CHAPTER NO. 11(HEAT AND THERMODYNAMICS)

Question 11.1:- Estimate the average speed of nitrogen molecules in air under standard conditions of pressure and temperature.

Solution:- Molar mass of nitrogen molecule = M = 28 g = 28 x 10^{-3} kg

Number of nitrogen molecules in one mole = N_A = 6.022 x 10^{23}

Mass of one nitrogen molecule = m = M/N_A = (28 x 10^{-3})/(6.022 x 10^{23}) = 4.65 x 10^{-26} kg

Standard temperature = T = 0 ^oC = 273 K

We know that $T = \frac{2}{3k} < \frac{1}{2} m v^2 >$

$<v^2> = \frac{3kT}{m}$ = (3)(1.38 x 10^{-23})(273)/(4.65 x 10^{-26})

$<v^2>$ = 243 x 10^3 $m^2 s^{-2}$

$<v>$ = 493 m s^{-1}

Question 11.2:- Show that ratio of the root mean square speeds of molecules of two different gases at a certain temperature is equal to the root of the inverse ratio of their masses.

Solution:- We assume that there are two gases with molecular masses m_1 and m_2, respectively. Suppose, that root mean square velocity of first gas is $<v_1>$ and that of second gas is $<v_2>$. Both gases are kept at same temperature.

FIRST GAS: $T = \frac{2}{3k} < \frac{1}{2} m_1 v_1^2 >$

SECOND GAS: $T = \frac{2}{3k} < \frac{1}{2} m_2 v_2^2 >$

Equating right sides of both these equations gives $\frac{2}{3k} < \frac{1}{2} m_1 v_1^2 > = \frac{2}{3k} < \frac{1}{2} m_2 v_2^2 >$

$<v_1^2>/<v_2^2> = m_2/m_1$

Taking square root on both sides gives

$$\frac{<v_1>}{<v_2>} = \sqrt{\frac{m_2}{m_1}}$$

It is proved that ratio of the root mean square speeds of molecules of two different gases at a certain temperature is equal to the root of the inverse ratio of their masses.

Question 11.3:- A sample of gas is compressed to one half of its initial volume at constant pressure of 1.25 x 10 ^{5} N m^{-2}. During the compressions, 100 J of work is done on the gas. Determine the final volume of the gas.

Solution:- Initial volume of the gas = V_1 = V

Final volume of the gas = V_2 = V/2

Change in volume of the gas = $\Delta V = V_1 - V_2$ = V/2

Applied pressure = P = 1.25 x 10^5 N m^{-2}

Work done on the gas = W = 100 J

We know that W = P ΔV

ΔV = W/P = (100)/(1.25 x 10^5) = 80 x 10^{-5} m^3 = 8.0 x 10^{-4} m^3

We also know that $\Delta V = V_2$ = V/2

Final volume = V_2 = 8.0 x 10^{-4} m^3

Question 11.4:- A thermodynamic system undergoes a process in which its internal energy decreases by 300 J. If at the same time 120 J of work is done on the system, find the heat lost by the system.

Solution:- Change in internal energy = ΔU = -300 J ∴Negative sign indicates decrease

Work done on the system = W = -120 J ∴Negative sign indicates work done on the system

First law of thermodynamics states that Q = W + ΔU

Q = (-120) + (-300)

Q = -420 J ∴Negative sign indicates that heat is lost by the system

Question 11.5:- A Carnot engine utilize an ideal gas. The source temperature is 227 °C and the sink temperature is 127 °C. Find the efficiency of the engine. Also find the heat input from the source and the heat rejected to the sink when 10000 J of work done.

Solution:- Temperature of the source = T_1 = 227 °C = 227 + 273 = 500 K

Temperature of the sink = T_2 = 127 °C = 127 + 273 = 400 K

Efficiency of the Carnot engine = $\eta = (1 - \frac{T_2}{T_1})$ x 100 %

$\eta = (1 - \frac{400}{500})$ x 100 % = 0.20 x 100 %

η = 20 %

Work done by the engine = W = 10000 J

Efficiency can also be expressed as $\eta = \frac{W}{Q_1}$ x 100 %

20 % = $\frac{10000}{Q_1}$ x 100 %

$\frac{10000}{Q_1} = 0.20$

Heat input from the source = Q_1 = 10000/0.20

Q_1 = 50000 J

Mathematical form of second law of thermodynamics is W = $Q_1 - Q_2$

Heat rejected by the engine = $Q_2 = Q_1 - W$ = 50000 – 10000

Q_2 = 40000 J

Question 11.6:- A reversible engine works between two temperatures whose difference is 100 °C. If it absorbs 746 J of heat from the source and rejects 546 J to the sink, calculate the temperature of the source and the sink.

Solution:- Difference of the temperature in degree celsius = $\Delta t = t_1 - t_2 = 100\ ^oC$

Difference of the temperature in kelvin = $\Delta T = T_1 - T_2 = (t_1 + 273) - (t_2 + 273) = t_1 - t_2$

$\Delta T = T_1 - T_2 = 100$ K -- Eq. (1)

Heat absorbed by the engine = $Q_1 = 746$ J

Heat rejected by the system = $Q_2 = 546$ J

Efficiency of the engine = $\eta = (1 - \frac{Q_2}{Q_1}) = (1 - \frac{546}{746})$

$\eta = 0.268$

Efficiency of the engine = $\eta = (1 - \frac{T_2}{T_1})$

$0.268 = 1 - \frac{T_2}{T_1}$

$\frac{T_2}{T_1} = 0.732$

$T_2 = 0.732\ T_1$ --- Eq. (2)

Put value of T_2 in Eq. (1)

$T_1 - 0.732\ T_1 = 100$

$0.268\ T_1 = 100$

$T_1 = 373\ K = 100\ ^oC$

Put value of T_1 in Eq. (2)

$T_2 = 0.732\ T_1 = 0.732\ (373)$

$T_2 = 273\ K = 0\ ^oC$

Question 11.7: -A mechanical engineer develops an engine, working between 327°C and 27°C and claims to have an efficiency of 52%. Does he claim correctly? Explain.

Solution:- Temperature of hot reservoir = $T_1 = 327\ ^oC = 327 + 273 = 600$ K

Temperature of cold reservoir = $T_2 = 27\ ^oC = 27 + 273 = 300$ K

Efficiency of the Carnot engine = $\eta = (1 - \frac{T_2}{T_1}) \times 100\ \%$

$\eta = (1 - \frac{300}{600}) \times 100\ \% = (1 - 0.50) \times 100\ \%$

$\eta = 50\ \%$

The claim of mechanical engineer, that efficiency is 52 %, is not correct.

Question 11.8:- A heat engine performs 100 J of work and at the same time rejects 400 J of heat energy to the cold reservoirs. What is the efficiency of the engine?

Solution:- Work done by the engine = W = 100 J

Heat rejected by the engine = Q_2 = 400 J

Mathematical form of second law of thermodynamics is $W = Q_1 - Q_2$

$Q_1 = W + Q_2 = 100 + 400$

$Q_1 = 500$ J

Efficiency of the engine = $\eta = (1 - \frac{Q_2}{Q_1}) \times 100\ \% = (1 - \frac{400}{500}) \times 100\ \%$

$\eta = (1 - 0.80) \times 100\ \%$

$\eta = 20\ \%$

Question 11.9:- A Carnot engine whose low temperature reservoir is at 7 °C has an efficiency of 50 %. It is desired to increase the efficiency to 70 %. By how many degrees the temperature of the source be increased?

Solution:- Temperature of the sink = $T_2 = 7\ ^oC = 7 + 273 = 280$ K

CASE 1: Initial efficiency of the engine = $\eta_1 = 50\ \% = 0.50$

Temperature of hot reservoir = T_1

Efficiency of the engine = $\eta = (1 - \frac{T_2}{T_1})$

$0.50 = (1 - \frac{280}{T_1})$

$\frac{280}{T_1} = 1 - 0.50 = 0.50$

$T_1 = 280 / 0.50$

$T_1 = 560\ K = 287\ ^oC$

CASE 2: Desired efficiency of the engine = $\eta_2 = 70\ \% = 0.70$

Temperature of hot reservoir = T_1'

Efficiency of the engine = $\eta_2 = (1 - \frac{T_2}{T_1'})$

$0.70 = (1 - \frac{280}{T_1'})$

$\frac{280}{T_1'} = 1 - 0.70 = 0.30$

$T_1' = 280 / 0.30$

$T_1' = 933\ K = 660\ ^oC$

Required increase in temperature of hot reservoir = $\Delta T_1 = T_1' - T_1 = 660 - 287$

$\Delta T_1 = 373\ ^oC$

Question 11.10:- A steam engine has a boiler that operates at 450 K. The heat changes water to steam, which drives the piston. The exhaust temperature of the outside air is about 300 K. What is the maximum efficiency of this steam engine?

Solution:- Temperature of the boiler = $T_1 = 450$ K

Temperature of the exhaust air = $T_2 = 300$ K

Efficiency of the engine = η = $(1 - \frac{T_2}{T_1})$ x 100 %

η = $(1 - \frac{300}{450})$ x 100 % = (1 - 0.67) x 100 %

η = 33 %

Question 11.11:- 336 J of energy is required to melt 1 g of ice at 0 °C. What is the change in entropy of 30 g of water at 0 °C as it is changed to ice at 0 °C by a refrigerator?

Solution:- We know that heat absorbed by one gram of solid to convert to its liquid phase is called latent heat of fusion. Conversely, heat extracted from one gram of liquid to convert it to its solid phase is called latent heat of freezing.

Latent heat of fusion = L_f = 336 J g^{-1}

Latent heat of freezing = -(Latent heat of fusion) = -336 J g^{-1}

Mass of water = m = 30 g

Temperature = T = 0 ^oC = 273 K

Change in entropy = ΔS = ΔQ/T

ΔQ = (Mass) (Latent heat of freezing) = (30) (-336) = -10080 J

ΔS = (-10080)/273

ΔS = -36.9 J K^{-1}