Question 6.4:- Water is flowing smoothly through a closed pipe system. At one point the speed of water is 3.0 m s^{-1}, while at another point 3.0 m higher, the speed is 4.0 m s^{-1}. If the pressure is 80 kPa at the lower point, what is the pressure at the upper point?

Solution:- Speed of water at lower point = v_1 = 3.0 m s^{-1}

Height of water at lower point = h_1 = 0 m

Pressure of water at lower point = P_1 = 80 kPa = 80000 Pa

Speed of water at upper point = v_2 = 4.0 m s^{-1}

Height of water at upper point = h_2 = 3.0 m

Density of pure water = ρ = 1000 kg m^{-3}

Pressure of water at upper point = P_2

The Bernoulli's equation states that $\boldsymbol{P_1 + \frac{1}{2} \rho\, v_1^2 + \rho\, g\, h_1 = P_2 + \frac{1}{2} \rho\, v_2^2 + \rho\, g\, h_2}$

$P_2 = P_1 + \rho\, g\, (h_1 - h_2) + \frac{1}{2} \rho\, (v_1^2 - v_2^2)$

$P_2 = 80000 + (1000)\,(9.8)\,(0 - 3) + \frac{1}{2}(1000)\,(9 - 16)$

$P_2 = 80000 - 29400 - 3500 = 47100$ Pa

P_2 = 47 kPa

Question 6.5:- An airplane wing is designed so that when the speed of air across the top of the wing is 450 m s^{-1}, the speed of air below the wing is 410 m s^{-1}. What is the pressure difference between the top and bottom of the wings? (Density of air = 1.29 kg m^{-3})

Solution:- Speed of air on upper part of wing = v_1 = 450 m s^{-1}

Speed of air on lower part of wing = v_2 = 410 m s^{-1}

Density of air = ρ = 1.29 kg m^{-3}

Pressure difference between the bottom and the top = $P_2 - P_1$

The Bernoulli's equation states that $\boldsymbol{P_1 + \frac{1}{2} \rho\, v_1^2 + \rho\, g\, h_1 = P_2 + \frac{1}{2} \rho\, v_2^2 + \rho\, g\, h_2}$

$P_2 - P_1 = \rho\, g\, (h_1 - h_2) + \frac{1}{2} \rho\, (v_1^2 - v_2^2)$

For horizontal flow of air, $h_1 = h_2$ i.e, $h_1 - h_2 = 0$

$P_2 - P_1 = \frac{1}{2} \rho\, (v_1^2 - v_2^2) = \frac{1}{2}(1.29)\,[(450)^2 - (410)^2]$

$P_2 - P_1 = (0.645)\,(34400) = 22188$ Pa

$P_2 - P_1$ = 22 kPa

Question 6.6:- The radius of aorta is about 1.0 cm and the blood flowing through it has a speed of about 30 cm s^{-1}. Calculate the average speed of the blood in the capillaries using the fact that although each capillary has a diameter of about 8.0 x 10^{-4} cm, there are literally millions of them so that their total cross section is about 2000 cm^2.

Solution:- Radius of aorta = r_1 = 1.0 cm = 0.01 m

Speed of blood through aorta = v_1 = 30 cm s^{-1} = 0.30 m s^{-1}

Area of cross section of aorta = $A_1 = \pi r_1^2 = 3.14 (0.01)^2 = 3.14 \times 10^{-4} m^2$

Area of cross section of capillaries = A_2 = 2000 cm^2 = 2000 x 10^{-4} m^2

Speed of blood in capillaries = v_2

We can use equation of continuity as $A_1 v_1 = A_2 v_2$

$(3.14 \times 10^{-4}) (0.30) = (2000 \times 10^{-4}) v_2$

$v_2 = (3.14)(0.30)/2000 = 0.00047 m s^{-1} = 4.7 \times 10^{-4} m s^{-1}$

$v_2 = 5 \times 10^{-4} m s^{-1}$

Question 6.7:- How large must a heating duct be if air moving 3.0 m s^{-1} along it can replenish the air in a room of 300 m^3 volume every 15 min? Assume that the air's density remains constant.

Solution:- Speed of the air = v = 3.0 m s^{-1}

Volume of the room = V = 300 m^3

Time = t = 15 min = 900 s

We know that A v = V/t

$\pi r^2 (v) = V/t$

$(3.14) r^2 (3.0) = 300/900$

$r^2 = 1/(3)(3.0)(3.14)$

$r^2 = 0.035 m^2$

r = 0.188 m

r = 19 cm

Question 6.8:- An airplane design calls for a "lift" due to the net force on the moving air on the wing of about 1000 N m^{-2} of wing area. Assume that air flows past the wing of an aircraft with streamline flow. If the speed of flow past the lower wing surface is 160 m s^{-1}, what is the required speed over the upper surface to give a "lift" of 1000 N m^{-2}? The density of air is 1.29 kg m^{-3} and assume maximum thickness of the wing to be one metre.

Solution:- Speed of air pas the lower surface = v_1 = 160 m s^{-1}

Density of air = ρ = 1.29 kg m^{-3}

Pressure difference = $P_1 - P_2$ = 1000 N m^{-2}

Thickness of wing = $h_2 - h_1 = 1$ m

The Bernoulli's equation states that $\boldsymbol{P_1 + \frac{1}{2}\rho\, v_1^2 + \rho\, g\, h_1 = P_2 + \frac{1}{2}\rho\, v_2^2 + \rho\, g\, h_2}$

$(P_1 - P_2) + \rho\, g\, (h_1 - h_2) + \frac{1}{2}\rho\, v_1^2 = \frac{1}{2}\rho\, v_2^2$

$\frac{1}{2}(1.29)\, v_2^2 = 1000 + (1.29)\,(9.8)\,(1) + \frac{1}{2}(1.29)\,(160)^2$

$0.645\, v_2^2 = 1000 + 11.61 + 16512$

$0.645\, v_2^2 = 17523.61$

$v_2^2 = 27168.39\ m^2\, s^{-2}$

$v_2 = 164.8\ m\, s^{-1}$

$v_2 = 165\ m\, s^{-1}$

Question 6.9:- What gauge pressure is required in the city mains for a stream from a fire hose connected to the mains to reach a vertical height of 15.0 m?

Solution:- Density of water = $\rho = 1000\ kg\, m^{-3}$

Gauge pressure = $P = P_2 - P_1$

Vertical height = $h = h_1 - h_2 = 15.0$ m

The Bernoulli's equation states that $\boldsymbol{P_1 + \frac{1}{2}\rho\, v_1^2 + \rho\, g\, h_1 = P_2 + \frac{1}{2}\rho\, v_2^2 + \rho\, g\, h_2}$

$P_2 - P_1 = \rho\, g\, (h_1 - h_2) + \frac{1}{2}\rho\, (v_1^2 - v_2^2)$

$P_2 - P_1 = \rho\, g\, (h_1 - h_2)$ $\quad \therefore \frac{1}{2}\rho\, (v_1^2 - v_2^2) = 0$ as speed remains same everywhere in mains

$P_2 - P_1 = (1000)(9.8)(15.0)$

$P_2 - P_1 = 147000$ Pa

$P = P_2 - P_1 = 1.47 \times 10^5$ Pa