CHAPTER NO. 3(MOTION AND FORCE)

Question 3.1:- A helicopter is ascending vertically at the rate of 19.6 m s^{-1}. When it is at a height of 156.8 m above the ground, a stone is dropped. How long does the stone take to reach the ground?

Solution:- When the stone is dropped from the vertically rising helicopter, it has initial momentum directed upwards. It moves upwards initially, comes to rest after reaching the highest point and moves downwards till it reaches the ground.

We will divide the journey of stone in two parts i.e. upwards and downwards.

Upward Journey:

Initial height = S = 156.8 m

Initial velocity directed upwards = v_i = 19.6 m s^{-1}

Time taken by stone to reach maximum height = t_1

Velocity at top of the path = $v_f = 0$

Acceleration = a = -g = -9.8 m s^{-2}

Apply first equation of motion $v_f = v_i + a\,t_1$

$0 = 19.6 + (-9.8)\,t_1$

$t_1 = 19.6/9.8 = 2$ s

The stone also covers some distance while rising upwards which can be calculated by applying third equation of motion as $2\,a\,S_1 = v_f^2 - v_i^2$

$2\,(-9.8)\,S_1 = (0)^2 - (19.6)^2$

$S_1 = 19.6$ m

Downward journey:

Total height before downward journey = Initial height + Distance travelled in moving upwards

$S_2 = S + S_1$

$S_2 = 156.8 + 19.6 = 176.4$ m

Now, we will consider the top point as initial (or reference) point.

Initial velocity at top point = $v_i = 0$

Acceleration downward = a = g = 9.8 m s^{-2}

Time taken by stone to reach ground level from maximum height = t_2

Apply second equation of motion as $S_2 = v_i\,t_2 + \frac{1}{2}\,g\,t_2^2$

$176.4 = (0)\,t_2 + \frac{1}{2}\,(9.8)\,t_2^2$

$t_2^2 = 36$

$t_2 = 6$ s

Total time taken by stone = $t = t_1 + t_2$

$t = 2 + 6$

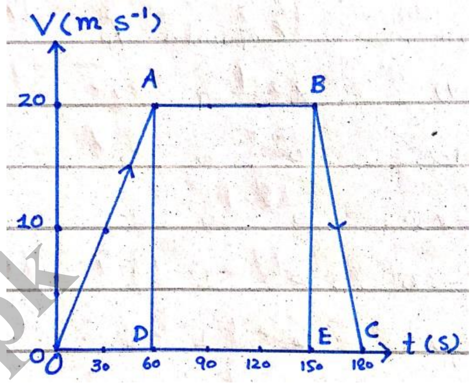
$t = 8$ s

Question 3.2:- Using the following data, draw a velocity-time graph for a short journey on a straight road of a motorbike.

Velocity (m s^{-1})	0	10	20	20	20	20	0
Time (s)	0	30	60	90	120	150	180

Use the graph to calculate (a) the initial acceleration (b) the final acceleration and (c) the total distance travelled by the motorcyclist.

Solution:- Graph:-



(a) The motorbike accelerated from O to A which can be measured by taking slope of line OA.

Time taken by motorbike to reach from O to A = $t_1 = 60$ s

Velocity at point O = $v_O = 0$

Velocity at point A = $v_A = 20$ m s^{-1}

Initial acceleration from O to A = $a_i = \frac{v_A - v_O}{t_1} = \frac{10-0}{60}$

$a_i = 0.33$ m s^{-2}

(b) The motorbike decelerated from B to C which can be measured by taking slope of line BC.

Time taken by motorbike to reach from B to C = $t_3 - t_2 = 180 - 150 = 30$ s

Velocity at point B = $v_B = 20$ m s^{-1}

Velocity at point C = $v_C = 0$

Final acceleration from B to C = $a_f = \frac{v_C - v_B}{t_3 - t_2} = \frac{0-20}{30} = -0.67$ m s^{-2}

$a_f = -0.67$ m s^{-2}

(c) The total distance can be determined by measuring the area between the velocity time graph and the time axis.

Total distance travelled by motor cyclist = S = Area of triangle OAD + Area of rectangle ABDE + Area of triangle BCE

$S = \frac{1}{2}$(base)(altitude) + (length)(width) + $\frac{1}{2}$(base)(height)

$S = \frac{1}{2}(OD)(DA) + (DA)(DE) + \frac{1}{2}(EB)(EC)$

$S = \frac{1}{2}(60)(20) + (20)(90) + \frac{1}{2}(20)(30) = 600 + 1800 + 300$

$S = 2700$ m = 2.7 km

Question 3.3:- A proton moving with speed of 1.0×10^{7} m s^{-1} passes through a 0.020 cm thick sheet of paper and emerges with a speed of 2.0×10^{6} m s^{-1}. Assuming uniform deceleration, find retardation and time taken to pass through the paper.

Solution:- Initial velocity of proton = $v_i = 1.0 \times 10^{7}$ m s^{-1}

Final velocity of proton = $v_f = 2.0 \times 10^{6}$ m s^{-1}

Distance travelled in paper sheet = S = 0.020 cm = 0.020×10^{-2} m = 2.0×10^{-4} m

Apply third equation of motion to find retardation as $2aS = v_f^2 - v_i^2$

$2(a)(2 \times 10^{-4}) = (2.0 \times 10^{6})^2 - (1.0 \times 10^{7})^2$

$a(4 \times 10^{-4}) = (4.0 \times 10^{12}) - (1.0 \times 10^{14}) = -9.6 \times 10^{13}$

$a = (-9.6 \times 10^{13}) / (4 \times 10^{-4})$

$a = -2.4 \times 10^{17}$ m s^{-2}

Retardation = Magnitude of negative acceleration = 2.4×10^{17} m s^{-2}

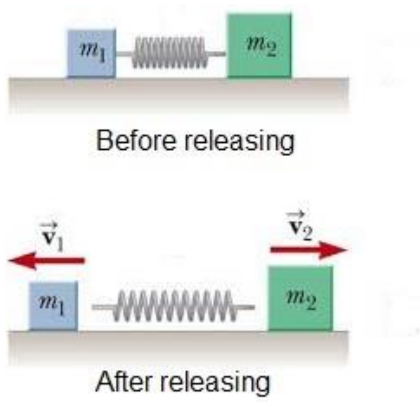
Time taken by proton to pass through paper sheet = t

Time taken can be determined by applying first equation of motion as $v_f = v_i + at$

$t = = \frac{v_f - v_i}{a} = \frac{(2.0 \times 10^{6}) - (1.0 \times 10^{7})}{-2.4 \times 10^{17}} = \frac{-8.0 \times 10^{6}}{-2.4 \times 10^{17}}$

$t = 3.3 \times 10^{-11}$ s

Question 3.4:- Two masses m_1 and m_2 are initially at rest with a spring compressed between them. What is the ratio of magnitude of their velocities after the spring has been released?



Solution:- Both masses are initially at rest with spring compressed between them. When the spring is released, both masses move in opposite direction. Assume that m_1 moves towards left with velocity v_1 while m_2 moves towards right with velocity v_2.

We can apply law of conservation of linear momentum.

Total initial momentum of the system = 0

Total final momentum of the system = $-m_1v_1 + m_2v_2$

Velocities towards right are considered as positive and vice versa.

We can apply law of conservation of linear momentum as under:-

Total final momentum of the system = Total initial momentum of the system

$$-m_1v_1 + m_2v_2 = 0$$

$$\frac{v_1}{v_2} = \frac{m_2}{m_1}$$

Question 3.5:- An amoeba of mass 1.0×10^{-12} kg propels itself through water by blowing a jet of water through a tiny orifice. The amoeba ejects water with a speed of 1.0×10^{-4} m s^{-1} and at

a rate of 1.0 x 10^{-13} kg s^{-1}. Assume that the water is being continuously replenished so that the mass of the amoeba remains the same.

(a) If there were no force on amoeba other than the reaction force caused by the emerging jet, what would be the acceleration of the amoeba?

(b) If amoeba moves with constant velocity through water, what is force of surrounding water (exclusively of jet) on the amoeba?

Solution:- Mass of amoeba = M = 1.0 x 10^{-12} kg

Speed of ejecting water jet = v = 1.0 x 10^{-4} m s^{-1}

Mass flow rate of water from jet = Mass per second = m = 1.0 x 10^{-13} kg s^{-1}

(a) Acceleration of the amoeba = a = $\frac{m\,v}{M} = \frac{1.0\;x\;10^{-13} x\;1.0\;x\;10^{-4}}{1.0\;x\;10^{-12}}$

<u>**a = 1.0 x 10^{-5} m s^{-2}**</u>

(b) Force on the amoeba = F = m v = 1.0 x 10^{-13} x 1.0 x 10^{-4}

<u>**F = 1.0 x 10^{-17} N**</u>

Question 3.6:- A boy places a firecracker of negligible mass in an empty can of 40 g mass. He then plugs the end with a wooden block of mass 200 g. After igniting the firecracker, he throws the can straight up. It explodes at the top of its path. If the block shoots out with a speed of 3.0 m s^{-1}, how fast will the can be going?

Solution:- Mass of the can = m_1 = 40 g

Mass of the wooden block = m_2 = 200 g

When both the can and wooden block are top, their velocity is zero. After the explosion, both the can and wooden block shoot out in opposite directions in order to conserve the momentum.

Speed of the wooden block = v_2 = 3.0 m s^{-1}

Speed of the can = v_1

We will apply law of conservation of linear momentum as under:-

Total final momentum of the system = Total initial momentum of the system

$-m_1v_1 + m_2v_2 = 0$

$-(40)(v_1) + (200)(3.0) = 0$

$v_1 = 600/40$

<u>**v_1 = 15 m s^1**</u>

Question 3.7:- An electron (m = 9.1 x 10^{-31} kg) travelling at 2.0 x 10^7 m s^{-1} undergoes a head on collision with a hydrogen atom (m = 1.67 x 10^{-27} kg) which is initially at rest. Assuming the collision to be perfectly elastic and motion to be along a straight line, find the velocity of hydrogen atom.

Solution:- Mass of the electron = $m_1 = 9.1 \times 10^{-31}$ kg

Mass of the hydrogen atom = $m_2 = 1.67 \times 10^{-27}$ kg

Initial velocity of the electron = $v_1 = 2.0 \times 10^{7}$ m s^{-1}

Initial velocity of the hydrogen atom = $v_2 = 0$

Using the results of elastic collision in one dimension, we can determine final velocity of the hydrogen atom as under:-

$$v_2' = \frac{2m_1}{m_1+m_2}v_1 + \frac{m_2-m_1}{m_1+m_2}v_2$$

Put $v_2 = 0$

$$v_2' = \frac{2m_1}{m_1+m_2}v_1 = \frac{2 \times (9.1 \times 10^{-31})}{(9.1 \times 10^{-31})+(1.67 \times 10^{-27})}(2.0 \times 10^{7})$$

$$v_2' = \frac{36.4 \times 10^{-24}}{10^{-27}(0.00091+1.67)}$$

$v_2' = 2.2 \times 10^{4}$ m s^{-1}

Question 3.8:- A truck weighing 2500 kg and moving with a velocity of 21 m s^{-1} collides with stationary car weighing 1000 kg. The truck and the car move together after the impact. Calculate their common velocity.

Solution:- Mass of the truck = m_1 = 2500 kg

Mass of the car = 1000 kg

Initial velocity of the truck = v_1 = 21 m s^{-1}

Initial velocity of the car = $v_2 = 0$

The truck and the car move with the combined velocity v after the collision. Apply law of conservation of linear momentum as under:-

Total momentum after the collision = Total momentum before the collision

$$m_1v_1 + m_2v_2 = (m_1+m_2)v$$

$$v = \frac{m_1v_1+m_2v_2}{m_1+m_2}$$

$$v = \frac{(2500)(21)+(1000)(0)}{(2500+1000)}$$

v = 15 m s^{-1}

Question 3.9:- Two blocks of masses 2.0 kg and 0.50 kg are attached at the two ends of a compressed spring. The elastic potential energy stored in the spring is 10 J. Find the velocities of the blocks if the spring delivers its energy to the blocks when released.

Solution:- Mass of the first block = m_1 = 2.0 kg

Mass of the second block = m_2 = 0.50 kg

Elastic potential energy of the spring = P.E. = 10 J

Initial momentum of system before release of spring = 0

When the spring is released, it delivers its P.E. to both blocks.

Velocity of first block after release of spring = v_1

Velocity of second block after release of spring = v_2

Apply law of conservation of momentum as under:-

Total momentum after the release = Total momentum before the release

$m_1v_1 + m_2v_2 = 0$

$2\ v_1 + 0.50\ v_2 = 0$

$v_2 = -4\ v_1$ -- Eq. (1)

Apply law of conservation of energy as under:-

Loss in P.E. of the spring = Gain in K.E. of the blocks

$10 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$

$10 = \frac{1}{2} (2) v_1^2 + \frac{1}{2} (0.50) (4 v_1)^2$

$10 = v_1^2 + 4 v_1^2$

$5 v_1^2 = 10$

$v_1^2 = 2$

$v_1 = 1.4$ m s^{-1}

Put value if v_1 in Eq. (1)

$v_2 = -4 v_1 = -4 (1.4)$

$v_2 = -5.6$ m s^{-1}

Question 3.10:- A football is thrown upward with an angle of 30^o with respect to horizontal. To throw a 40 m pass what must be the initial speed of the ball?

Solution:- Angle of projection with respect to horizontal = θ = 30^o

Required range of the projectile = R = 40 m

Range of the projectile = R = $\frac{v_i^2}{g}$ sin 2θ

Rearranging gives $v_i^2 = \frac{g\ R}{\sin 2\theta}$

$v_i = \sqrt{\frac{g\ R}{\sin 2\theta}} = \sqrt{\frac{(9.8)(40)}{\sin 60^o}}$

$v_i = \sqrt{\frac{392}{0.866}}$

$v_i = 21$ m s^{-1}

Question 3.11:- A ball is thrown horizontally from a height of 10 m with a velocity of 21 m s^{-1}. How far off it hit the ground and with what velocity?

Solution:- Initial height of the projectile = Y = 10 m

Initial horizontal velocity of the projectile = $v_x = 21\ m\ s^{-1}$

Initial vertical velocity of the projectile = $v_{iy} = 0$

We know that for a projectile which is projected horizontally from some height Y, we can express $Y = \frac{1}{2} g\, t^2$

$t = \sqrt{\frac{2\,Y}{g}} = \sqrt{\frac{2\,(10)}{9.8}}$

t = 1.43 s

We can determine horizontal distance X by using the equation S = v t as under:-

$X = v_x\, t = (21)\ (1.4)$

<u>X = 30 m</u>

We can determine final vertical velocity by applying first equation of motion as under:-

$v_{fy} = v_{iy} + a\, t$

$v_{fy} = 0 + (9.8)\ (1.43)$

$v_{fy} = 14\ m\ s^{-1}$

Final velocity before hitting the ground = $v_f = \sqrt{{v_x}^2 + {v_{fy}}^2}$

$v_f = \sqrt{(21)^2 + (14)^2} = \sqrt{637}$

<u>$v_f = 25\ m\ s^{-1}$</u>

Question 3.12:- A bomber dropped a bomb from a height of 490 m when its velocity along the horizontal was 300 km h^{-1}. (a) How long was it in air? (b) At what distance from the point vertically below the bomber at the instant the bomb was dropped, did it strike the ground?

Solution:- Initial height of the bomb = Y = 490 m

Initial horizontal velocity of the bomb = $v_x = 300\ km\ h^{-1} = 300 \times \frac{1000}{3600}\ m\ s^{-1} = 83.3\ m\ s^{-1}$

We know that for a projectile which is projected horizontally from some height Y, we can express $Y = \frac{1}{2} g\, t^2$

$t = \sqrt{\frac{2\,Y}{g}} = \sqrt{\frac{2\,(490)}{9.8}}$

<u>t = 10 s</u>

We can determine horizontal distance X by using the equation S = v t as under:-

$X = v_x\, t = (83.3)\ (10)$

<u>X = 833 m</u>

Question 3.13:- Find the angle of projection of a projectile for which its maximum height and horizontal range are equal.

Solution:- Height of the projectile = $H = \frac{v_i^2 sin^2\theta}{2\,g}$

Range of the projectile = R = $\frac{v_i^2 \sin 2\theta}{g}$

Height of the projectile = Range of the projectile

H = R

$\frac{v_i^2 sin^2\theta}{2\,g} = \frac{v_i^2 \sin 2\theta}{g}$

$\frac{v_i^2 sin^2\theta}{2\,g} = \frac{v_i^2\,(2\sin\theta\cos\theta)}{g}$

$\frac{\sin\theta}{\cos\theta} = 4$

tan θ = 4

θ = $\tan^{-1}$ (4)

θ = 76°

Question 3.14:- Prove that for angles of projection, which exceed or fall short of 45° by equal amounts, the ranges are equal.

Solution:- According to statement of question, we have to prove that range of projectile at an angle which exceed 45° by some amount is equal to its range at an angle which falls short of 45° by the same amount.

We choose two angles as under:-

$\theta_1 = 45^o + \theta$ (θ_1 is the angle which exceeds 45° by amount θ)

$\theta_2 = 45^o - \theta$ (θ_2 is the angle which falls short of 45° by amount θ)

$R_1 = \frac{v_i^2 \sin 2\theta_1}{g} = \frac{v_i^2 \sin 2(45^o+\theta)}{g} = \frac{v_i^2 \sin(90^o+2\theta)}{g} = \frac{v_i^2 \cos 2\theta}{g}$ $\quad\therefore \sin(90^o + 2\theta) = \cos 2\theta$

$R_2 = \frac{v_i^2 \sin 2\theta_2}{g} = \frac{v_i^2 \sin 2(45^o-\theta)}{g} = \frac{v_i^2 \sin(90^o-2\theta)}{g} = \frac{v_i^2 \cos 2\theta}{g}$ $\quad\therefore \sin(90^o - 2\theta) = \cos 2\theta$

We can see the $R_1 = R_2$

Alternately, we can show it by using particular value of θ (say θ = 15°)

$\theta_1 = 45^o + 15^o = 60^o$

$\theta_2 = 45^o - 15^o = 30^o$

$R_1 = \frac{v_i^2 \sin 2\theta_1}{g} = \frac{v_i^2 \sin 2(60^o)}{g} = \frac{v_i^2 \sin(120^o)}{g} = \frac{v_i^2 (0.866)}{g}$

$R_2 = \frac{v_i^2 \sin 2\theta_2}{g} = \frac{v_i^2 \sin 2(30^o)}{g} = \frac{v_i^2 \sin(60^o)}{g} = \frac{v_i^2 (0.866)}{g}$

We can see that $R_1 = R_2$

Question 3.15:- A SLBM(submarine launched ballistic missile) is fired from a distance of 3000 km. If the Earth is considered flat and the angle of launch is 45° with horizontal, find the velocity with which the missile is fired and the time taken by SLBM to hit the target.

Solution:- Range of the projectile = R = 3000 km = 3 x 10^6 m

Angle of projection with horizontal = $\theta = 45^o$

Range of the projectile = $R = \frac{v_i^2}{g} \sin 2\theta$

Rearranging gives $v_i^2 = \frac{g\,R}{\sin 2\theta}$

$v_i = \sqrt{\frac{g\,R}{\sin 2\theta}} = \sqrt{\frac{(9.8)(3\ x\ 10^6)}{\sin 90^o}}$

$v_i = \sqrt{\frac{29.4\ x\ 10^6}{1}} = 5.42 \text{ x } 10^3 \text{ m s}^{-1}$

v_i = 5.42 km s^{-1}

Time of flight of projectile = $t = \frac{2\,v_i}{g} \sin \theta$

$t = \frac{2\ x\ 5.42\ x10^3}{9.8}\ \sin 45^o = \frac{2\ x\ 5.42\ x10^3}{9.8}\ (0.707) = 782 \text{ s}$

t = 13 minutes