CHAPTER NO. 4(WORK AND ENERGY)

Question 4.1:- A man pushes a lawn mower with a 40 N force directed at an angle of 20° downward from the horizontal. Find the work done by the man as he cuts the strip of grass 20 m long.

Solution:- Applied force = F = 40 N

Displacement covered = d = 20 m

Angle between force and displacement = θ = 20°

Work done = W = F d cos θ

W = (40)(20)(cos 20°)

W = (40)(20)(0.94) = 750 J

$W = 7.5 \times 10^2$ J

Question 4.2:- A rain drop ($m = 3.35 \times 10^{-5}$ kg) falls vertically at a constant speed under the influence of forces of gravity and friction. In falling through 100 m, how much work is done by (a) gravity and (b) friction.

Solution:- Mass of rain drop = m = 3.35×10^{-5} kg

Distance covered in falling = h = 100 m

Force of gravity = F_g = mg

The rain droplet falls with constant speed, its acceleration is zero which means net force on it is also zero.

$F_f + F_g = 0$

$F_f = - F_g = -mg$

(a) Work done by gravity = $W_g = F_g (h) = mgh = (3.35 \times 10^{-5})(9.8)(100)$

$W_g = 0.0328$ J

(a) Work done by friction = $W_f = F_f (h) = -mgh = -(3.35 \times 10^{-5})(9.8)(100)$

$W_g = -0.0328$ J

Question 4.3:- Ten bricks, each 6.0 cm thick and mass 1.5 kg, lie flat on a table. How much work is required to stack them on the top of another?

Solution:- Number of bricks = N = 10

Thickness of each brick = h = 6.0 cm = 0.06 m

Mass of each brick = m = 1.5 kg

We can consider the case of stacking bricks one on top of another. First brick is already lying flat, no work is done on it. In order to stack second brick over first brick, the work is done to place it over first brick is mgh, two place third brick over first two bricks is 2mgh and so on.

Work done in stacking N^{th} brick = W_N = (N-1)mgh

Total work = (work done on 1st brick) + (work done on 2nd brick) + (work done on 3rd brick) + (work done on 4th brick) + (work done on 5th brick) + (work done on 6th brick) + (work done on 7th brick) + (work done on 8th brick) + (work done on 9th brick) + (work done on 10th brick)

$W = W_1 + W_2 + W_3 + W_4 + W_5 + W_6 + W_7 + W_8 + W_9 + W_{10}$

$W = 0 + mgh + (2mgh) + (3mgh) + (4mgh) + (5mgh) + (6mgh) + (7mgh) + (8mgh) + (9mgh)$

$W = 45mgh = 45(1.5)(9.8)(0.06)$

$\underline{W = 40\ J}$

Question 4.4:- A car of mass 800 kg travelling at 54 km h^{-1} is brought to rest in 60 metres. Find the average retarding force on the car. What has happened to the original kinetic energy?

Solution:- Mass of the car = m = 800 kg

Initial velocity of the car = v_i =54 km h^{-1} = $54 \frac{1000}{3600}$ m s^{-1} = 15 m s^{-1}

Final velocity of the car = v_f = 0

Displacement covered = d = 60 m

Work energy principle state that:-

Work done = Change in kinetic energy

$F\ d = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$

$F\ (60) = \frac{1}{2} (800)\ (0) - \frac{1}{2} (800)\ (15)^2$

$F\ (60) = -90,000$

$F = -1500\ N$

The negative sign indicates retardation i.e. opposing force.

Average retarding force = 1500 N

Original kinetic energy has utilized in doing work against road friction and braking force.

Question 4.5:- A 1000 kg automobile at the top of an incline 10 metre high and 100 m long is released and rolls down the hill. What is its speed at the bottom of the incline if the average retarding force due to friction is 480 N?

Solution:- Mass of the car = m = 1000 kg

Height of the inclined plane = h = 10 m

Length of the inclined plane = S = 100 m

Average force of friction = f = 480 N

According to law of conservation of kinetic energy:-

Potential energy of car at top = Kinetic energy of car at bottom + Work done against friction

$m\,g\,h = \frac{1}{2}\,m\,v^2 + f\,S$

$\frac{1}{2}\,m\,v^2 = m\,g\,h - f\,S$

$\frac{1}{2}\,(1000)\,v^2 = (1000)\,(9.8)\,(10) - (480)\,(100) = 50{,}000$

$500\,v^2 = 50{,}000$

$v^2 = 100$

$v = 10$ m s^{-1}

Question 4.6:- 100 m^3 of water is pumped from a reservoir into a tank, 10 m higher than the reservoir, in 20 minutes. If density of water is 1000 kg m^{-3}, find (a) the increase in P.E. (b) the power delivered by the pump.

Solution:- Volume of water pumped = V = 100 m^3

Height of tank = h = 10 m

Time taken in pumping water = t = 20 minutes = 1200 s

Density of water = ρ = 1000 kg m^{-3}

(a) Increase in P.E. = Work done = m g h = ρ V g h = 1000 x 100 x 9.8 x 10

Increase in P.E. = 9.8 x 10^6 J

(b) Power delivered by the pump = P = Work done / time taken

P = m g h / t = ρ V g h / t = (1000 x 100 x 9.8 x 10)/(1200)

P = 8166.66 W

P = 8.2 kW

Question 4.7:- A force (thrust) of 400 N is required to overcome road friction and air resistance in propelling an automobile at 80 km h^{-1}. What power (kW) must the engine develop?

Solution:- Required force of the engine = F = 400 N

Velocity of automobile = v = 80 km h^{-1} = 80 $\frac{1000}{3600}$ m s^{-1} = 22.22 m s^{-1}

Power of the engine = P = Force x velocity = F v

P = (400) (22.22) = 8888 W

P = 8.9 kW

Question 4.8:- How large a force is required to accelerate an electron (m = 9.1 x 10^{-31} kg) from rest to a speed of 2.0 x 10^7 m s^{-1} through a distance of 5.0 cm?

Solution:- Mass of the electron = m = 9.1 x 10^{-31} kg

Initial velocity of the electron = v_i = 0

Final velocity of the electron = v_f = 2.0 x 10^7 m s^{-1}

Distance = d = 5.0 cm = 0.05 m

Work energy principle state that:-

Work done = Change in kinetic energy

$F\ d = \frac{1}{2}\ m\ v_f^2 - \frac{1}{2}\ m\ v_i^2$

$F\ (0.05) = \frac{1}{2}\ (9.1 \times 10^{-31})\ (2.0 \times 10^7)^2 - \frac{1}{2}\ (9.1 \times 10^{-31})\ (0)^2$

$F\ (0.05) = 9.1 \times 10^{-17}$

$F = (9.1 \times 10^{-17})\ /\ 0.05$

$F = 3.6 \times 10^{-15}$ N

Question 4.9:- A diver weighing 750 N dives from a board 10 m above the surface of a pool of water. Use the conservation of mechanical energy to find his speed at a point 5.0 m above the water surface, neglecting air friction.

Solution:- Weight of the diver = W = mg = 750 N

Initial height of the diver above the water surface = h_1 = 10 m

Final height of the diver above the water surface = h_2 = 5 m

Initial velocity of the diver at height $h_1 = v_1$

Final velocity of the diver at height $h_2 = v_2 = v$

According to law of conservation of energy:-

Loss in potential energy = Gain in kinetic energy

$m\ g\ h_1 - m\ g\ h_2 = \frac{1}{2}\ m\ v_2^2 - \frac{1}{2}\ m\ v_1^2$

$m\ g\ (h_1 - h_2) = \frac{1}{2}\ m\ v^2$

$v^2 = 2\ g\ (h_1 - h_2) = 2\ (9.8)\ (10 - 5)$

$v^2 = 98$

v = 9.9 m s^{-1}

Question 4.10:- A child starts from rest at the top of a slide of height 4.0 m. (a) What is his speed at the bottom if the slide is frictionless? (b) if he reaches the bottom, with a speed of 6 m s^{-1}, what percentage of his total energy at the top of the slide is lost as a result of friction?

Solution:- Height of the slide = h = 4.0 m

(a) Law of conservation of energy states that loss in P.E. = Gain in K.E.

Potential energy at top = Kinetic energy at bottom

$m\ g\ h = \frac{1}{2}\ m\ v^2$

$v = \sqrt{2\ g\ h} = \sqrt{2\ (9.8)\ (4)}$

v = 8.8 m s^{-1}

(a) Speed at the bottom in the presence of friction = v = 6 m s^{-1}

Percentage loss of energy $= \frac{Loss\ of\ energy}{Total\ energy}$ x 100%

Percentage loss of energy $= \frac{Initial\ potential\ energy - Final\ kinetic\ energy}{Initial\ potential\ energy}$ x 100%

Percentage loss of energy $= \frac{mgh - \frac{1}{2}mv^2}{mgh}$ x 100% $= \frac{gh - \frac{1}{2}v^2}{gh}$ x 100%

Percentage loss of energy $= \frac{(9.8)(4) - \frac{1}{2}(6)^2}{(9.8)(4)}$ x 100%

Percentage loss of energy = 54 %