CHAPTER NO. 10(OPTICAL INSTRUMENTS)

Question 10.1:- A converging lens of focal length 5.0 cm is used as a magnifying glass. If the near point of observer is 25 cm and the lens is held close to the eye, calculate (i) the distance of object from the lens (ii) the angular magnification. What is the angular magnification when the final image is formed at infinity?

Solution:- Focal length = f = 5.0 cm

Near point = d = 25 cm

Distance of virtual image from the lens = q = d = 25 cm

(i) The lens formula for virtual image is $\frac{1}{f} = \frac{1}{p} - \frac{1}{q}$

$\frac{1}{p} = \frac{1}{f} + \frac{1}{q} = \frac{1}{5} + \frac{1}{25}$

$\frac{1}{p} = \frac{5+1}{25} = \frac{6}{25}$

$p = \frac{25}{6}$

p = 6.2 cm

(ii) Angular magnification = $M = 1 + \frac{d}{f} = 1 + \frac{25}{5} = 1 + 5$

M = 6

When object is placed at focus i.e. p = f, the image will be formed at infinity.

$M = \frac{Distance\ of\ image\ from\ the\ lens}{Distance\ of\ object\ from\ the\ lens} = \frac{q}{p} = \frac{d}{f}$

$M = \frac{25}{5}$

M = 5

Question 10.2:- A telescope objective has focal length 96 cm and diameter 12 cm. Calculate focal length and minimum diameter of a simple eye piece lens for use with the telescope, if the linear magnification required is 24 times and all the light transmitted by an objective from a distant point on the telescope axis is to fall on the eye piece.

Solution:- Focal length of objective lens = f_o = 96 cm

Diameter of objective lens = D_o = 12 cm

Magnification = M = 24

$M = \frac{f_o}{f_e}$

$24 = \frac{96}{f_e}$

$f_e = \frac{96}{24}$

f_e = 4 cm

All the light transmitted by an objective from a distant point on the telescope axis will fall on the eye piece if ratio of focal length of both lense is equal to ratio of diameters of both lens.

$$\frac{f_o}{f_e} = \frac{D_o}{D_e}$$

$$\frac{24}{4} = \frac{12}{D_e}$$

$D_e = (12)(4)/(48)$

$D_e = 0.50$ cm

Question 10.3:- A telescope is made of an objective of focal length 20 cm and an eye piece of 5.0 cm, both convex lenses. Find the angular magnification.

Solution:- Focal length of objective lens = f_o = 20 cm

Focal length of eyepiece = f_e = 5.0 cm

Magnification = M

$M = \frac{f_o}{f_e} = \frac{20}{5.0}$

M = 4

Question 10.4:- A simple astronomical telescope in normal adjustment has an objective of focal length 100 cm and an eye piece of 5.0 cm. (i) where the final image formed? (ii) calculate the angular magnification.

Solution:- Focal length of objective lens = f_o = 100 cm

Focal length of eyepiece = f_e = 5.0 cm

In normal adjustment, intermediate image of the object is formed on focus of eyepiece. This intermediate image works as object for eyepiece. When object is on focus, image is formed at infinity.

Magnification = M

$M = \frac{f_o}{f_e} = \frac{100}{5.0}$

M = 5

Question 10.5:- A point object is placed on the axis of and 3.6 cm from a thin convex lens of focal length 3.0 cm. A second thin convex lens of focal length 16.0 cm is placed coaxial with the first and 26.0 cm from it on the side away from the object. Find the position of final image produced by two lenses.

Solution:- Focal length of first lens = f_1 = 3.0 cm

Distance of object from first lens = p_1 = 3.6 cm

Focal length of second lens = f_2 = 16.0 cm

Distance between both lenses = L = $q_1 + p_2$ = 26.0 cm

First lens will produce real image between two lenses. We will apply lens formula to first lens as $\frac{1}{f_1} = \frac{1}{p_1} + \frac{1}{q_1}$

$\frac{1}{q_1} = \frac{1}{f_1} - \frac{1}{p_1}$

$\frac{1}{q_1} = \frac{1}{3} - \frac{1}{3.6} = \frac{3.6 - 3.0}{(3.0)(3.6)}$

$\frac{1}{q_1} = \frac{0.6}{(3.0)(3.6)}$

$q_1 = 18$ cm

$L = q_1 + p_2 = 26.0$ cm

$L - q_1 = p_2 = 26 - 18$

$p_2 = 8$ cm (This image will act as object for second lens)

We will apply lens formula to first lens as $\frac{1}{f_2} = \frac{1}{p_2} + \frac{1}{q_2}$

$\frac{1}{q_2} = \frac{1}{f_2} - \frac{1}{p_2} = \frac{1}{16} - \frac{1}{8} = \frac{1-2}{16}$

$q_2 = -16$ cm

The image is formed at a distance of 16 cm from the second lens. The negative sign indicates that image formed by second lens is virtual.

Question 10.6:- A compound microscope has lenses of focal length 1.0 cm and 3.0 cm. An object is placed 1.2 cm from the object lens. If a virtual image is formed, 25 cm from the eye, calculate separation of the lenses and the magnification of the instrument.

Solution:- Focal length of objective = $f_o = 1.0$ cm

Focal length of eyepiece = $f_e = 3.0$ cm

Distance of object from objective = $p_e = 1.2$ cm

We will apply lens formula to objective as $\frac{1}{f_o} = \frac{1}{p_o} + \frac{1}{q_o}$

$\frac{1}{q_o} = \frac{1}{f_o} - \frac{1}{p_o}$

$\frac{1}{q_o} = \frac{1}{1} - \frac{1}{1.2} = \frac{1.2 - 1.0}{1.2}$

$\frac{1}{q_o} = \frac{0.2}{1.2}$

$q_o = 6$ cm

Distance of final image from eyepiece = $q_e = -25$ cm (Negative sign indicates virtual image)

We will apply lens formula to eyepiece as $\frac{1}{f_e} = \frac{1}{p_e} + \frac{1}{q_e}$

$\frac{1}{p_e} = \frac{1}{f_e} - \frac{1}{q_e}$

$\frac{1}{p_e} = \frac{1}{3} - \frac{1}{-25} = \frac{1}{3} + \frac{1}{25}$

$\frac{1}{p_e} = \frac{25+3}{75} = \frac{28}{75}$

$p_e = 2.7$ cm

The separation between both lenses is $L = q_o + p_e$

$L = 6 + 2.7$

$L = 8.7$ cm

Magnification = $M = \frac{q_o}{p_o}(1 + \frac{d}{f_e})$

$M = \frac{6}{1.2}(1 + \frac{25}{3})$

$M = (5)(9.33)$

$M = 47$

Question 10.7:- Sodium light of wavelength 589 nm is used to view an object under microscope. If the aperture of the objective is 0.90 cm, (i) find the limiting angle of resolution, (ii) using visible light of any wavelength, what is the maximum limit of resolution for this microscope.

Solution:- Wavelength of light = λ = 589 nm = 5.89×10^{-7} m

Aperture of objective lens = D = 0.90 cm = 0.90×10^{-2} m

(i) $\alpha_{min} = 1.22\frac{\lambda}{D} = 1.22\frac{5.89 \times 10^{-7}}{0.90 \times 10^{-2}}$

$\alpha_{min} = 8.0 \times 10^{-5}$ rad

(ii) Maximum limit of resolution will be obtained with blue light of wavelength λ = 400 nm.

$\lambda = 400 \times 10^{-9}$ m $= 4 \times 10^{-7}$ m

$\alpha_{min} = 1.22\frac{\lambda}{D} = 1.22\frac{4.0 \times 10^{-7}}{0.90 \times 10^{-2}}$

$\alpha_{min} = 5.4 \times 10^{-5}$ rad

Question 10.8:- An astronomical telescope having magnifying power of 5 consist of two thin lenses 24 cm apart. Find the focal length of the lenses.

Solution:- Magnification = M = 5

Distance between lenses = L = 24 cm

$M = \frac{f_o}{f_e} = 5$

$f_o = 5 f_e$ -- Eq. (1)

$L = f_o + f_e = 24$ -- Eq. (2)

Put value of f_o in Eq. (2)

$5 f_e + f_e = 24$

$6 f_e = 24$

$f_e = 4$ cm

Put value of f_e in Eq. (1)

$f_o = 5\ (4)$

$f_o = 20$ cm

Question 10.9:- A glass light pipe in air will totally internally reflect a light ray if its angle of incidence is at least 39°. What is the minimum angle of total internal reflection if pipe is in water? (Refractive index of water=1.33).

Solution:- Critical angle for air-glass boundary = $\theta_C = 39.0^o$

Refractive index of glass = $n_g = \frac{1}{\sin\theta_C} = 1/\sin 39^o$

$n_g = 1.59$

Refractive index of water = $n_w = 1.33$

Apply Snell's law for glass-water boundary as $n_1 \sin\theta_1 = n_2 \sin\theta_2$

$n_1 = n_g$ & $n_2 = n_w$

$\theta_1 = \theta_C$ and $\theta_2 = 90^o$ for total internal reflection

$n_g \sin\theta_C = n_w \sin 90^o$

$\sin\theta_C = (1.33)/(1.59)$

$\theta_c = \sin^{-1}(0.84)$

$\theta_C = 57^o$

Question 10.10:- The refractive index of core and cladding of an optical fiber 1.6 and 1.4 respectively. Calculate (i) the critical angle for the interface (ii) the maximum angle of incidence in the air of a ray which enters the fiber and is incident at the critical angle on the interface.

Solution:- Refractive index of core = $n_{core} = 1.6$

Refractive index of cladding = $n_{cladding} = 1.4$

(i) Apply Snell's law for core-cladding interface as $n_1 \sin\theta_1 = n_2 \sin\theta_2$

$n_1 = n_{core}$ and $n_2 = n_{cladding}$

$\theta_1 = \theta_C$ and $\theta_2 = 90^o$ for total internal reflection

$(1.6) \sin\theta_C = (1.4) \sin 90^o$

$\theta_C = \sin^{-1}(1.4/1.6)$

$\theta_C = 61^o$

(ii) The critical angle for core-cladding interface is 61^o. It means the angle of refraction in core should be $\theta_1' = 90^o - 61^o = 29^o$ when ray of light enters in it from air.

Apply Snell's law on air-core boundary

$n_{air} \sin\theta_i = n_{core} \sin\theta_1'$

$(1) \sin\theta_i = (1.6) \sin 29^o$

$\sin \theta_i = 0.776$

$\theta_i = \sin^{-1} (0.776)$

$\theta_i = 51^o$