CHAPTER NO. 5(CIRCULAR MOTION)

Question 5.1:- A tiny laser beam is directed from the Earth to the Moon. If the beam is to have a diameter of 2.50 at the Moon, how small must the divergence angle be for the beam? The distance of Moon from the Earth is 3.8×10^8 m.

Solution:- Diameter of beam on moon = Arc length = S = 2.50 m

The distance between Moon and Earth = Radius of circular curve = r = 3.8×10^8 m

$S = r\,\theta$

$\theta = S / r$

$\theta = (2.50)/(3.8 \times 10^8)$

$\theta = 0.66 \times 10^{-8}$ rad

$\theta = 6.6 \times 10^{-9}$ rad

Question 5.2:- A gramophone record turntable accelerates from rest to an angular velocity of 45.0 rev min^{-1} in 1.60 s. What is its average angular acceleration?

Solution:- Initial velocity = $\omega_i = 0$ rad s^{-1}

Final velocity = ω_f = 45.0 rev min^{-1} = $45.0 \times \frac{2\pi}{60}$ rad s^{-1}

$\omega_f = 4.71$ rad s^{-1}

Time taken = $\Delta t = 1.60$ s

Average angular acceleration = $\alpha = \Delta\omega/\Delta t = 4.71/1.60$

$\alpha = 2.94$ rad s^{-2}

Question 5.3:- A body of moment of inertia I = 0.80 kg m^2 about a fixed axis, rotates with a constant angular velocity of 100 rad s^{-1}. Calculate its angular momentum L and the torque to sustain this motion.

Solution:- Moment of inertia = I = 0.80 kg m^2

Angular velocity = ω = 100 rad s^{-1}

Angular momentum = $L = I\,\omega = (0.80)\,(100)$

L = 80 J s

Torque = $\tau = I\,\alpha = I\,(\Delta\omega/\Delta t) = (0.80)(0/\Delta t)$ ∴ ω is constant so $\Delta\omega$ is zero.

$\tau = 0$ N m

Question 5.4:- Consider the rotating cylinder shown in figure. Suppose that m = 5.0 kg, F = 0.60 N and r = 0.20 m. Calculate (a) the torque acting on the cylinder, (b) the angular acceleration of the cylinder. (Moment of inertia of the cylinder = $\frac{1}{2}$ m r^2)

Solution:- Mass of the cylinder = M = 5.0 kg

Force = F = 0.60 N

Radius = r = 0.20 m

(a) Torque = $\tau = r\,F = (0.20)(0.60)$

$\tau = 0.12$ N m

(b) Angular acceleration = α

Moment of inertia = $I = \frac{1}{2}\,m\,r^2 = \frac{1}{2}\,(5)\,(0.20)^2 = 0.1$ kg m^{-2}

$\tau = I\,\alpha$

$\alpha = \tau/I = 0.12/0.1$

$\alpha = 1.2$ rad s^{-2}

Question 5.5:- Calculate the angular momentum of a star of mass 2.0 x 10^{30} kg and radius 7.0 x 10^5 km. If it makes one complete rotation about its axis once in 20 days, what is its kinetic energy?

Solution:- Mass of the star = m = 2.0 x 10^{30} kg

Radius of the star = R = 7.0 x 10^5 km = 7.0 x 10^8 m

Time period about its own axis = T = 20 days = 20 x 86400 s = 1.728 x 10^6 s

Angular velocity = $\omega = 2\pi/T = {}^{2\pi}/_{(1.728\,x\,10^6)} = 3.63 \times 10^{-6}$ rad s^{-1}

Angular momentum = $L = I\,\omega = (\frac{2}{5}\,M\,R^2)\,\omega = \frac{2}{5}\,(2.0 \times 10^{30})\,(7.0 \times 10^8)^2\,(3.63 \times 10^{-6})$

L = 142 x 10^{40} J s

L = 1.42 x 10^{42} J s

Rotational kinetic energy = $K.E._{rot} = \frac{1}{2}\,I\,\omega^2 = \frac{1}{2}\,(\frac{2}{5}\,M\,R^2)\,\omega^2 = \frac{1}{5}\,M\,R^2\,\omega^2$

$K.E._{rot} = \frac{1}{5}\,(2.0 \times 10^{30})\,(7.0 \times 10^8)^2\,(3.63 \times 10^{-6})^2 = 258 \times 10^{34}$ J

$K.E._{rot} = 2.58 \times 10^{36}$ J

Question 5.6:- A 1000 kg car travelling with a speed of 144 km h^{-1} round a curve of radius 100 m. Find the necessary centripetal force.

Solution:- Mass of the car = m = 1000 kg

Speed of the car = v = 144 km h^{-1} = 144 $\frac{1000}{3600}$ m s^{-1} = 40 m s^{-1}

Radius of circular path = r = 100 m

Centripetal force = $F_C = m\,\frac{v^2}{r} = (1000)\,(\frac{40^2}{100}) = 16000$ N

$F_C = 1.6 \times 10^4$ N

Question 5.7:- What is the least speed at which an aeroplane can execute a vertical loop of 1.0 km radius so that there will be no tendency for the pilot to fall down at the highest point?

Solution:- Radius of vertical circular loop = r = 1.0 km = 1000 m

The pilot will not topple over at the highest point of vertical loop if all the required centripetal force is provided by its weight at that point i.e. m $\frac{v^2}{r}$ = m g

$v^2 = g\,r$

$v = \sqrt{g\,r} = \sqrt{(9.8)\,(1000)} = \sqrt{9800}$

v = 99 m s^{-1}

Question 5.8:- The Moon orbits the Earth so that same side always faces the Earth. Determine the ratio of its spin angular momentum (about its own axis) and its orbital angular momentum. (In this case, treat the Moon as a particle orbiting the Earth). Distance between the Earth and the Moon is 3.85 x 10^8 m. Radius of the Moon is 1.74 x 10^6 m.

Solution:- Distance between the Earth and the Moon = R = 3.85 x 10^8 m

Radius of the Moon = R_M = 1.74 x 10^6 m

$L_S = I_S\,\omega = (\frac{2}{5} M\, R_M^2)\,(\omega)$

$L_O = I\,\omega = (M\,R^2)\,(\omega)$

$$\frac{L_S}{L_O} = \frac{\frac{2}{5}\,M\,R_M^2\,\omega}{M\,R^2\,\omega}$$

$$\frac{L_S}{L_O} = \frac{\frac{2}{5}\,R_M^2}{R^2}$$

$\frac{L_S}{L_O} = \frac{2}{5}(1.74 \times 10^6)^2/(3.85 \times 10^8)^2$

$\frac{L_S}{L_O} = 0.082 \times 10^{-4}$

$\frac{L_S}{L_O}$ = **8.2 x 10^{-6}**

Question 5.9:- The Earth rotates on its axis once a day. Suppose, by some process the Earth contracts so that its radius is only half as large as at present. How fast will it be rotating then?

Solution:- Initial time period of rotation of the Earth = T_1 = 24 h

Initial radius of the Earth = R_1 = R

Radius of the Earth after contraction = R_2 = R/2

Time period of rotation of the Earth after contraction = T_2

According to law of conservation of angular momentum $I_1\,\omega_1 = I_2\,\omega_2$

$\frac{2}{5} M\,R_1^2\,\omega_1 = \frac{2}{5} M\,R_2^2\,\omega_2$

$R_1^2\,(\frac{2\pi}{T_1}) = R_2^2\,(\frac{2\pi}{T_2})$

Put value of R_1 and R_2 in above equation.

$R\,(\frac{2\pi}{T_1}) = (\frac{R^2}{4})\,(\frac{2\pi}{T_2})$

$T_2 = T_1/4 = 24\ h/4$

$T_2 = 6\ h$

Question 5.10:- What should be the orbiting speed to launch a satellite in a circular orbit 900 km above the surface of the Earth? (Take mass of the Earth as 6.0×10^{24} kg and its radius as 6400 km).

Solution:- Mass of the Earth = M = 6.0×10^{24} kg

Radius of the Earth = R = 6400 km = 6.4×10^{6} m

Height of orbit above the surface of the Earth = h = 900 km = 9.0×10^{5} m

Distance of the satellite from the centre of the Earth = r = R + h = 6400 km + 900 km

r = 7300 km = 7.3×10^{6} m

Orbital velocity = v = $\sqrt{\frac{G\ M}{r}} = \sqrt{\frac{(6.67\ x\ 10^{-11})(6.0\ x\ 10^{24})}{7.3\ x\ 10^{6}}}$

v = $\sqrt{54.8\ x\ 10^{6}}$

v = 7.4×10^{3} m s^{-1}

v = 7.4 km s^{-1}