CHAPTER NO. 1(MEASUREMENTS)

Question 1.1:- A light year is the distance light travels in one year. How many metres are there in one light year: (speed of light = 3.0×10^8 m s^{-1}).

Solution:- Speed of light = c = 3.0×10^8 m s^{-1}

Time = t = 1 year = 365 days = 365 x 24 hours = 365 x 24 x 60 mints = 365 x 24 x 60 x 60 s

t = 31,536,000 s = 3.1536×10^7 s

Distance = S = Speed x Time = c x t = (3.0×10^8 m s^{-1}) x (3.1536×10^7 s) = 9.4608×10^{15} m

<u>One light year = 9.5×10^{15} m (rounded off to two significant figures)</u>

Question 1.2:- a) How many seconds are there in 1 year?

b) How many nanoseconds in 1 year?

c) How many years in 1 second?

Solution:- a) Time = t = 1 year = 365 days = 365 x 24 hours = 365 x 24 x 60 minutes = 365 x 24 x 60 x 60 s

<u>1 year = 31,536,000 s = 3.1536×10^7 s</u>

b) Time = t = 1 year = 365 days = 365 x 24 hours = 365 x 24 x 60 minutes = 365 x 24 x 60 x 60 s

1 year = 31,536,000 s = 3.1536×10^7 s

We know that 1 ns = 10^{-9} s

Rearranging gives 1 s = 10^9 s

<u>1 year = $3.1536 \times 10^7 \times 10^9$ ns = 3.1536×10^{16} ns</u>

c) We have already calculated number of seconds in 1 year as under:-

1 year = 3.1536×10^7 s

Divide both sides by 3.1536×10^7

1 s = $\frac{1\, year}{3.1536\, x\, 10^7}$ = 0.31×10^{-7} year = 3.1×10^{-8} year

<u>1 s = 3.1×10^{-8} year</u>

Question 1.3:- The length and width of a rectangular plate are measured to be 15.3 cm and 12.80 cm, respectively. Find the area of the plate.

Solution:- Length = L = 15.3 cm

Width = W = 12.80 cm

Area = A = Length x Width = L x W = (15.3 cm) x (12.80 cm) = 195.84 cm^2

<u>Area = A = 196 cm^2 (rounded off to three significant figures)</u>

Question 1.4:- Add the following masses given in kg upto appropriate precision. 2.189, 0.089, 11.8 and 5.32.

Solution:- $m_1 = 2.189$ kg

$m_2 = 0.089$ kg

$m_3 = 11.8$ kg

$m_4 = 5.32$ kg

$M = m_1 + m_2 + m_3 + m_4 = 2.189 + 0.089 + 11.8 + 5.32 = 19.398$ kg

M = 19.4 kg (only one digit will retained after decimal point)

Question 1.5:- Find the value of 'g' and its uncertainty using $T = 2\pi\sqrt{\frac{l}{g}}$ from the following measurements made during an experiment

Length of simple pendulum = l = 100 cm

Time for 20 vibrations = 40.2 s

Length was measured by a metre scale of accuracy upto 1 mm and time by stop watch of accuracy upto 0.1 s.

Solution:- $T = 2\pi\sqrt{\frac{l}{g}}$

Squaring both sides of $T = 2\pi\sqrt{\frac{l}{g}}$ and rearranging gives $g = 4\pi^2\frac{l}{T^2}$

Time for 20 vibrations = t = 40.2 s

Number of vibrations = n = 20

Time period = $T = {}^{t}/_{n} = {}^{40.2}/_{20} = 2.01$ s

Uncertainty in time = Δt = 0.1 s

Uncertainty in time period = ΔT = Δt/n = 0.1/20 = 0.005 s

Length of simple pendulum = l = 100 cm = 1 m

Uncertainty in length = Δl = 1 mm = 0.001 m

$g = 4\,(3.14)^2 \frac{1}{(2.01)^2} = 9.76$ m s^{-2}

Percentage error of length = $\frac{\Delta l}{l}$ x 100 % = $\frac{0.001}{1}$ x 100 % = 0.1 %

Percentage error of time period = $\frac{\Delta T}{T}$ x 100 % = $\frac{0.005}{2.01}$ x 100 % = 0.25 %

Percentage error of g = (Percentage error of length) + 2 (Percentage error of time period)

Percentage error of g = (0.1 %) + 2 (0.25 %) = 0.6 %

Absolute error of g = $\frac{Percentage\ error\ of\ g}{100}$ x Value of g = $\frac{0.6}{100}$ x 9.76 = 0.06 m s^{-2}

g = (9.76 ± 0.06) m s^{-2}

Question 1.6:- What are the dimensions and units of gravitational constant G in the formula $F = G\frac{m_1 m_2}{r^2}$?

Solution:- Given that $F = G\frac{m_1 m_2}{r^2}$

$G = F\frac{r^2}{m_1 m_2}$

Unit of G $= \frac{(Unit\ of\ F)(Unit\ of\ r)^2}{(Unit\ of\ m_1)(Unit\ of\ m_2)} = \frac{(N)\,(m)^2}{(kg)(kg)}$

Unit of G = $N\ m^2\ kg^{-2}$

Dimension of G = [G] $= \frac{[F][r^2]}{[m_1 m_2]} = \frac{[M\ L\ T^{-2}]\,[L^2]}{[M^2]}$

Dimension of G = [G] = $[M^{-1}\ L^3\ T^{-2}]$

Question 1.7:- Show that the expression $v_f = v_i + at$ is dimensionally correct, where v_i is the velocity at t = 0, a is the acceleration and v_f is the velocity at time t.

Solution:- According to principle of homogeneity of dimensions, an equation is correct dimensionally if dimensions of quantities on both sides of the equation are same.

Given equation $v_f = v_i + at$ contains three terms i.e. v_f, v_i and at.

Dimension of $v_f = [v_f] = [L\ T^{-1}]$

Dimension of $v_i = [v_i] = [L\ T^{-1}]$

Dimension of at = [at] = $[L\ T^{-2}]\ [T] = [L\ T^{-1}]$

It is clear that all the terms in the equation $v_f = v_i + at$ have same dimensions, so equation is correct dimensionally.

Question 1.8:- The speed v of sound waves through a medium may be assumed to depend on (a) the density ρ of the medium and (b) its modulus of elasticity E which is the ratio of stress to strain. Deduce by the method of dimensions, the formula for the speed of sound.

Solution:- Dimension of speed = [v] = $[L\ T^{-1}]$

Dimension of density = [ρ] $= \frac{Dimension\ of\ mass}{Dimension\ of\ volume} = \frac{[M]}{[L^3]} = [M\ L^{-3}]$

Dimension of modulus of elasticity = [E] $= \frac{Dimension\ of\ stress}{Dimension\ of\ strain}$

$$= \frac{Dimension\ of\ force \,/\, Dimension\ of\ area}{Dimension\ of\ volume \,/\, Dimension\ of\ change\ in\ volume} = \frac{[M\ L\ T^{-2}]/[L^2]}{[L^3]/[L^3]} = [M\ L^{-1}\ T^{-2}]$$

Given that $v \propto \rho^a E^b$

$v = Constant\ \rho^a E^b$ -- Eq. (1)

Using dimensional analysis, we can apply dimensions on both sides of above equation as under:-

$[v] = [\rho]^a\ [E]^b$ ∴Constant appearing in dimensional analysis is always dimensionless

$[L\ T^{-1}] = [M\ L^{-3}]^a\ [M\ L^{-1}\ T^{-2}]^b$

$[M^0\ L\ T^{-1}] = [M]^{a+b}\ [L^{-1}]^{-3a-b}\ [T]^{-2b}$

Comparing powers of M, L and T on both sides gives:-

Power of M: $0 = a + b$

Rearranging gives $a = -b$ -- Eq. (2)

Power of L: $1 = -3a - 2b$

Power of T: $-1 = -2b$ -- Eq. (3)

Rearranging Eq. (3) gives $\mathbf{b = \frac{1}{2}}$

Putting the value of b from Eq. (3) in Eq. (2) gives $\mathbf{a = -\frac{1}{2}}$

Put values of a and b in Eq. (1) gives v = Constant $\rho^{-\frac{1}{2}} E^{\frac{1}{2}}$

v = Constant $\frac{E^{\frac{1}{2}}}{\rho^{\frac{1}{2}}}$

v = Constant $\sqrt{\frac{E}{\rho}}$

Question 1.9:- Show that the famous "Einstein equation" $E = mc^2$ is dimensionally consistent.

Solution:- $E = m c^2$

L.H.S. of equation is energy, which can be found by multiplying force with displacement.

Dimension of E = $[E] = [F] [d] = [M L T^{-2}] [L] = [M L^2 T^{-2}]$

R.H.S. of equation is product of mass with square of speed of light.

Dimension of $m c^2 = [m c^2] = [M] [L T^{-1}]^2 = [M L^2 T^{-2}]$

Dimension of L.H.S. = Dimension of R.H.S.

The famous "Einstein equation" is dimensionally consistent.

Question 1.10:- Suppose, we are told that the acceleration of a particle moving in a circle of radius r with uniform speed v is proportional to some power of r, say r^n, and some power of v, say v^m, determine the powers of r and v?

Solution:- Dimension of acceleration = $[a] = [L T^{-2}]$

Dimension of radius = $[r] = [L]$

Dimension of speed = $[v] = [L T^{-1}]$

Given that $a \alpha r^n v^m$

a = Constant $r^n v^m$ -- Eq. (1)

Using dimensional analysis, we can apply dimensions on both sides of above equation as under:-

$[a] = [r]^n [v]^m$ ∴Constant appearing in dimensional analysis is always dimensionless

$[L T^{-2}] = [L]^n [L T^{-1}]^m$

$[L T^{-2}] = [L]^{n+m} [T]^{-m}$

Comparing powers of L and T on both sides gives:-

Power of L: $1 = n + m$

Rearrange as n = 1 – m -- Eq. (2)

Power of T: -2 = -m –--- Eq. (3)

Rearranging Eq. (3) gives **m = 2**

After putting the value of **m** from Eq. (3) in Eq. (2) gives n = 1 – 2 = -1.

n = -1, m = -2