## CHAPTER NO. 2 (VECTORS AND EQUILIBRIUM)

**Question 2.1:- Define the terms (i) unit vector (ii) Position vector and (iii) Components of a vector.**

**Answer:- Unit Vector:-** A vector whose magnitude is one in a specific direction is called unit vector. It is represented by $\hat{A}$. *Mathematically*, $\hat{\boldsymbol{A}} = \overrightarrow{\boldsymbol{A}}\ /\ A$.

**Position Vector:-** A vector which describes the position of a point with respect to origin is called position vector. It is represented by $\vec{\boldsymbol{r}}$. In a plane $\vec{\boldsymbol{r}}$ = x $\hat{\imath}$ + y $\hat{\jmath}$ while in space $\vec{\boldsymbol{r}}$ = x $\hat{\imath}$ + y $\hat{\jmath}$ + z $\hat{k}$.

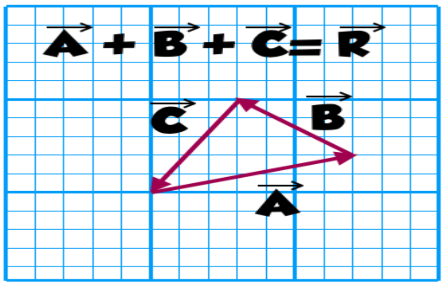
**Component of a Vector:-** Component of a vector is its effective value in a given direction.

Normally, vector is divided in to two components which are mutually perpendicular.

The components of a vector which are perpendicular to each other are called **rectangular components.** In xy plane, component of vector along x-axis is called x-component or horizontal component while component of vector along y-axis is called y-component or vertical component.



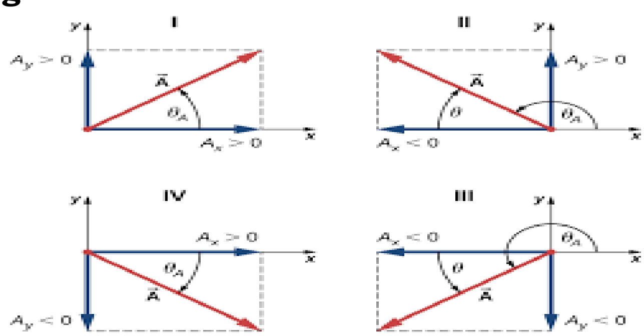
**Question 2.2:- The vector sum of three vectors gives a zero resultant. What can be the orientation of the vectors?**




**Answer:-** If vector sum of three vectors is zero, the vectors are arranged in the form of a triangle. When three vectors are arranged in head to tail configuration and the orientation forms a triangular geometry, the vector sum is zero as tail of first vector is joined with the head of third vector in this case.

$\vec{R} = \vec{A} + \vec{B} + \vec{C} = \vec{0}$.

**Question 2.3:- Vector A lies in the xy plane. For what orientation will both of its rectangular components be negative? For what orientation will its components have opposite signs?**




**Answer:-** In xy plane, there are four quadrants.

**i)** When a vector lies in third quadrant, both of its rectangular components will be negative.

**ii)** When a vector lies in second or forth quadrant, both of its rectangular components will have opposite signs.

**Question 2.4:- If one of the rectangular components of a vector is not zero, can its magnitude be zero? Explain.**

**Answer:-** No, if one of the rectangular component of a vector is not zero, its magnitude is not zero. For a vector lying in xy plane, there are two cases:-

**1)** If $A_x = 0$ and $A_y \neq 0$ then $A = \sqrt{A_x{}^2 + A_y{}^2} = \sqrt{0^2 + A_y{}^2} = A_y \neq 0$.

**2)** If $A_x \neq 0$ and $A_y = 0$ then $A = \sqrt{A_x{}^2 + A_y{}^2} = \sqrt{A_x{}^2 + 0^2} = A_x \neq 0$.

**Question 2.5:- Can a vector have component greater than the vector's magnitude?**

**Answer:-** No, a vector cannot have components greater than the vector's magnitude. For a vector $\vec{A}$ lying in xy plane:-

| | |
|---|---|
| $A_x = A\cos\theta$<br>$\lvert A_x\rvert = \lvert A\cos\theta\rvert = \lvert A\rvert\ \lvert\cos\theta\rvert$<br>We know that $\lvert\cos\theta\rvert \leq 1$<br>So $\lvert A_x\rvert = \lvert A\rvert\ \lvert\cos\theta\rvert \leq \lvert A\rvert$ | $A_y = A\sin\theta$<br>$\lvert A_y\rvert = \lvert A\sin\theta\rvert = \lvert A\rvert\ \lvert\sin\theta\rvert$<br>We know that $\lvert\sin\theta\rvert \leq 1$<br>So $\lvert A_y\rvert = \lvert A\rvert\ \lvert\sin\theta\rvert \leq \lvert A\rvert$ |

It is clear that magnitude of components of a vector is always less than or equal to a vector's magnitude and cannot be greater than vector's magnitude.

**Question 2.6:- Can the magnitude of a vector have negative value?**

**Answer:-** No, the magnitude of a vector can never be negative. Magnitude of a vector is either zero or positive. The magnitude of a vector in xy-plane is given as $A = \sqrt{A_x{}^2 + A_y{}^2}$. We know that square of a negative number is also positive, therefore, magnitude of a vector can never be zero.

**Question 2.7:- If A + B = 0, what can you say about the components of the two vectors?**

**Answer:-** If $\vec{A} + \vec{B} = \vec{0}$, We can say that $\vec{A} = -\vec{B}$

$(A_x\,\hat{\imath} + A_y\,\hat{\jmath} + A_z\,\hat{k}) = -(B_x\,\hat{\imath} + B_y\,\hat{\jmath} + B_z\,\hat{k})$ **or** $(A_x + B_x)\,\hat{\imath} + (A_y + B_y)\,\hat{\jmath} + (A_z + B_z)\,\hat{k} = \vec{0}$.

This implies that:-

**i)** Corresponding components of both the vectors are equal in magnitude but opposite in direction.

**ii)** Sum of corresponding components of both the vectors is also zero.

**Question 2.8:- Under what circumstances would a vector have components that are equal in magnitude?**

**Answer:-** The components of vector will be equal in magnitude when the vector makes an angle of $45^o$, $135^o$, $225^o$ and $315^o$ with respect to positive x-axis.

Mathematically, we can prove this as following:-

We know that $A_x = A \cos\theta$ and $A_y = A \sin\theta$

$|A_y| = |A_x|$ only when $|A \sin\theta| = |A \cos\theta|$

$|\tan\theta| = 1$ which means $\tan\theta = \pm 1$

$\tan\theta = +1$ implies that $\theta = \tan^{-1}(1) = 45^o$ & $225^o$ $\tan\theta = -1$ implies that $\theta = \tan^{-1}(-1) = 135^o$ & $315^o$.

**Question 2.9:- Is it possible to add a vector quantity to a scalar quantity? Explain.**

**Answer:-** No, a vector cannot be added to a scalar quantity.

The rules for addition of vectors and scalars are different. Scalars are added by using normal addition while for adding vectors, their directions are also taken into account.



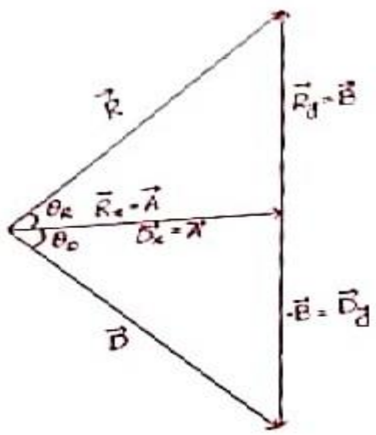
**Question 2.10:- Can you add zero to a null vector?**

**Answer:-** No, we cannot add zero to a null vector. Zero is a scalar and cannot be added to a vector because rules for addition of both are different.

**Question 2.11:- Two vectors have unequal magnitudes. Can their sum be zero? Explain.**

**Answer:-** No, the sum of two vectors of unequal magnitude can never be zero. The sum of two vectors is only zero if they are equal in magnitude but opposite in direction i.e. $\vec{A} + \vec{B} = \vec{0}$, We can say that $\vec{A} = -\vec{B}$ which means vectors should be equal in magnitude but opposite in direction.

**Question 2.12:- Show that the sum and difference of two perpendicular vectors of equal lengths are also perpendicular and of the same length.**



**Answer:-** Suppose we have two vector **A** and **B** which are equal in magnitude and mutually perpendicular. Assume **A** is along x-axis and **B** is along y-axis. A = B (say)

__SUM__**:-** $R_x = A$ and $R_y = B$

$\vec{R} = \vec{A} + \vec{B}$ implies that R = $\sqrt{R_x{}^2 + R_y{}^2}$ = $\sqrt{A^2 + B^2}$ = $\sqrt{2}$ A and $\theta_R = \tan^{-1} (R_y/R_x)$ = $\tan^{-1}$ (B/A) = $\tan^{-1}$ (1) = 45° w.r.t. x-axis.

**<u>DIFFERENCE</u>:-** $D_x$ = A and $D_y$ = -B

$\vec{D} = \vec{A} - \vec{B}$ implies that D = $\sqrt{D_x{}^2 + D_y{}^2}$ = $\sqrt{A^2 + (-B)^2}$ = $\sqrt{2}$ A and $\theta_D = \tan^{-1}$ $(D_y/D_x)$ = $\tan^{-1}$ (-B/A) = $\tan^{-1}$ (1) = -45° w.r.t. x-axis.

This show that R = D while angle between $\vec{R}$ and $\vec{D}$ is $\theta = \theta_R - \theta_D$ = 45° – (-45°) = 90°

So the sum and difference of two perpendicular vectors of same length are also equal in magnitude and perpendicular to each other.

**Question 2.13:- How would the two vectors of the same magnitude have to be oriented, if they were to be combined to give a resultant equal to a vector of the same magnitude?**

**Answer:-** When two vectors of same magnitude are oriented at an angle of 120⁰ with respect to each other, their resultant is also equal in magnitude to either of the vector. In this case, the geometry becomes an equilateral triangle.

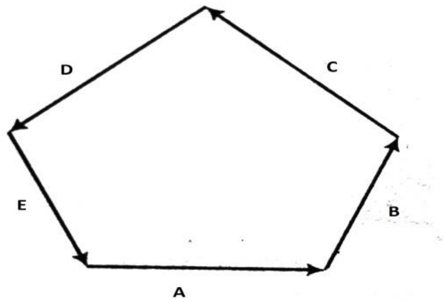
Alternately, magnitude of resultant vector R of two vectors A and B is determined as R = $\sqrt{A^2 + B^2 + 2\,A\,B\cos\theta}$. Using the condition R = A = B, We find that cos θ = -0.5 and θ = $\cos^{-1}$ (-0.5) = 120°.

**Question 2.14:- The two vectors to be combined have magnitudes 60 N and 35 N. Pick the correct answer from those given below and tell why is it the only one of the three that is correct? i) 100 N ii) 70 N iii) 20 N**

**Answer:-** When two vectors are oriented in the same direction, the magnitude of resultant vector is maximum and is equal to sum of magnitudes of both vectors. When two vectors are oriented in opposite direction, the magnitude of resultant vector is minimum and is equal to difference of magnitudes of both vectors.

In case 60 N and 35 N are parallel, maximum resultant force is 60 + 35 = 95 N and when 60 N and 35 N are anti-parallel, minimum resultant force is 60 – 35 = 25 N. The resultant of 60 N and 35 N lies between 25 N to 95 N. The resultant can neither be less than 25 N and nor be greater than 95 N, hence the correct answer is **70 N.**

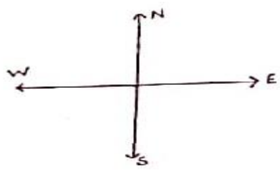
**Question 2.15:- Suppose the sides of a closed polygon represent vectors arranged head to tail. What is the sum of these vectors?**



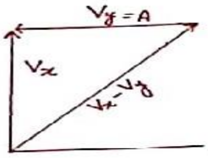

**Answer:-** When sides of a closed polygon represent the vectors arranged in head to tail configuration, the sum of these vectors will be a null or zero vector. In this case, tail of first vector is joined with the head of last vector, therefore, resultant vector is a zero vector.

**Question 2.16:- Identify the correct answer.**

**i) Two ships X and Y are travelling in different directions at equal speeds. The actual direction of motion of X is due north but to an observer on Y, the apparent direction of motion of X is north-east. The actual direction of motion of Y as observed from the shore will be (A) East (B) West (C) south-east (D) south-west.**




$V_R =$




$(\vec{V_x} - \vec{V_y}) + \vec{A} = \vec{V_x}$

$\Rightarrow \vec{A} = \vec{V_y}$

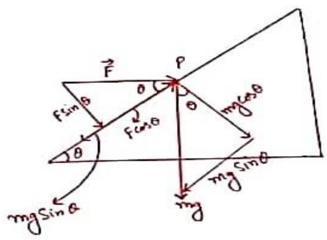
West

**ii) A horizontal force F is applied to a small object P of mass m at rest on a smooth plane inclined at angle θ to the horizontal as shown in figure. The magnitude of the resultant force acting up and along the surface of the plane, on the object is**

**a) F cos θ – mg sin θ b) F sin θ – mg cos θ c) F cos θ + mg cos θ d) F sin θ + mg sin θ e) mg tan θ**

**Answer:- i)** West

**ii)** F cos θ – mg sin θ




$F_s = F\cos\theta - mg\sin\theta$

**Question 2.17:- If all the components of the vectors, $\vec{A_1}$ and $\vec{A_2}$ were reversed, how would this alter $\vec{A_1} \times \vec{A_2}$?**

**Answer:-** If all the components of the vectors $A_1$ and $A_2$ are reversed, there would be no effect on $\vec{A_1} \times \vec{A_2}$

We know that $\vec{A_1} \times \vec{A_2} = (A_1 A_2 \sin\theta)\, \hat{n}$

After reversing all components of $\mathbf{A_1}$ and $\mathbf{A_2}$, we evaluate vector product as $(-\vec{A_1}) \times (-\vec{A_2}) = (-A_1)(-A_2) \sin\theta\, \hat{n} = (A_1 A_2 \sin\theta)\, \hat{n} = \vec{A_1} \times \vec{A_2}$ . We can also prove it by using determinant form to evaluate cross product:-

$$-\overrightarrow{A_1}\,x\,-\overrightarrow{A_2} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -A_{1x} & -A_{1y} & -A_{1z} \\ -A_{2x} & -A_{2y} & -A_{2z} \end{vmatrix} = (-1)(-1)\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_{1x} & A_{1y} & A_{1z} \\ A_{2x} & A_{2y} & A_{2z} \end{vmatrix} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_{1x} & A_{1y} & A_{1z} \\ A_{2x} & A_{2y} & A_{2z} \end{vmatrix} = \overrightarrow{A_1}\,x\,\overrightarrow{A_2}$$

**Question 2.18:- Name the three different conditions that could make $\overrightarrow{A_1}\,x\,\overrightarrow{A_2}$ = $\vec{0}$.**

**Answer:-** We know that $\overrightarrow{A_1}\,x\,\overrightarrow{A_2}$= ($A_1$ $A_2$ sin θ) $\hat{n}$. The three different conditions which could make $\overrightarrow{A_1}\,x\,\overrightarrow{A_2} = \vec{0}$ are:-

**i)** $\overrightarrow{A_1}$ is a null vector i.e. $A_1$ = 0.

**ii)** $\overrightarrow{A_2}$ is a null vector i.e. $A_2$ = 0.

**iii)** $\overrightarrow{A_1}$ and $\overrightarrow{A_2}$ are parallel and anti-parallel to each other i.e. θ = $0^o$ or $180^o$.

**Question 2.19:- Identify true or false statements and explain the reason.**

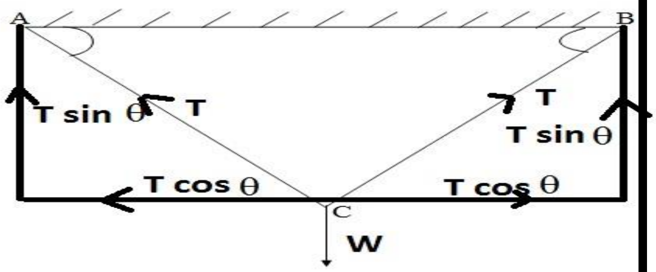
**i) A body in equilibrium implies that it is neither moving nor rotating.**

**ii) If coplanar forces acting on the body form a closed polygon, then the body is said to be in equilibrium.**

**Answer:- i)** The statement is false. When a body is in equilibrium, it is either at rest or moving (rotating) with uniform velocity.

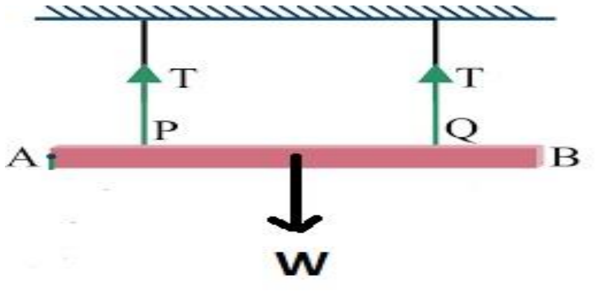
**ii)** The statement is true. When coplanar forces acting on the body form a closed polygon, the resultant force is zero and it satisfies first condition of equilibrium. The body will be in translational equilibrium in this case.

**Question 2.20:- A picture is suspended from a wall by two strings. Show by diagram the configuration of the string for which the tension in the string will be minimum.**




**Answer:-** Consider the picture of weight W is suspended from a wall by two strings. We suppose that suspended picture produces equal tensions in the both strings as shown:-

Resolve the tension in the string in rectangular components and apply first condition of equilibrium in vertical direction as T sin θ + T sin θ = W, T = W / (2 sin θ).




Tension in the string will be minimum when sin θ is maximum. Maximum value of sin θ = 1 so θ = $\sin^{-1}$ (1) = $90^o$.

$T_{min}$ = W/2 and the orientation is as shown.

**Question 2.21:- Can a body rotate about its centre of gravity under the action of its weight?**

**Answer:-** No, a body cannot rotate about its centre of gravity under the action of its weight because torque is zero. The moment arm in this case is zero as the line of action of force (weight) passes through the pivot point (centre of gravity).

We know that torque = (moment arm) (force) = (zero) (mg) = zero.