Linear Inequalities in one variable:

The inequalities of the form $ax + b < 0, ax + b > c$

$ax + b \geq c$ $and\ ax + by \leq c\ are\ called$ $linear$ $inequalities$

In one variable.

Linear inequalities in two variables:

The inequalities of the form $ax + by + c,$

$ax + b > c\ and\ ax + by \geq c$ $and\ ax + by \leq 0$ $are\ called\ linear\ inqualities\ in\ two\ variables$ $ca\ nd\ y\ where\ a, b\ and\ c\ are\ constants.$

Corresponding Equation/ associated Equation:

(i) The corresponding equation to nay inequality is an equation formed by replacing the inequality symbol with an equal sign. For example

$Corresponding\ equation$ $of\ x + 2y < 6\ is$ $x + 2y = 6$

(ii) Corresponding equations of $x \geq 0\ and$ $2x + y \geq 2\ are\ x = 0\ and\ 2x + y = 2$ Respectively.

Graphing of a linear inequalities in two variables:

i. The corresponding equation is useful for graphing inequalities, because this equation forms the boundary line to the graph of given inequality.

ii. A vertical line (line $||to\ y\ axis)divides$ $the\ xy\ plane\ into\ two\ regions\ called\ half\ plane$ $(left\ half\ plane\ x - axis\ (x \leq 0)and\ right\ half$ $plane\ x \geq 0$

iii. A non- vertical line ($line\ ||to\ x - axis)divides$ $xy\ plane$ in two regions called "half planes". $(Upper\ half\ plane\ x \geq 0\ and\ lower\ half\ plane\ x \leq 0$

iv. If the inequality is strict ($<\ or\ >$) then we draw dashed or broken boundary line.

v. If the inequality is non-strict ($\leq\ or\ \geq\ then$ we draw a solid boundary line.

Procedure for graphing a linear inequality in two variables:

i. Graph the corresponding equation of given inequality.

ii. Select any test point(not on the graph of corresponding equation of inequality) The region $(0,0)is$ most convenient point to choose as a test pt.

iii. Put the coordinates of the test pt.in the inequality.

iv. If the test point satisfied the given inequality, then shade the half plane containing the test point.

v. If the test point does not satisfied the given inequality then the shade the half plane that does not contain the test point.

Solution set of linear inequalities:

The ordered pair $(a, b)which\ satisfy\ the\ linear$ inequality in two variables $x\ and\ y$ form the solution.

Solution Region:

Solution region of system of inequalities is the common region that satisfies all given inequality in the system.

Corner point / vertex:

A point of the solution region where two of its boundary lines intersect is called the corner point or vertex of the solution region.

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Exercise 5.1

1. Graph the solution of each of the following linear inequality in $xy - plane$.

(i) $2x + y \leq 6$

Solution:

$$2x + y \leq 6 \rightarrow (i)$$

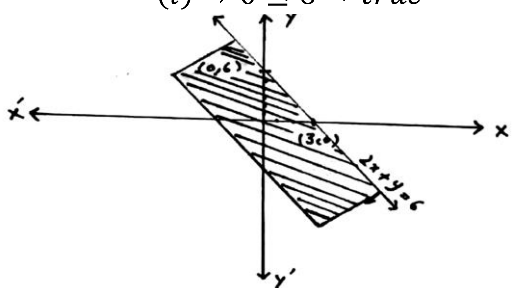
The associated eq. of (i) is $2x + y = 6 \rightarrow (ii)$

$(ii) \Rightarrow putx = 0, y = 6\ so\ that\ pt.(0,6)$

Put $y = 0, x = 3\ so\ that\ pt.(3,0)$

$Test\ pt(0,0): We\ test\ (i)at\ (0,0)$

$(i) \Rightarrow 0 \leq 6 \rightarrow true$



(ii) $3x + 7y \geq 21$

Solution:

$$3x + 7y \geq 21 \rightarrow (i)$$

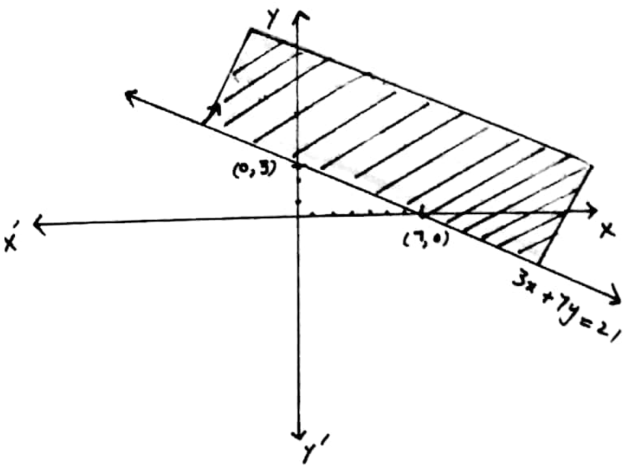
The associated eq. $of\ (i)is\ 3x + 7y = 21$

$(ii) \Rightarrow Putx = 0\ y = 3\ so\ that\ pt(0,3)$

$\Rightarrow put\ y = 0\ x = 7\ so\ pt(7,0)$

Test $pt(0,0)\ we\ test\ (i)at\ (0,0)$

$(i) \Rightarrow 0 \geq 21 \rightarrow false.$



(iii) $3x-2y\geq 6$

Solution:

$$3x-2y\geq 6\rightarrow(i)$$

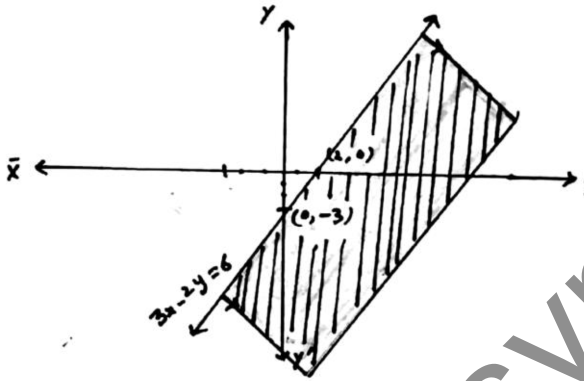
the associatd eq.of (i)is $3x-2y=6$

$(ii)\Rightarrow put x=0,y=-3$ *so the pt.* $(o,-3)$

put $y=0,x=2$ *so the pt.* $(2,0)$

Test $pt(0,0)$: *We test (i)at (o,o)so*

$(i)\Rightarrow 0\geq 6\rightarrow false$



(iv) $5x-4y\leq 20$

Solution:

$$5x-4y\leq 20\rightarrow(i)$$

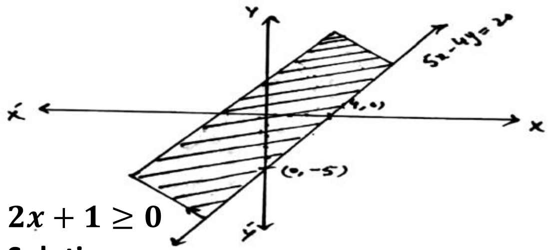
The associated $eq.of\ (i)is\ 5x-4y=20\rightarrow(ii)$

$(ii)\Rightarrow Put\ x=0,y=$
-5 *so the* $pt(0,-5)$

put $y=0,x=4$ *so the* $pt(4,o)$

Test $pt(o,o)$: *we test* $(i)at\ (0,0)so$

$(i)\Rightarrow 0\leq 20\rightarrow true$



(v) $2x+1\geq 0$

Solution:

$$2x+1\geq 0\rightarrow(i)$$

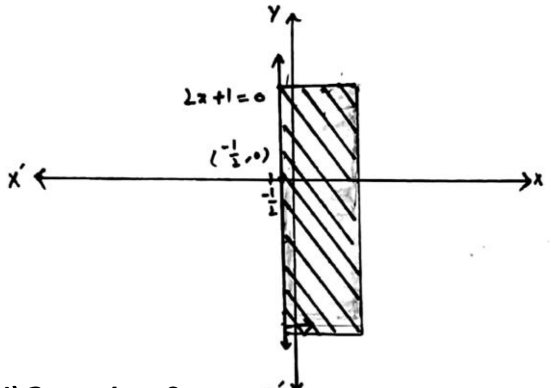
The associated eq. of $(i)is\ 2x+1=0$

$\Rightarrow 2x=-1\Rightarrow x$
$=-\frac{1}{2}(line\ ||to\ y-axis\ passing$

Through $\left(-\frac{1}{2},0\right)$

Test $pt(0,0)$: *we test (i)at (0,0) so*

$(i)\Rightarrow 2(0)+1\geq 0\Rightarrow 1\geq 0\rightarrow true$



(vi) $3y-4\leq 0$

Solution:

$$3y-4\leq 0\rightarrow(i)$$

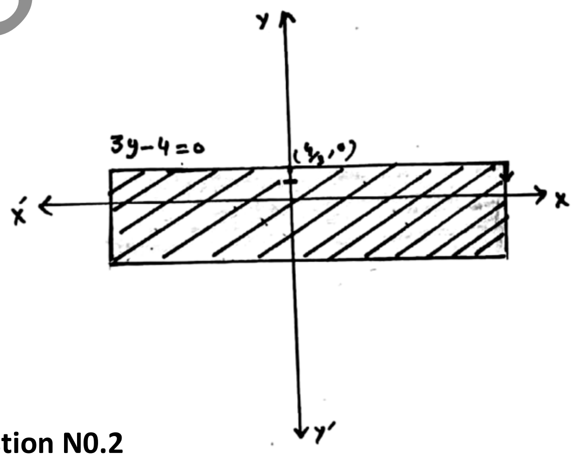
The associated $eq.of\ (i)\ is\ 3y-4=0$

$\Rightarrow 3y=4\Rightarrow y=\frac{4}{3}(line\ ||to\ x-axis\ passing$

Through $\left(0,\frac{4}{3}\right)$

Test $pt(0,0)$: *we test (i)at (0,0) so*

$(i)\Rightarrow 3(0)-4\leq 0\Rightarrow -4\leq 0\rightarrow true$



Question N0.2

Indicate the solution set of the following systems of linear inequalities of shading:

(i) $2x-3y\leq 6$
$2x+3y\leq 12$

Solution:

$$\mathbf{2x-3y\leq 6\rightarrow(i)}$$
$$\mathbf{2x+3y\leq 12\rightarrow(ii)}$$

the associated eqs. of (i)and (ii)are

$l_1; 2x-3y=6\rightarrow(iii) l_2; 2x+3y=12\rightarrow(iv)$

$(iii)\Rightarrow put x=0,y=-2$ *so the* $pt(0,-2)$

put $y=0,x=3$ *so the* $pt(3,0)$

$(iv)\Rightarrow put x=0,y=4$ *so the* $pt(0,4)$

put $y=0,x=6$ *so the* $pt(6,0)$

Test $pt(0,0)$: *we test (i)and (ii)at (0,0)so*

$(i)\Rightarrow o\leq 6\rightarrow true$

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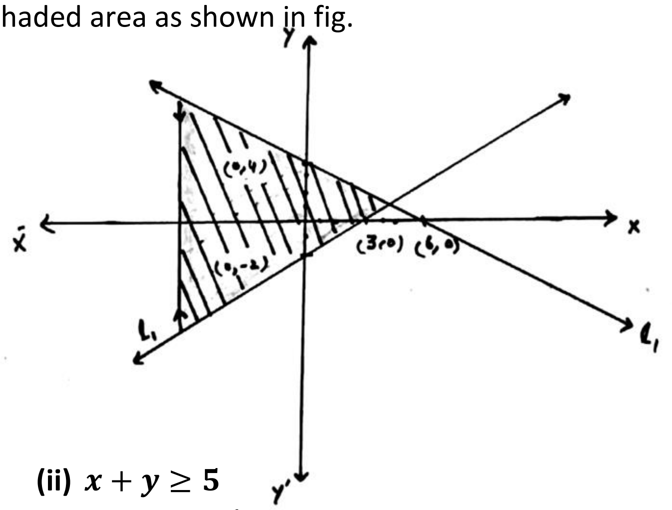
$(ii) \Rightarrow 0 \leq 12 \rightarrow true$

Solution region:

The solution of the given is intersection of the graphs of (i) and (ii). so solution region is shaded area as shown in fig.



(ii) $\boldsymbol{x + y \geq 5}$

$\boldsymbol{-y + x \leq 1}$

Solution:

$$\boldsymbol{x + y \rightarrow (i)}$$

$$\boldsymbol{-y + x \leq 1 \rightarrow (ii)}$$

$$the\ associated\ eqs.of\ (i) and\ (ii) are$$

$$\boldsymbol{l_1}; x + y \rightarrow (iii) l_2; -y + x \leq 1 \rightarrow (iv)$$

$$(iii) \Rightarrow put x = 0, y = 5\ so\ the\ pt(0,5)$$

$$put\ y = 0, x = 5\ so\ the\ pt(5,0)$$

$$(iv) \Rightarrow put x = 0, y = -1\ so\ the\ pt(0,-1)$$

$$put\ y = 0, x = 1\ so\ the\ pt(1,0)$$

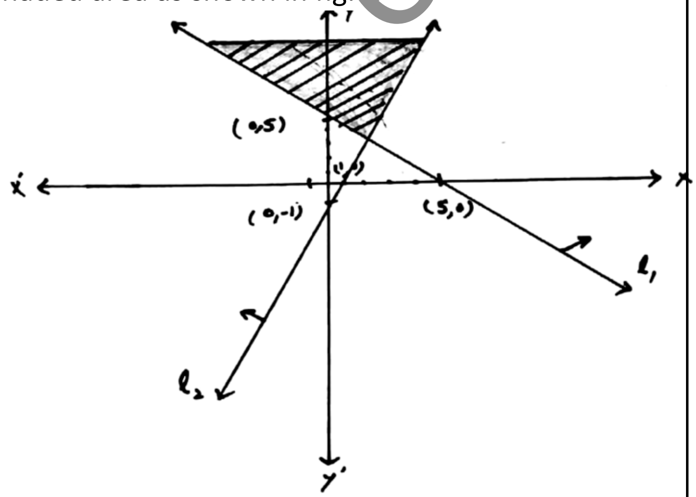
Test $pt(0,0)$: $we\ test\ (i) and\ (ii) at\ (0,0) so$

$$(i) \Rightarrow o \geq 5 \rightarrow true$$

$$(ii) \Rightarrow 0 \leq 1 \rightarrow true$$

Solution region:

The solution of the given is intersection of the graphs of (i) and (ii). so solution region is shaded area as shown in fig.



(iii) $\boldsymbol{3x + 7y \geq 21}$

$\boldsymbol{x - y \geq 2}$

Solution:

$$\boldsymbol{3x + 7y \geq 21 \rightarrow (i)}$$

$$\boldsymbol{x - y \geq 2 \rightarrow (ii)}$$

$$the\ associated\ eqs.of\ (i) and\ (ii) are$$

$$\boldsymbol{l_1}; 3x + 7y \geq 21 \rightarrow (iii) l_2; x - y \geq 2 \rightarrow (iv)$$

$$(iii) \Rightarrow put x = 0, y = 3\ so\ the\ pt(0,3)$$

$$put\ y = 0, x = 7\ so\ the\ pt(7,0)$$

$$(iv) \Rightarrow put x = 0, y = -2\ so\ the\ pt(0,-2)$$

$$put\ y = 0, x = 2\ so\ the\ pt(2,0)$$

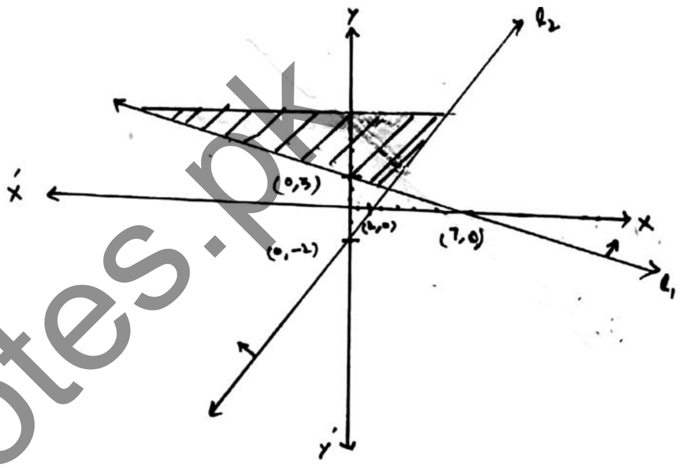
Test $pt(0,0)$: $we\ test\ (i) and\ (ii) at\ (0,0) so$

$$(i) \Rightarrow 0 \geq 21 \rightarrow true$$

$$(ii) \Rightarrow 0 \leq 2 \rightarrow true$$

Solution region:

The solution of the given system of the graphs of (i) and (ii). so solution region is shaded area as shown in fig.



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(iv) $\boldsymbol{4x - 3y \leq 12}$ $\boldsymbol{, x \geq -\frac{3}{2}}$

Solution:

$$4x - 3y \leq 12 \rightarrow (i)$$

$$x \geq -\frac{3}{2} \rightarrow (ii)$$

$$the\ associated\ eqs.of\ (i) and\ (ii) are$$

$$\boldsymbol{l_1}; 4x - 3y = 12 \rightarrow (iii) l_2; x \geq -\frac{3}{2} \rightarrow (iv)$$

$$(iii) \Rightarrow put x = 0, y = -4\ so\ the\ pt(0,-4)$$

$$put\ y = 0, x = 3\ so\ the\ pt(3,0)$$

$(iv) \Rightarrow put x = -\frac{3}{2}, (line\ || to\ y - axis\ through$

$pt\left(-\frac{3}{2}, o\right)$

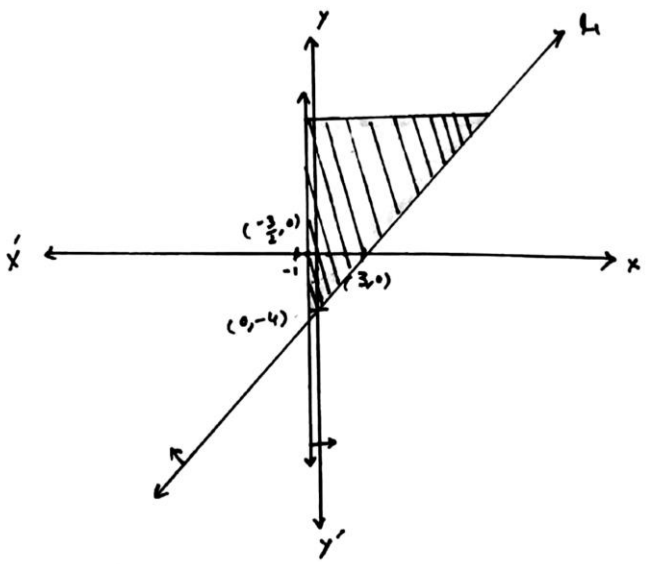
Test $pt(0,0)$: $we\ test\ (i) and\ (ii) at\ (0,0) so$

$$(i) \Rightarrow o \leq 12 \rightarrow true$$

$$(ii) \Rightarrow 0 \leq -\frac{3}{2} \rightarrow true$$

Solution region:

The solution of the given system is intersection of the graph of (i) and (ii). so solution region is shaded area as shown in fig.



(v) $\boldsymbol{3x+7y\geq 21,\ y\leq 4}$

Solution:

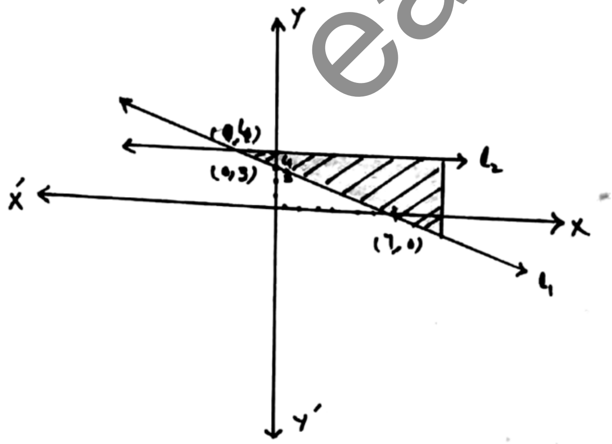
$$3x+7y\geq 21\rightarrow(i)$$
$$y\leq 4\rightarrow(ii)$$
$$the\ associated\ eqs.of\ (i)and\ (ii)are$$
$$\boldsymbol{l_1};3x+7y=21\rightarrow(iii)l_2;y=4\rightarrow(iv)$$
$$(iii)\Rightarrow putx=0,y=3\ so\ the\ pt(0,3)$$
$$put\ y=0,x=7\ so\ the\ pt(7,0)$$
$$(iv)\Rightarrow put\ y=4\ (line\ ||to\ x-axis\ through\ pt(0,4)$$

Test $pt(0,0)$: $we\ test\ (i)and\ (ii)at\ (0,0)so$

$$(i)\Rightarrow o\geq 21\rightarrow false$$
$$(ii)\Rightarrow 0\leq 4\rightarrow true$$

Solution region:

The solution of the given system is intersection of the graph of $(i)\ and\ (ii)$. so solution region is shaded area as shown in fig.



Question No.3

Indicate the solution region of the following systems of linear inequalities of shading.

(i) $\boldsymbol{2x-3y\leq 6\quad ,2x+3y\leq 12;y\geq 0}$

Solution:

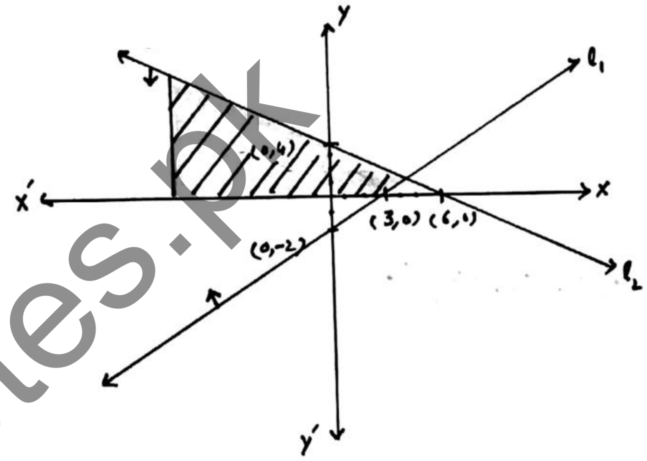
$$2x-3y\leq 6\rightarrow(i)$$
$$2x+3y\leq 12\rightarrow(ii)$$
$$the\ associated\ eqs.of\ (i)and\ (ii)are$$
$$\boldsymbol{l_1};2x-3y=6\rightarrow(iii)l_2;2x+3y\leq 12\rightarrow(iv)$$
$$(iii)\Rightarrow putx=0,y=-2so\ the\ pt(0,-2)$$
$$put\ y=0,x=3\ so\ the\ pt(3,0)$$
$$(iv)\Rightarrow putx=0,y=4\ so\ the\ pt(0,4)$$
$$put\ y=0,x=6\ so\ the\ pt(6,0)$$

Test $pt(0,0)$: $we\ test\ (i)and\ (ii)at\ (0,0)so$

$$(i)\Rightarrow 0\geq 6\rightarrow true$$
$$(ii)\Rightarrow 0\leq 12\rightarrow true$$

Solution region:

The solution of the given system is intersection of the graphs of $(i)\ and\ (ii)$. so solution set is upper half plane including the graph of boundary line $y=0$ as shown in fig.



(ii) $\boldsymbol{x+y\leq 5\ ,y-2x\leq 2\ ;x\geq 0}$

Solution:

$$x+y\leq 5\rightarrow(i)\ \ y-2x\leq 2\rightarrow(ii)$$
$$the\ associated\ eqs.of\ (i)and\ (ii)are$$
$$\boldsymbol{l_1};x+y\leq 5\rightarrow(iii)l_2;y-2x=2\rightarrow(iv)$$
$$(iii)\Rightarrow putx=0,y=5so\ the\ pt(0,5)$$
$$put\ y=0,x=5\ so\ the\ pt(5,0)$$
$$(iv)\Rightarrow putx=0,y=2\ so\ the\ pt(0,2)$$
$$put\ y=0,x=-1\ so\ the\ pt(-1,0)$$

Test $pt(0,0)$: $we\ test\ (i)and\ (ii)at\ (0,0)so$

$$(i)\Rightarrow 0\leq 5\rightarrow true$$
$$(ii)\Rightarrow 0\leq 2\rightarrow true$$

Solution region:

The solution of the given system is intersection of the graphs of $(i)\ and\ (ii)$.

Also $x\geq 0\ shows\ that\ the\ graph$ the solution set of right half plane including the graph of boundary line

$x=0$ As shown in fig.

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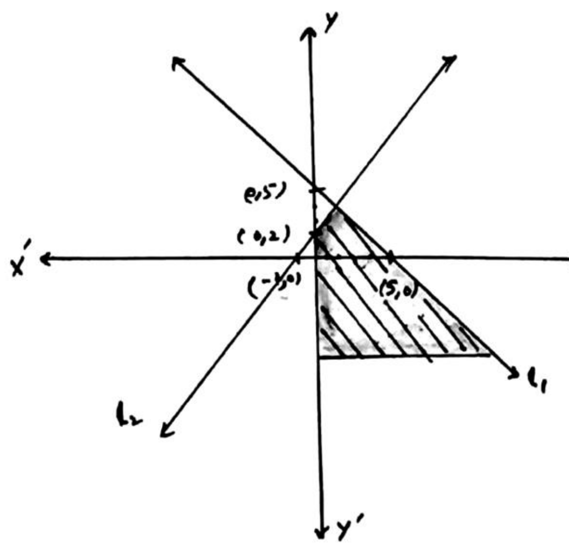


(iii) $\boldsymbol{x+y\geq 5}$; $\boldsymbol{x-y\geq 1}$; $\boldsymbol{y\geq 0}$

Solution:

$x+y\geq 5\rightarrow(i)\ \ x-y\geq 1\rightarrow(ii)$

$the\ associated\ eqs.of\ (i)and\ (ii)are$

$\boldsymbol{l_1}; x+y\geq 5\rightarrow(iii) l_2; x-y=1\rightarrow(iv)$

$(iii)\Rightarrow put x=0, y=5 so\ the\ pt(0,5)$

$put\ y=0, x=5\ so\ the\ pt(5,0)$

$(iv)\Rightarrow put x=0, y=-1\ so\ the\ pt(0,-1)$

$put\ y=0, x=1\ so\ the\ pt(1,0)$

Test $pt(0,0)$: $we\ test\ (i)and\ (ii)at\ (0,0)so$

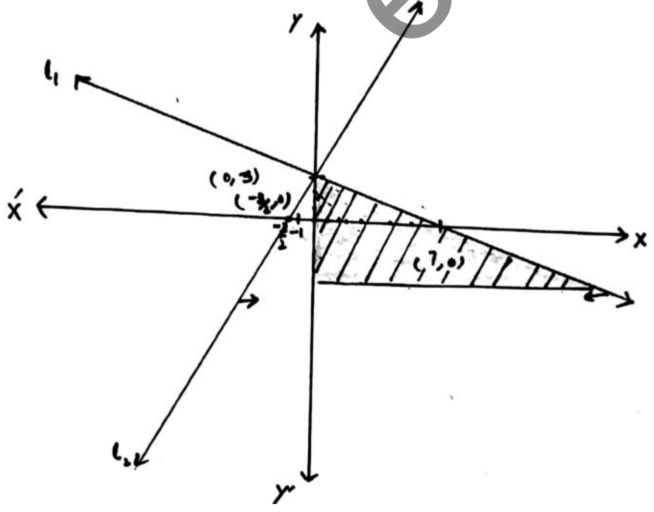
$(i)\Rightarrow 0\geq 5\rightarrow false$

$(ii)\Rightarrow 0\geq 1\rightarrow true$

Solution region:

The solution of the given system is intersection of the graphs of (i) and (ii).

Also $y\geq 0$ $shows\ that$ the solution set is upper half plane including the graph of boundary line $y=0$ as shown in fig.



(iv) $\boldsymbol{3x+7y\leq 21}$, $\boldsymbol{x-y\leq 2}$, $\boldsymbol{x\geq 0}$

Solution:

$3x+7y\leq 21\rightarrow(i)\ \ x-y\leq 2\rightarrow(ii)$

$the\ associated\ eqs.of\ (i)and\ (ii)are$

$\boldsymbol{l_1}; \boldsymbol{3}x+7y\leq 21\rightarrow(iii) l_2; x-y\leq 2\rightarrow(iv)$

$(iii)\Rightarrow put x=0, y=3\ so\ the\ pt(0,3)$

$put\ y=0, x=7\ so\ the\ pt(7,0)$

$(iv)\Rightarrow put x=0, y=-2\ so\ the\ pt(0,-2)$

$put\ y=0, x=2\ so\ the\ pt(2,0)$

Test $pt(0,0)$: $we\ test\ (i)and\ (ii)at\ (0,0)so$

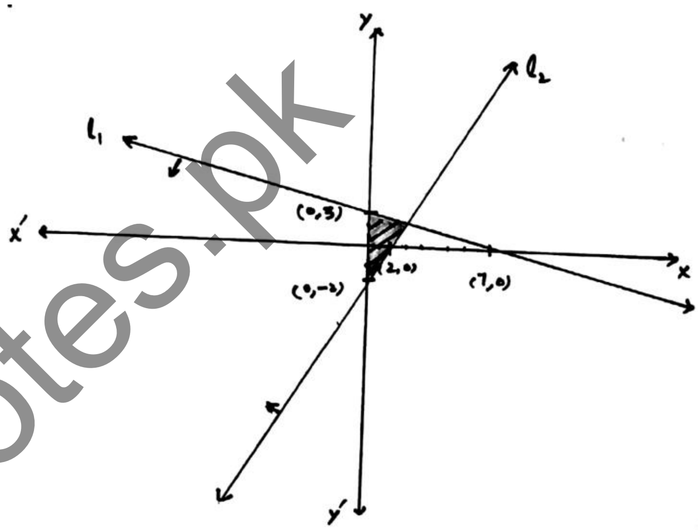
$(i)\Rightarrow 0\leq 21\rightarrow true$

$(ii)\Rightarrow 0\leq 2\rightarrow true$

Solution region:

The solution of the given system is intersection of the graphs of (i) and (ii).

Also $x\geq 0$ $shows\ that$ the solution set of right half plane including the graph of boundary line $x=0$ as shown in fig.



(v) $3x+7y\leq 21$; $x-y\leq 2$; $y=0$

Solution:

$3x+7y\leq 21\rightarrow(i)\quad x-y\leq 2\rightarrow(ii)$

$the\ associated\ eqs.of\ (i)and\ (ii)are$

$\boldsymbol{l_1}; 3x+7y\leq 21\rightarrow(iii) l_2; x-y\leq 2\rightarrow(iv)$

$(iii)\Rightarrow put x=0, y=3\ so\ the\ pt(0,3)$

$put\ y=0, x=7\ so\ the\ pt(7,0)$

$(iv)\Rightarrow put x=0, y=-2\ so\ the\ pt(0,-2)$

$put\ y=0, x=2\ so\ the\ pt(2,0)$

Test $pt(0,0)$: $we\ test\ (i)and\ (ii)at\ (0,0)so$

$(i)\Rightarrow 0\leq 21\rightarrow true$

$(ii)\Rightarrow 0\leq 2\rightarrow true$

Solution region:

The solution of the given system is intersection of the graphs of (i) and (ii).

Also $y\geq 0$ $shows\ that$ the solution set is upper half plane including the graph of boundary line $y=0$ as shown in fig.

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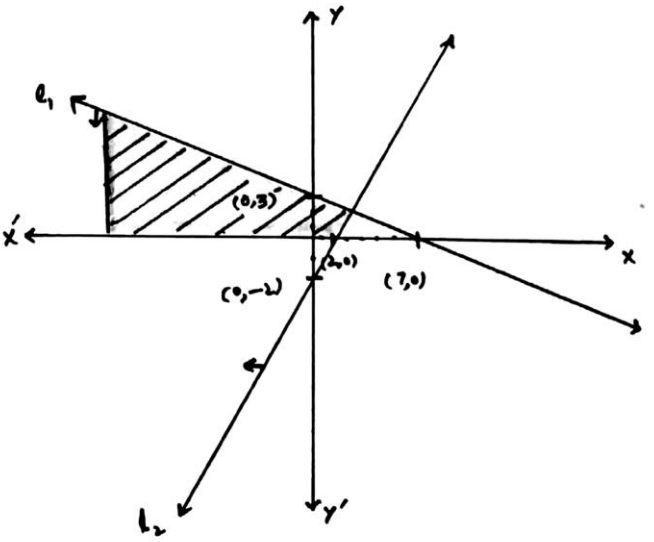


(vi) $\mathbf{3x+7y\le 21\ ,\ \ 2x-y\ge -3\ ,x\ge 0}$

Solution:

$$3x+7y\le 21\rightarrow(i)\ 2x-y\ge -3\rightarrow(ii)$$
$$the\ associated\ eqs.of\ (i)and\ (ii)are$$
$$\boldsymbol{l_1};3x+7y\le 21\rightarrow(iii)l_2;2x-y\le -3\rightarrow(iv)$$
$$(iii)\Rightarrow put x=0,y=3\ so\ the\ pt(0,3)$$
$$put\ y=0,x=7\ so\ the\ pt(7,0)$$
$$(iv)\Rightarrow put x=0,y=3 so\ the\ pt(0,3)$$
$$put\ y=0,x=-\tfrac{3}{2}\ so\ the\ pt(-\tfrac{3}{2},0)$$

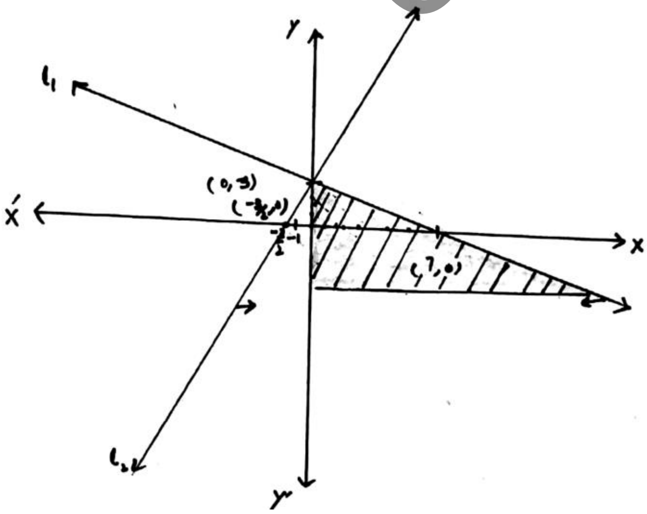
Test $pt(0,0)$: $we\ test\ (i)and\ (ii)at\ (0,0)so$

$$(i)\Rightarrow 0\le 21\rightarrow true$$
$$(ii)\Rightarrow 0\ge -3\rightarrow true$$

Solution region:

The solution of the given system is intersection of the graphs of (i) and (ii).

Also $x\ge 0$ $shows\ that$ the solution set of right half plane including the graph of boundary line $x=0$ as shown in fig.



Question No.4

Graph the solution region of the following system of linear inequalities and find the corner points in each case.

(i) $2x-3y\le 6\ ;2x+3y\le 12\ ;x\ge 0$

Solution:

$$2x-3y\le 6\rightarrow(i)\ 2x+3y\le 12\rightarrow(ii)$$
$$the\ associated\ eqs.of\ (i)and\ (ii)are$$
$$\boldsymbol{l_1};\mathbf{2}x-3y=6\rightarrow(iii)l_2;2x+3y\le 12\rightarrow(iv)$$
$$(iii)\Rightarrow put x=0,y=-2\ \ so\ the\ pt(0,-2)$$
$$put\ y=0,x=3\ \ so\ the\ pt(3,0)$$
$$(iv)\Rightarrow put x=0,y=4\ so\ the\ pt(0,4)$$
$$put\ y=0,x=6\ so\ the\ pt(6,0)$$

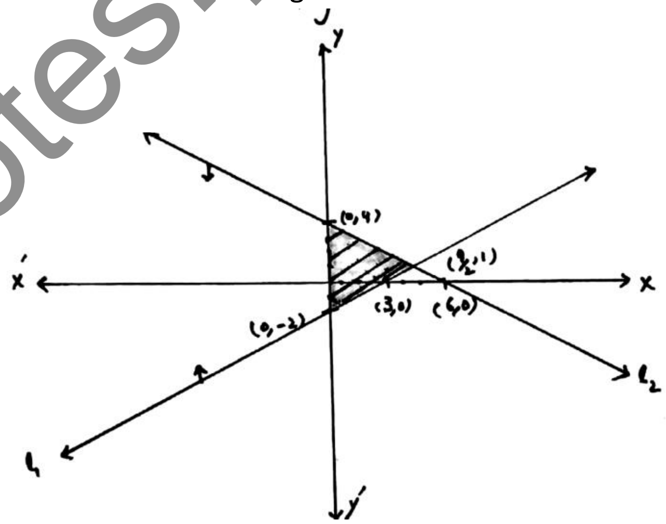
Test $pt(0,0)$: $we\ test\ (i)and\ (ii)at\ (0,0)so$

$$(i)\Rightarrow 0\le 6\rightarrow true$$
$$(ii)\Rightarrow 0\le 12\rightarrow true$$

Solution region:

The solution of the given system is intersection of the graphs of (i) and (ii).

Also $x\ge 0$ $shows\ that$ the solution set of right half plane including the graph of boundary line $x=0$ as shown in fig.



Corner point:

$$2x-3y=6\rightarrow(i)\ 2x+3y=12\rightarrow(ii)$$
$$by\ (i)+(ii)\Rightarrow 4x=18\Rightarrow x=\frac{9}{2}\ put\ in(ii)$$
$$2\left(\frac{9}{2}\right)+3y=12\Rightarrow 3y=12-9\Rightarrow y=1$$

$so\ \left(\frac{9}{2},1\right)\ is\ pt.of\ intersection\ of\ lines\ (i)and$ $(ii).hence\ corner\ points\ are\ (0,-2)(o,4)\left(\frac{9}{2},1\right)$

(ii) $\boldsymbol{x+y\le 5}\ ;\boldsymbol{-2x+y\le 2,\ \ y\ge 0}$

Solution:

$$x+y\le 5\rightarrow(i)\ \ and-2x+y\le 2\rightarrow(ii)$$
$$the\ associated\ eqs.of\ (i)and\ (ii)are$$
$$\boldsymbol{l_1};x+y=5\rightarrow(iii)l_2;-2x+y=2\rightarrow(iv)$$
$$(iii)\Rightarrow put x=0,y=5\ so\ the\ pt(0,5)$$

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$put\ y = 0, x = 5\ so\ the\ pt(5,0)$

$(iv) \Rightarrow putx = 0, y = 2\ so\ the\ pt(0,2)$

$put\ y = 0, x = -1\ so\ the\ pt(-1,0)$

Test $pt(0,0): we\ test\ (i)and\ (ii)at\ (0,0)so$

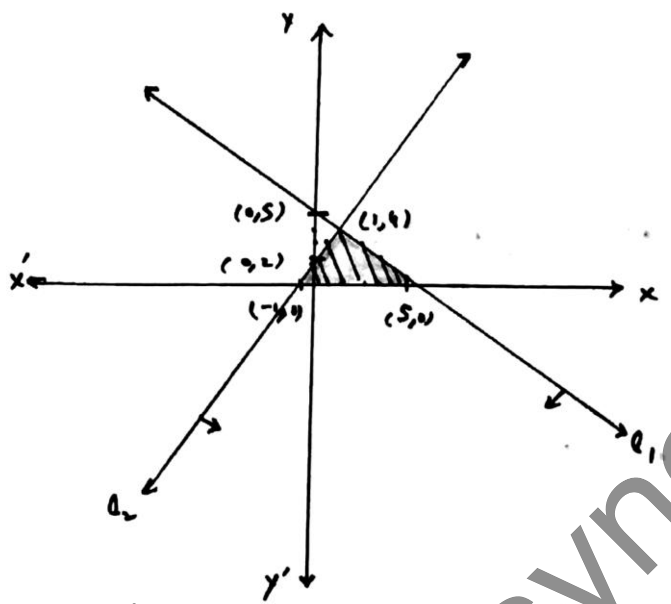
$(i) \Rightarrow 0 \leq 5 \rightarrow true$

$(ii) \Rightarrow 0 \leq 2 \rightarrow true$

Solution region:

The solution of the given system is intersection of the graphs of $(i)\ and\ (ii)$.

Also $y \geq 0\ shows\ that$ the solution set is upper half plane including the graph of boundary line $y = 0$ as shown in fig.



Corner point:

$x + y = 5 \rightarrow (i) - 2x + y = 2 \rightarrow (ii)$

$by\ (ii) - (i) \Rightarrow 3x = 3 \Rightarrow x = 1\ put\ in(i)1 + y = 5$

$\Rightarrow y = 4\ so\ (1,4)is\ the\ pt.of\ intersection\ of$ Lines

$(i)and(ii). hence\ corner\ point\ are\ (-1,0)$ $(5,0), (1,4)$

(iii) $\boldsymbol{3x + 7y \leq 21\ \ ; 2x - y \leq -3\ \ , y \geq 0}$

Solution:

$3x + 7y \leq 21 \rightarrow (i)\ \ 2x - y \leq -3 \rightarrow (ii)$

$the\ associated\ eqs.of\ (i)and\ (ii)are$

$\boldsymbol{l_1}; 3x + 7y = 21 \rightarrow (iii)l_2; 2x - y = -3 \rightarrow (iv)$

$(iii) \Rightarrow putx = 0, y = 3\ so\ the\ pt(0,3)$

$put\ y = 0, x = 7\ so\ the\ pt(7,0)$

$(iv) \Rightarrow putx = 0, y = 3\ so\ the\ pt(0,3)$

$put\ y = 0, x = -\frac{3}{2}\ so\ the\ pt(-\frac{3}{2}, 0)$

Test $pt(0,0): we\ test\ (i)and\ (ii)at\ (0,0)so$

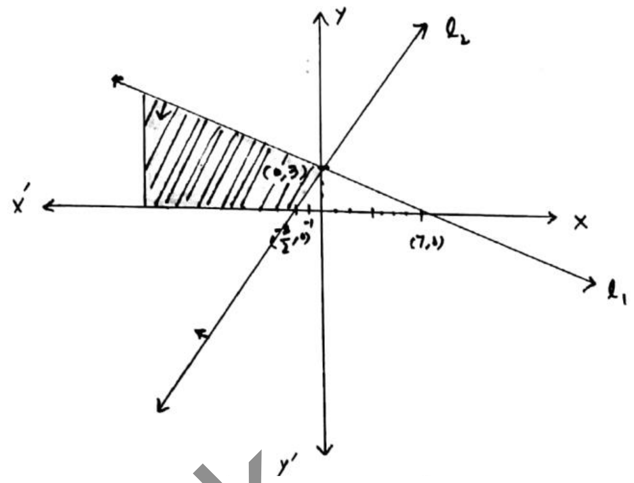
$(i) \Rightarrow 0 \leq 21 \rightarrow true$

$(ii) \Rightarrow 0 \leq -3 \rightarrow false$

Solution region:

The solution of the given system is intersection of the graphs of $(i)\ and\ (ii)$.

Also $y \geq 0\ shows\ that$ the solution set is upper half plane including the graph of boundary line $y = 0$ as shown in fig.



Corner point:

Corner pts. $are\ (0,3)and\ \left(-\frac{3}{2}, 0\right)$

(iv) $\boldsymbol{3x + 2y \geq 6 \rightarrow (i)\ \ x + 3y \leq 6\ \ ; y \geq 0}$

Solution:

$3x + 2y \geq 6 \rightarrow (i)\ \ and\ x + 3y \leq 6 \rightarrow (ii)$

$the\ associated\ eqs.of\ (i)and\ (ii)are$

$\boldsymbol{l_1}; 3x + y = 6 \rightarrow (iii)l_2; x + 3y = 6 \rightarrow (iv)$

$(iii) \Rightarrow put\ x = 0, y = 3\ \ so\ the\ pt(0,3)$

$put\ y = 0, x = 2so\ the\ pt(2,0)$

$(iv) \Rightarrow putx = 0, y = 2\ so\ the\ pt(0,2)$

$put\ y = 0, x = 6\ so\ the\ pt(6,0)$

Test $pt(0,0): we\ test\ (i)and\ (ii)at\ (0,0)so$

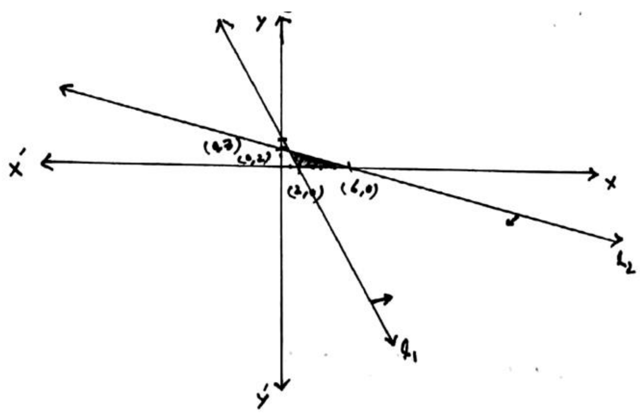
$(i) \Rightarrow 0 \geq 6 \rightarrow false$

$(ii) \Rightarrow 0 \leq 6 \rightarrow true$

Solution region:

The solution of the given system is intersection of the graphs of $(i)\ and\ (ii)$.

Also $y \geq 0\ shows\ that$ the solution set is upper half plane including the graph of boundary line $y = 0$ as shown in fig.



Corner point:

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$$as\ 3x + 2y = 6 \rightarrow (i)$$
$$\boldsymbol{x + 3y = 6 \rightarrow (ii)}$$

By $3(ii) - (i) \Rightarrow \begin{array}{c} 3x+9y=18 \\ \underline{\pm 3x \pm 2y = \pm 6} \\ 7y=12 \end{array}$

$$\Rightarrow y = \frac{12}{7}\ put\ in(ii)$$
$$\Rightarrow x = 6 - \frac{36}{7} = \frac{42-36}{7} = \frac{6}{7}$$

So $pt.\,of\ intersection\ of\ lines\ (i) and\ (ii) is$ $\left(\frac{6}{7}, \frac{12}{7}\right)$ $hence\ corner\ pts\ are\ (2,0), (6,0), \left(\frac{6}{7}, \frac{12}{7}\right)$

(v) $\boldsymbol{5x + 7y \le 35\ ;\ -x + 3y \le 3\ ; x \ge 0}$

Solution:

$$5x + 7y \le 35 \rightarrow (i)\quad -x + 3y \le 3 \rightarrow (ii)$$
$$the\ associated\ eqs.\ of\ (i) and\ (ii) are$$
$$\boldsymbol{l_1}; 5x + 7y = 35 \rightarrow (iii) l_2; -x + 3y = 3 \rightarrow (iv)$$
$$(iii) \Rightarrow put x = 0, y = 5\ so\ the\ pt(0,5)$$
$$put\ y = 0, x = 7\ so\ the\ pt(7,0)$$
$$(iv) \Rightarrow put x = 0, y = 1\ so\ the\ pt(0,1)$$
$$put\ y = 0, x = -3\ so\ the\ pt(-3,0)$$

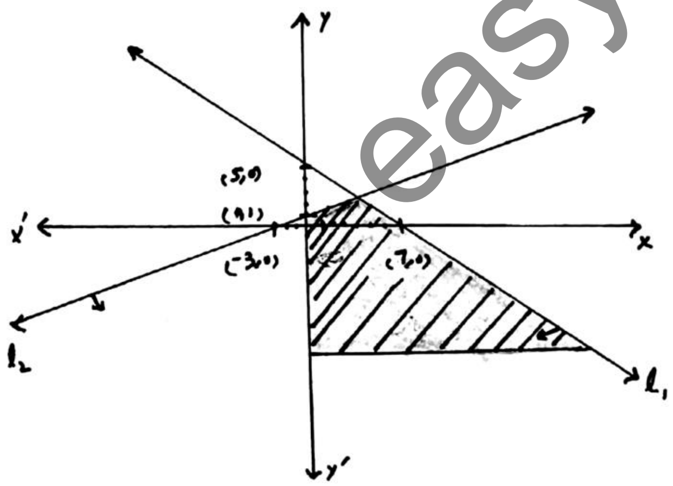
Test $pt(0,0)$: $we\ test\ (i) and\ (ii) at\ (0,0) so$

$$(i) \Rightarrow 0 \le 35 \rightarrow true$$
$$(ii) \Rightarrow 0 \le 3 \rightarrow true$$

Solution region:

The solution of the given system is intersection of the graphs of (i) and (ii).

Also $x \ge 0$ $shows\ that$ the solution set of right half plane including the graph of boundary line $x = 0$ as shown in fig.



Corner point: as $5x + 7y = 35 \rightarrow (i)$

$$-x + 3y = 3 \rightarrow (ii)$$

By $5(ii) + (i) \Rightarrow 22y - 50 \Rightarrow y = \frac{50}{22}$ $put\ in\ (ii)$

$$-x + 3\left(\frac{50}{22}\right) = 3$$
$$\Rightarrow -x = 3 - \frac{150}{22} = 3 - \frac{75}{11} = \frac{33-75}{11} = -\frac{42}{11}$$
$$\Rightarrow -x = -\frac{42}{11} \Rightarrow x = \frac{42}{11}$$

So $pt.\,of\ intersection\ of\ lines\ (i) and\ (ii) is$ $\left(\frac{42}{11}, \frac{25}{11}\right)$ $so\ corner\ points\ are$ $(0,1), (7,0), \left(\frac{42}{11}, \frac{25}{11}\right)$.

(vi) $\boldsymbol{5x + 7y \le 35;\quad x - 2y \le 2;\quad x \ge 0}$

Solution:

$$5x + 7y \le 35 \rightarrow (i)\quad x - 2y \le 2 \rightarrow (ii)$$
$$the\ associated\ eqs.\ of\ (i) and\ (ii) are$$
$$\boldsymbol{l_1}; 5x + 7y \le 35 \rightarrow (iii) l_2; x - 2y \le 2 \rightarrow (iv)$$
$$(iii) \Rightarrow put x = 0, y = 5\ so\ the\ pt(0,5)$$
$$put\ y = 0, x = 7\ so\ the\ pt(7,0)$$
$$(iv) \Rightarrow put x = 0, y = -1\ so\ the\ pt(0,-1)$$
$$put\ y = 0, x = 2\ so\ the\ pt(2,0)$$

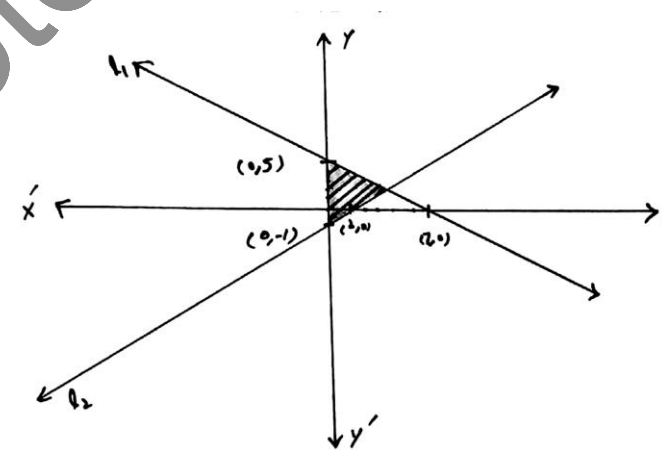
Test $pt(0,0)$: $we\ test\ (i) and\ (ii) at\ (0,0) so$

$$(i) \Rightarrow 0 \le 35 \rightarrow true$$
$$(ii) \Rightarrow 0 \le 2 \rightarrow true$$

Solution region:

The solution of the given system is intersection of the graphs of (i) and (ii).

Also $x \ge 0$ $shows\ that$ the solution set of right half plane including the graph of boundary line $x = 0$ as shown in fig.



Corner points:

$$as\ 5x + 7y = 35 \rightarrow (i)$$
$$\boldsymbol{x - 2y = 2 \rightarrow (ii)}$$

By $\boldsymbol{5(ii) - (i)} \Rightarrow$

$$\begin{array}{r} 5x - 10y = 10 \\ \underline{\pm 5x \pm 7y = \pm 35} \\ -17y = -25 \Rightarrow y = \frac{25}{17} \end{array}$$

$$put\ in(ii) \Rightarrow x - 2\left(\frac{25}{17}\right) = 2 \Rightarrow x = 2 + \frac{50}{17}$$
$$x = \frac{34+50}{17} \Rightarrow x = \frac{84}{17}$$

So $pt.\,of\ intersection\ of\ lins\ (i) and\ (ii) are$ $\left(\frac{84}{17}, \frac{25}{17}\right)$ $.\,so\ corner\ pts.\ are\ (0,-2), (0,5)$ $\left(\frac{84}{17}, \frac{25}{17}\right)$.

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Question No.5

Graph the solution region of the following system of linear inequalities by shading

(i) $3x - 4y \le 12$; $3x + 2y \ge 3$, $x + 2y \le 9$

Solution:

$3x - 4y \le 12 \rightarrow (i)\ ; 3x + 2y \ge 3 \rightarrow (ii)$

$x + 2y \le 9 \rightarrow (iii)$

The associated $eqs.\ of\ (i), (ii) and\ (iii) are$

$l_1; 3x - 4y = 12 \rightarrow (iv)\quad , l_2; 3x + 2y = 3 \rightarrow (ii)$

$l_3\ ;\ \ x + 2y = 9 \rightarrow (vi)$

$(iv) \Rightarrow put x = 0, y = -3\quad so\ that\ pt. (0, -3)$

$put\ y = 0, x = 4\ \ so\ the\ pt. (4,0)$

$(v) \Rightarrow put x = 0, y = -\frac{3}{2}\ \ so\ the\ pt. \left(0, \frac{3}{2}\right)$

$put\ y = 0, x = 1\ so\ the\ pt(1,0)$

$(vi) \Rightarrow put x = 0, y = \frac{9}{2}\ so\ the\ pt\ \left(0, \frac{9}{2}\right)$

$put\ y = 0, x = 9\ so\ the\ pt. (9,0)$

Test $pt(0,0)$: $we\ test\ (i), (ii) and\ (iii) at\ (0,0)$

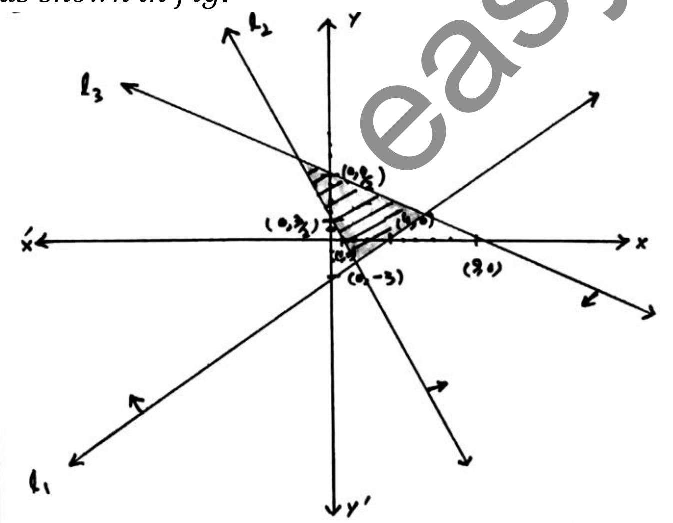
$so\ (i) \Rightarrow 0 \le 12 \rightarrow true\quad (ii) \Rightarrow 0 \ge 3 \rightarrow false$

$(iii) \Rightarrow 0 \le 9 \rightarrow true$

Solution region:

The solution of the given system is intersection of

$(i), (ii) and\ (iii) so\ solution\ region\ is\ shaded\ area\ as\ shown\ in\ fig.$



(ii) $3x - 4y \le 12\ ; x + 2y \le 6; x + y \ge 1$

Solution:

$3x - 4y \le 12 \rightarrow (i)\ ; x + 2y \ge 6 \rightarrow (ii)$

$x + y \ge 1 \rightarrow (iii)$

The associated $eqs.\ of\ (i), (ii) and\ (iii) are$

$l_1; 3x - 4y = 12 \rightarrow (iv)\quad , l_2; x + 2y = 6 \rightarrow (ii)$

$l_3\ ;\ \ x + y = 1 \rightarrow (vi)$

$(iv) \Rightarrow put x = 0, y = -3\quad so\ that\ pt. (0, -3)$

$put\ y = 0, x = 4\ \ so\ the\ pt. (4,0)$

$(v) \Rightarrow put x = 0, y = 3\ so\ the\ pt. (0,3)$

$put\ y = 0, x = 6\ so\ the\ pt(6,0)$

$(vi) \Rightarrow put\ x = 0, y = 1\ so\ the\ pt\ (0,1)$

$put\ y = 0, x = 1\ so\ the\ pt. (1,0)$

Test $pt(0,0)$: $we\ test\ (i), (ii) and\ (iii) at\ (0,0)$

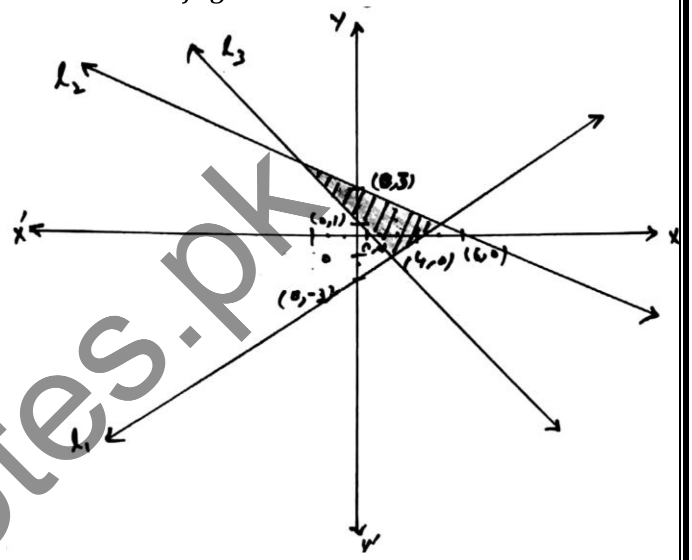
$so\ (i) \Rightarrow 0 \le 12 \rightarrow true\quad (ii) \Rightarrow 0 \le 6 \rightarrow true$

$(iii) \Rightarrow 0 \ge 1 \rightarrow false$

Solution region:

The solution of the given system is intersection of

$(i), (ii) and\ (iii) so\ solution\ region\ is\ shaded\ area\ as\ shown\ in\ fig.$



(iii) $2x + y \le 4\ ; 2x - 3y \ge 12; x + 2y \le 12$

$x + 2y \le 6$

Solution:

$2x + y \le 4 \rightarrow (i)\ ; 2x - 3y \ge 12 \rightarrow (ii);$

$x + 2y \le 12 \rightarrow (iii)$

The associated $eqs.\ of\ (i), (ii) and\ (iii) are$

$l_1; 2x + y = 4 \rightarrow (iv)\quad , l_2; 2x - 3y = 12 \rightarrow (ii)$

$l_3\ ;\ \ 2x - 3y = 12 \rightarrow (vi)$

$(iv) \Rightarrow put x = 0, y = 4\quad so\ that\ pt. (0,4)$

$put\ y = 0, x = 2\ \ so\ the\ pt. (2,0)$

$(v) \Rightarrow put\ x = 0, y = -4\ so\ the\ pt. (0, -4)$

$put\ y = 0, x = 6\ so\ the\ pt(6,0)$

$(vi) \Rightarrow put\ x = 0, y = 3\ so\ the\ pt\ (0,3)$

$put\ y = 0, x = 6\ so\ the\ pt. (6,0)$

Test $pt(0,0)$: $we\ test\ (i), (ii) and\ (iii) at\ (0,0)$

$so\ (i) \Rightarrow 0 \le 4 \rightarrow true\quad (ii) \Rightarrow 0 \ge 12 \rightarrow false$

$(iii) \Rightarrow 0 \le 6 \rightarrow true$

Solution region:

The solution of the given system is intersection of

$(i), (ii) and\ (iii) so\ solution\ region\ is\ shaded\ area\ as\ shown\ in\ fig.$

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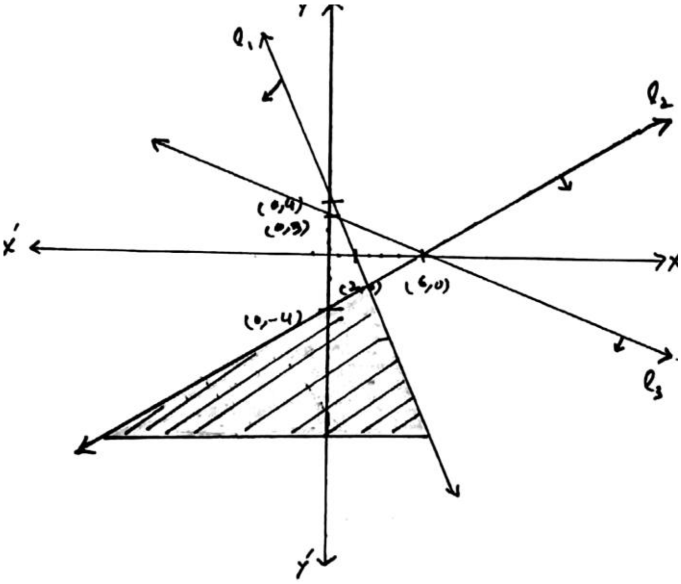


(iv) $\boldsymbol{2x + y \leq 10;\ \ x + y \leq 7;\ -2x + y \leq 4}$

Solution:

$2x + y \leq 10 \rightarrow (i);\ \ x + y \leq 7 \rightarrow (ii);$
$-2x + y \leq 4 \rightarrow (iii)$

The associated $eqs. of\ (i), (ii) and\ (iii) are$

$l_1; 2x + y = 10 \rightarrow (iv)\ \ \ , l_2; x + y = 7 \rightarrow (ii)$

$l_3\ ;\ -2x + y = 4 \rightarrow (vi)$

$(iv) \Rightarrow putx = 0, y = 10\ \ so\ that\ pt. (0,4)$

$put\ y = 0, x = 5\ \ so\ the\ pt. (5,0)$

$(v) \Rightarrow put\ x = 0, y = 7\ so\ the\ pt. (0, 7)$

$put\ y = 0, x = 7\ so\ the\ pt(7,0)$

$(vi) \Rightarrow put\ x = 0, y = 4\ so\ the\ pt\ (0,4)$

$put\ y = 0, x = -2\ so\ the\ pt. (-2,0)$

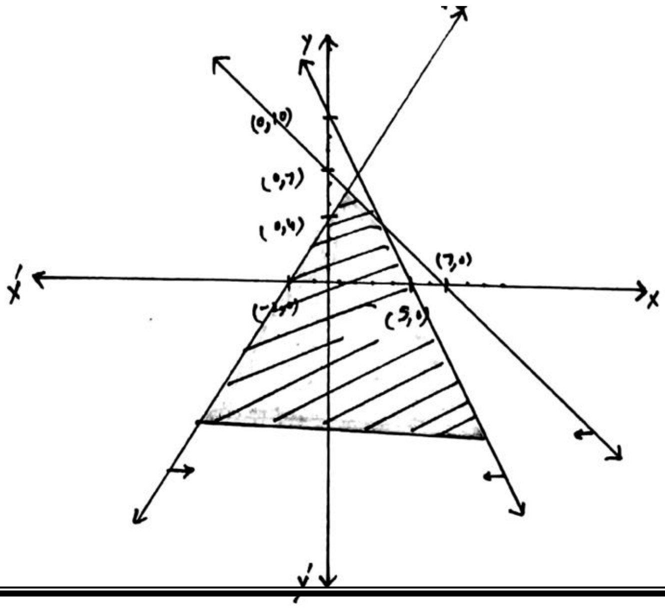
Test $pt(0,0)$: $we\ test\ (i), (ii) and\ (iii) at\ (0,0)$

$so\ (i) \Rightarrow 0 \leq 10 \rightarrow true\ \ \ (ii) \Rightarrow 0 \leq 7 \rightarrow true$

$(iii) \Rightarrow 0 \leq 4 \rightarrow true$

Solution region:

The solution of the given system is intersection of $(i), (ii) and\ (iii) so\ solution\ region\ is\ shaded\ area\ as\ shown\ in\ fig.$



(v) $\boldsymbol{2x + 3y \leq 18; 2x + y \leq 10\ ;\ -2x + y \leq 2}$

Solution:

$2x + 3y \leq 18 \rightarrow (i)\ 2x + y \leq 10 \rightarrow (ii)$
$;\ -2x + y \leq 2 \rightarrow (iii)$

The associated $eqs. of\ (i), (ii) and\ (iii) are$

$l_1; 2x + 3y = 18 \rightarrow (iv)\ \ \ , l_2; 2x + y = 10 \rightarrow (ii)$

$l_3\ ;\ -2x + y = 2 \rightarrow (vi)$

$(iv) \Rightarrow putx = 0, y = 6\ \ so\ that\ pt. (0,6)$

$put\ y = 0, x = 9\ so\ the\ pt. (9,0)$

$(v) \Rightarrow put\ x = 0, y = 10\ so\ the\ pt. (0, 10)$

$put\ y = 0, x = 5\ so\ the\ pt(5,0)$

$(vi) \Rightarrow put\ x = 0, y = 2\ so\ the\ pt\ (0,2)$

$put\ y = 0, x = -1\ so\ the\ pt. (-1,0)$

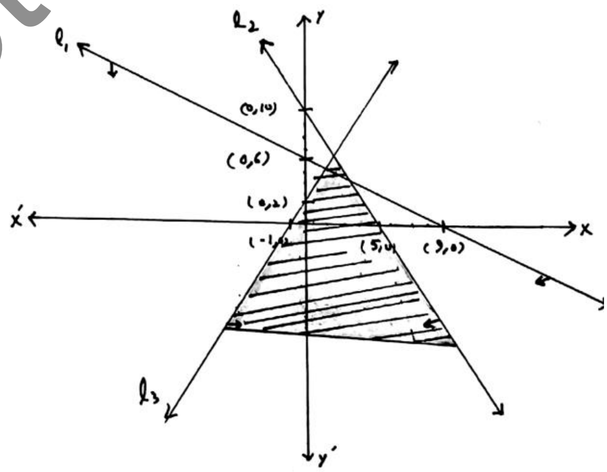
Test $pt(0,0)$: $we\ test\ (i), (ii) and\ (iii) at\ (0,0)$

$so\ (i) \Rightarrow 0 \leq 18 \rightarrow true\ \ \ (ii) \Rightarrow 0 \leq 10 \rightarrow true$

$(iii) \Rightarrow 0 \leq 2 \rightarrow true$

Solution region:

The solution of the given system is intersection of $(i), (ii) and\ (iii) so\ solution\ region\ is\ shaded\ area\ as\ shown\ in\ fig.$



(vi) $\boldsymbol{3x - 2y \geq 3;\ \ x + 4y \leq 12\ ; 3x + y \leq 12}$

Solution:

$3x - 2y \geq 3 \rightarrow (i)\ x + 4y \leq 12 \rightarrow (ii)$
$3x + y \leq 12 \rightarrow (iii)$

The associated $eqs. of\ (i), (ii) and\ (iii) are$

$l_1; 3x - 2y = 3 \rightarrow (iv)\ \ \ , l_2; x + 4y = 12 \rightarrow (ii)$

$l_3\ ;\ 3x + y = 12 \rightarrow (vi)$

$(iv) \Rightarrow putx = 0, y = -\frac{3}{2}\ \ so\ that\ pt. \left(0, -\frac{3}{2}\right)$

$put\ y = 0, x = 1\ so\ the\ pt. (1,0)$

$(v) \Rightarrow put\ x = 0, y = 3\ so\ the\ pt. (0, 3)$

$put\ y = 0, x = 12\ so\ the\ pt(12,0)$

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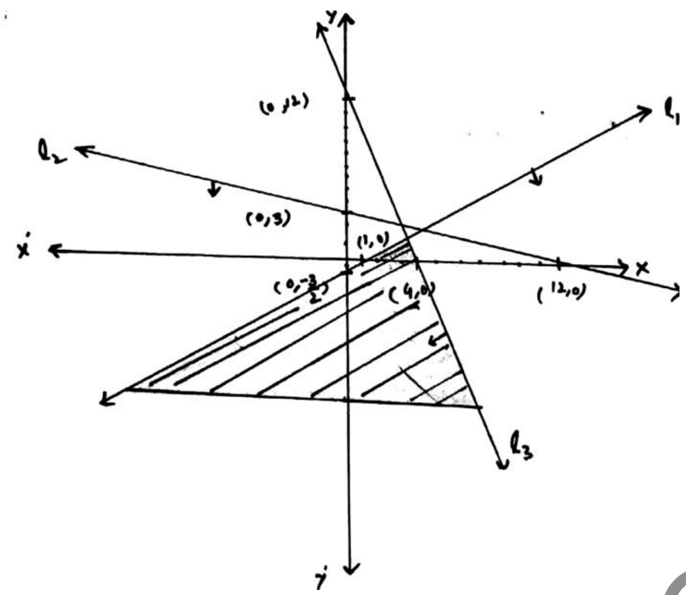
$(vi) \Rightarrow put\ x = 0, y = 12\ so\ the\ pt\ (0,12)$
$put\ y = 0, x = 4\ so\ the\ pt. (4,0)$

Test $pt(0,0)$: $we\ test\ (i), (ii) and\ (iii) at\ (0,0)$
$so\ (i) \Rightarrow 0 \geq 3 \rightarrow false \quad (ii) \Rightarrow 0 \leq 12 \rightarrow true$
$(iii) \Rightarrow 0 \leq 12 \rightarrow true$

Solution region:

The solution of the given system is intersection of
$(i), (ii) and\ (iii) so\ solution\ region\ is\ shaded\ area\ as\ shown\ in\ fig.$



Problem constraints:
The restrictions applied on the everyday life problems are called problem concentration.

Non- Negative constraints:
The constraints that are always satisfied are called natural constraints or non- negative constraints.

Decision variable:
The variable used in non- negative constraints are called decision variable.

Feasible region:
The solution region which is restricted to the first quadrant is called feasible region. We restricted the solution region by using non-negative constraints $x \geq 0\ and\ y \geq 0$

Feasible solution:
Each point of feasible region is called feasible solution of the system.

Feasible solution Set:
A set consists of all the feasible solution of the system is called feasible solution.

Exercise 5.2

Graph the feasible region of the following system of linear inequalities and find the corner points in each case

(i) $\mathbf{2x - 3y \leq 6\ ; 2x + 3y \leq 12\ ; x \geq 0, y \geq 0}$

Solution:

$2x - 3y \leq 6 \rightarrow (i)\ 2x + 5y \leq 12 \rightarrow (ii)$
$the\ associated\ eqs.of\ (i) and\ (ii) are$
$l_1; 2x - 3y = 6 \rightarrow (iii), \quad l_2;\ \ 2x + 5y = 12 \rightarrow (iv)$
$(iii) \Rightarrow putx = 0, y = -2\ so\ the\ pt(0,-2)$
$put\ x = 0, y = 3 \quad so\ the\ pt(3,0)$
$(iv) \Rightarrow put\ x = 0, y = 4\ so\ the\ pt\ (0,4)$
$put\ y = 0\ , x = 6\ so\ the\ pt(6,0)$

Test $pt(0,0)$: $we\ test\ (i) and\ (ii)\ at\ (0,0)\ so$
$(i) \Rightarrow 0 \leq 6 \rightarrow true \quad (ii) \Rightarrow 0 \leq 12 \rightarrow true.$

Feasible region:
The feasible region of the given system is the intersection of the graphs of (i) and (ii).
Also $x \geq 0, y \geq 0\ indicates\ that\ graph\ of\ solution$
$Set\ in\ 1st\ Quadrant\ as\ shown\ in\ fig.$

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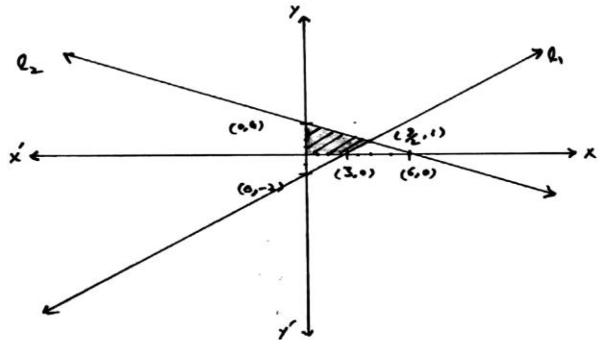


Corner point:

$$as\ 2x - 3y = 6 \rightarrow (i)$$
$$2x + 3y = 12 \rightarrow (ii)$$

By $(i) + (ii) \Rightarrow 4x = 18 \Rightarrow x = \frac{9}{2}\ put\ in\ (i)$

$$\Rightarrow 2\left(\frac{9}{2}\right) - 3y = 6 \Rightarrow -3y = 6 - 9$$
$$-3y = -3$$

$\Rightarrow y = 1\ so\ \left(\frac{9}{2}, 1\right)\ is\ \ the\ pt.of\ intersection\ of\ lines\ (i) and\ (ii)\ Thus\ cornerpoints\ of\ feasible\ region\ are\ (0,0), (3,0), \left(\frac{9}{2}, 1\right)\ and\ (0,4)$

(ii) $\boldsymbol{x + y \leq 5\ ;\ -2x + y \leq 2\ ; x \geq 0, y \geq 0}$

Solution:

$x + y \leq 5 \rightarrow (i);\ -2x + y \leq 2 \rightarrow (ii)$

$the\ associated\ eqs.of\ (i) and\ (ii) are$

$l_1; x + y = 5 \rightarrow (iii),\ \ l_2;\ -2x + y = 2 \rightarrow (iv)$

$(iii) \Rightarrow put\ x = 0, y = 5\ so\ the\ pt(0,5)$

$put\ x = 0, y = 5\ \ so\ the\ pt(5,0)$

$(iv) \Rightarrow put\ x = 0, y = 2\ so\ the\ pt\ (0,2)$

$put\ y = 0\ , x = -1 so\ the\ pt(-1,0)$

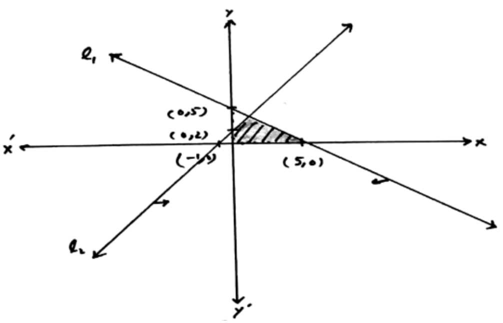
Test $pt(0,0): we\ test\ (i) and\ (ii)\ at\ (0,0)\ so$

$(i) \Rightarrow 0 \leq 5 \rightarrow true\ \ (ii) \Rightarrow 0 \leq 2 \rightarrow true.$

Feasible region:

The feasible region of the given system is the intersection of the graphs of (i) and (ii).

Also $x \geq 0, y \geq 0\ indicates\ that\ graph\ of\ solution$ Set in $1st\ Quadrant\ as\ shown\ in\ fig.$



Corner point:

$$as\ x + y = 5 \rightarrow (i)$$
$$-2x + y = 2 \rightarrow (ii)$$

By $(i) - (ii) \Rightarrow 3x = 3 \Rightarrow x = 1\ put\ in\ (i)$

$$\Rightarrow 1 + y = 5 \Rightarrow y = 5 - 1 = 4$$

$so\ (1,4) is\ pt.of\ intersection\ of\ lines\ (i) and\ (ii)$

$thus\ corner\ pts.of\ feasible\ region\ are\ (o,o), (5,0)\ (1,4) and\ (0,2)$

(iii) $\boldsymbol{x + y \leq 5\ ;\ -2x + y \geq 2; x \geq 0, y \geq 0}$

Solution:

$x + y \leq 5 \rightarrow (i), -2x + y \geq 2 \rightarrow (ii)$

$the\ associated\ eqs.of\ (i) and\ (ii) are$

$l_1; x + y = 5 \rightarrow (iii),\ \ l_2;\ -2x + y = 2 \rightarrow (iv)$

$(iii) \Rightarrow put\ x = 0, y = 5\ so\ the\ pt(0,5)$

$put\ x = 0, y = 5\ \ so\ the\ pt(5,0)$

$(iv) \Rightarrow put\ x = 0, y = 2\ so\ the\ pt\ (0,2)$

$put\ y = 0\ , x = -1 so\ the\ pt(-1,0)$

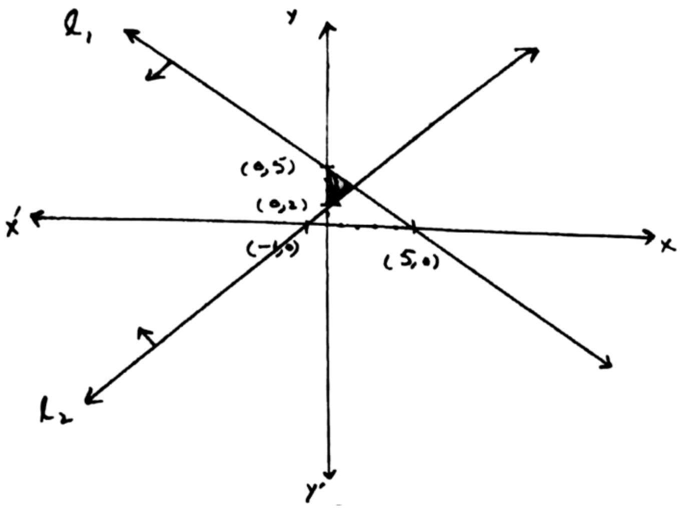
Test $pt(0,0): we\ test\ (i) and\ (ii)\ at\ (0,0)\ so$

$(i) \Rightarrow 0 \leq 5 \rightarrow true\ \ (ii) \Rightarrow 0 \geq 2 \rightarrow false.$

Feasible region:

The feasible region of the given system is the intersection of the graphs of (i) and (ii).

Also $x \geq 0, y \geq 0\ indicates\ that\ graph\ of\ solution$ Set in $1st\ Quadrant\ as\ shown\ in\ fig.$



Corner point:

$$as\ x + y = 5 \rightarrow (i)$$
$$-2x + y = 2 \rightarrow (ii)$$

By $(i) - (ii) \Rightarrow 3x = 3 \Rightarrow x = 1\ put\ in\ (i)$

$$\Rightarrow 1 + y = 5 \Rightarrow y = 5 - 1 = 4$$

$so\ (1,4) is\ pt.of\ intersection\ of\ lines\ (i) and\ (ii) thus\ corner\ pts.of\ feasible\ region\ are\ (0,5)(1,4) and\ (0,2)$

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(iv) $\boldsymbol{3x + 7y \leq 21}$; $\boldsymbol{x - y \leq 3}$; $\boldsymbol{x \geq 0, y \geq 0}$

Solution:

$3x + 7y \leq 21 \rightarrow (i)$; $x - y \leq 3 \rightarrow (ii)$

$the\ associated\ eqs.of\ (i)and\ (ii)are$

$l_1; 3x + 7y = 21 \rightarrow (iii)$, $l_2;\ x - y = 3 \rightarrow (iv)$

$(iii) \Rightarrow put\ x = 0, y = 3\ so\ the\ pt(0,3)$

$put\ x = 7, y = 0\ \ so\ the\ pt(7,0)$

$(iv) \Rightarrow put\ x = 0, y = -3\ so\ the\ pt\ (0,2)$

$put\ y = 0\ , x = 3\ so\ the\ pt(-1,0)$

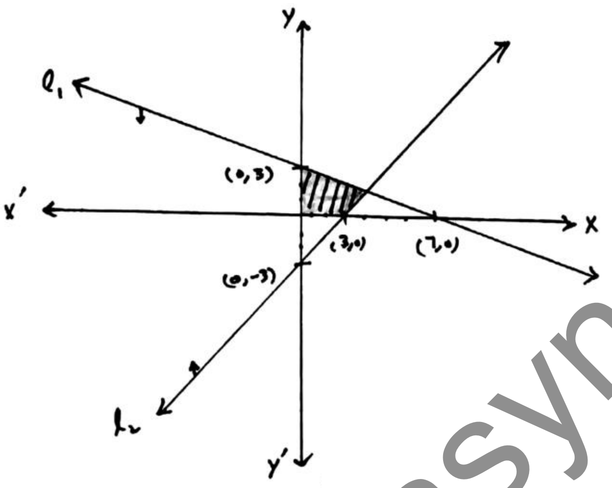
Test $pt(0,0)$: $we\ test\ (i)and\ (ii)\ at\ (0,0)\ so$

$(i) \Rightarrow 0 \leq 21 \rightarrow true\ \ (ii) \Rightarrow 0 \leq 3 \rightarrow false.$

Feasible region:

The feasible region of the given system is the intersection of the graphs of (i) and (ii).

Also $x \geq 0, y \geq 0$ $indicates\ that\ graph\ of\ solution$ Set in $1st\ Quadrant\ as\ shown\ in\ fig.$



Corner point:

$as\ 3x + 7y = 21 \rightarrow (i)$

$x - y = 3 \rightarrow (ii)$

By $7(ii) + (i) \Rightarrow 10x = 42 \Rightarrow x = \frac{21}{5}\ put\ in\ (ii)$

$$\Rightarrow \frac{21}{5} - y = 3 \Rightarrow y = \frac{21}{5} - 3 = \frac{6}{5}$$

$so\ \left(\frac{121}{5}, \frac{6}{5}\right)\ is\ pt.of\ intersection\ of\ lines\ (i)and\ (ii)thus\ corner\ pts.of\ feasible\ region\ are$ $(0,0), (3,0), \left(\frac{21}{5}, \frac{6}{5}\right)\ and\ (0,3)$

(v) $\boldsymbol{3x + 2y \geq 6}$; $\boldsymbol{x + y \leq 4}$; $\boldsymbol{x \geq 0, y \geq 0}$

Solution:

$3x + 2y \geq 6 \rightarrow (i)\ \ \ and\ x + y \leq 4 \rightarrow (ii)$

$the\ associated\ eqs.of\ (i)and\ (ii)are$

$l_1; 3x + 2y = 6 \rightarrow (iii)$, $l_2;\ x + y = 4 \rightarrow (iv)$

$(iii) \Rightarrow put\ x = 0, y = 3\ so\ the\ pt(0,3)$

$put\ x = 2, y = 0\ \ so\ the\ pt(2,0)$

$(iv) \Rightarrow put\ x = 0, y = 4\ so\ the\ pt\ (0,4)$

$put\ y = 0\ , x = 4 so\ the\ pt(4,0)$

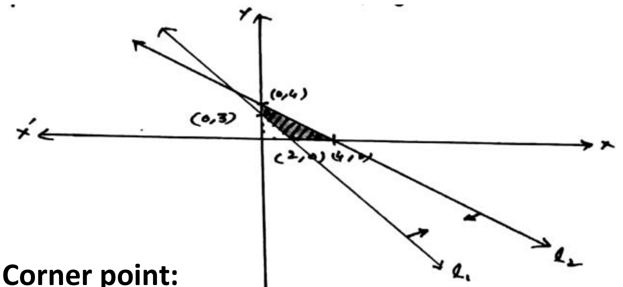
Test $pt(0,0)$: $we\ test\ (i)and\ (ii)\ at\ (0,0)\ so$

$(i) \Rightarrow 0 \geq 6 \rightarrow false\ \ (ii) \Rightarrow 0 \leq 4 \rightarrow true.$

Feasible region:

The feasible region of the given system is the intersection of the graphs of (i) and (ii).

Also $x \geq 0, y \geq 0$ $indicates\ that\ graph\ of\ solution$ Set in $1st\ Quadrant\ as\ shown\ in\ fig.$



Corner point:

$thus\ corner\ pts.of\ feasible\ region\ are$

$(2,0)(4,0)(0,0)and\ (0,3)$

(vi) $\boldsymbol{5x + 7y \leq 35}$; $\boldsymbol{x - 2y \leq 4}$; $\boldsymbol{x \geq 0, y \geq 0}$

Solution:

$5x + 7y \leq 35 \rightarrow (i)\ \ \ x - 2y \leq 4 \rightarrow (ii)$

$the\ associated\ eqs.of\ (i)and\ (ii)are$

$l_1; 5x + 7y = 35 \rightarrow (iii)$, $l_2;\ x - 2y = 4 \rightarrow (iv)$

$(iii) \Rightarrow put\ x = 0, y = 5\ so\ the\ pt(0,5)$

$put\ x = 7, y = 0\ \ so\ the\ pt(7,0)$

$(iv) \Rightarrow put\ x = 0, y = 4\ so\ the\ pt\ (0,-2)$

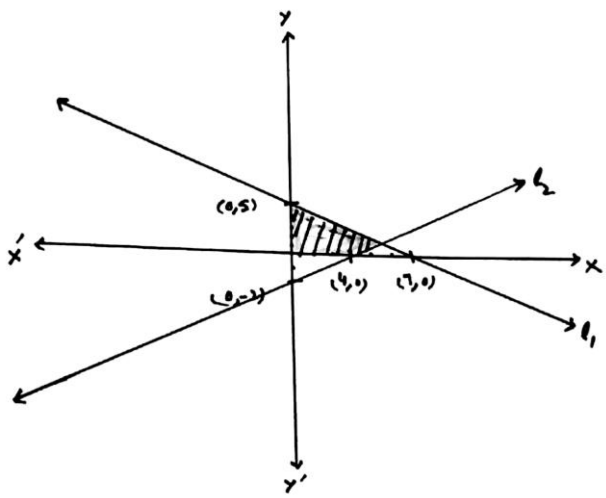
$put\ y = 0\ , x = 4\ so\ the\ pt(4,0)$

Test $pt(0,0)$: $we\ test\ (i)and\ (ii)\ at\ (0,0)$

Feasible region:

The feasible region of the given system is the intersection of the graphs of (i) and (ii).

Also $x \geq 0, y \geq 0$ $indicates\ that\ graph\ of\ solution$ Set in $1st\ Quadrant\ as\ shown\ in\ fig.$



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Corner point:

$$As\ 5x + 7y = 35 \rightarrow (i)$$
$$x - 2y = 4 \rightarrow (ii)$$

By $5(ii) - (i) \Rightarrow \quad 5x - 10y = 20$
$$\underline{\pm 5x \pm 7y = \pm 35}$$
$$-17y = -15$$
$$\Rightarrow y = \frac{15}{17}\ put\ in\ (ii)$$
$$\Rightarrow x - 2\left(\frac{15}{17}\right) = 4 \Rightarrow x = 4 + \frac{30}{17}$$

So,

$\left(\frac{99}{17}, \frac{15}{17}\right)$ *is the pt. of intersection of lines (i) and (ii)*

$$Hence\ corner\ pts.are\ (o,o), (4,0)\left(\frac{98}{17}, \frac{15}{17}\right)\ and\ (0,5)$$

Question No.2

Graph the feasible region of the following system of linear inequalities and find the corner points in each case.

(i) $2x + y \leq 10\ ; x + 4y \leq 12\ ; x + 2y \leq 10;$
$$x \geq 0, y \geq 0$$

Solution:

$$2x + y \leq 10 \rightarrow (i); x + 4y \leq 12 \rightarrow (ii)$$
$$; x + 2y \leq 0 \rightarrow (iii)$$

The associated $eqs.\ of\ (i), (ii) and\ (iii) are$

$l_1; 2x + y = 10 \rightarrow (iv)\quad , l_2; x + 4y = 12 \rightarrow (ii)$

$l_3;\ \ x + 2y = 10 \rightarrow (vi)$

$$(iv) \Rightarrow putx = 0, y = 10\ \ so\ that\ pt. (0,10)$$
$$put\ y = 0, x = 5\ \ so\ the\ pt. (5,0)$$
$$(v) \Rightarrow put\ x = 0, y = 3\ so\ the\ pt. (0, 3)$$
$$put\ y = 0, x = 12\ so\ the\ pt (12,0)$$
$$(vi) \Rightarrow put\ x = 0, y = 5\ so\ the\ pt\ (0,5)$$
$$put\ y = 0, x = 10\ so\ the\ pt. (10,0)$$

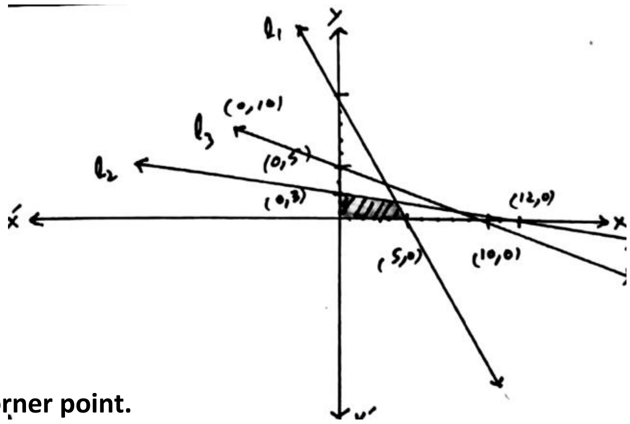
Test $pt(0,0)$: $we\ test\ (i), (ii) and\ (iii) at\ (0,0)$

$so\ (i) \Rightarrow 0 \leq 10 \rightarrow true\quad (ii) \Rightarrow 0 \leq 12 \rightarrow True$
$$(iii) \Rightarrow 0 \leq 10 \rightarrow true$$

Feasible region:

The feasible region of the given system is the intersection of the graphs of (i) and (ii).

Also $x \geq 0, y \geq$
$0\ indicates\ that\ graph\ of\ solution$
$Set\ in\ 1st\ Quadrant\ as\ shown\ in\ fig$



Corner point.

We find pt. of intersection of lines $l_1, l_2\ so$

$$l_1;\ \ 2x + y = 10 \rightarrow (i)$$
$$l_2; x + 4y = 12 \rightarrow (ii)$$

By $2(ii) - (i) \quad \Rightarrow \quad 2x + 8y = 24$
$$\underline{\pm 2x \pm y = -10}$$
$$7y = 14 \Rightarrow y = 2\ put\ in\ (i)$$
$$\Rightarrow x = 4$$

So$(4,2) is\ pt\ of\ intersection\ of\ lines\ (i)$
$and\ (ii) thus\ corner\ pt. of\ feasible\ region$
$are\ (0,0), (5,0), (4,2) and\ (0,3)$

(ii) $\mathbf{2x + 3y \leq 18\ ; 2x + y \leq 10\ ; x + 4y \leq 12}$
$$\mathbf{x \geq 0, y \geq 0}$$

Solution:

$$2x + 3y \leq 18 \rightarrow (i); 2x + y \leq 10 \rightarrow (ii)$$
$$; x + 4y \leq 12 \rightarrow (iii)$$

The associated $eqs.\ of\ (i), (ii) and\ (iii) are$

$l_1; 2x + 3y = 18 \rightarrow (iv)\quad , l_2; 2x + y = 10 \rightarrow (ii)$

$l_3;\ \ x + 4y = 12 \rightarrow (vi)$

$$(iv) \Rightarrow putx = 0, y = 6\ \ so\ that\ pt. (0,6)$$
$$put\ y = 0, x = 9\ so\ the\ pt. (9,0)$$
$$(v) \Rightarrow put\ x = 0, y = 10\ so\ the\ pt. (0, 10)$$
$$put\ y = 0, x = 5\ so\ the\ pt(5,0)$$
$$(vi) \Rightarrow put\ x = 0, y = 3\ so\ the\ pt\ (0,3)$$
$$put\ y = 0, x = 12\ so\ the\ pt. (12,0)$$

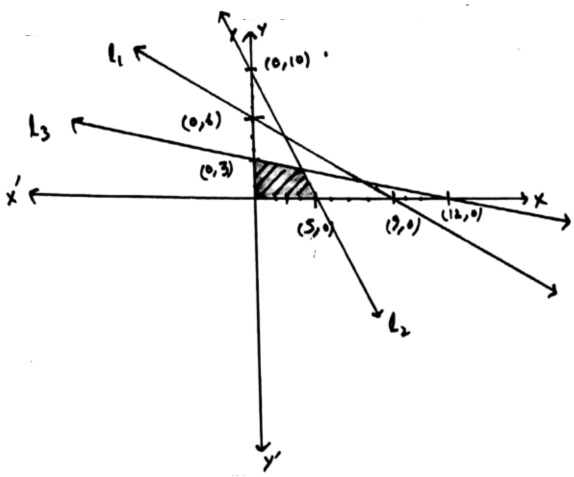
Test $pt(0,0)$: $we\ test\ (i), (ii) and\ (iii) at\ (0,0)$

$so\ (i) \Rightarrow 0 \leq 18 \rightarrow true\quad (ii) \Rightarrow 0 \leq 10 \rightarrow True$
$$(iii) \Rightarrow 0 \leq 12 \rightarrow true$$

Feasible region:

The feasible region of the given system is the intersection of the graphs of (i) and (ii).

Also $x \geq 0, y \geq$
$0\ indicates\ that\ graph\ of\ solution$
$Set\ in\ 1st\ Quadrant\ as\ shown\ in\ fig.$



Corner point.

We find pt. of intersection of lines $l_1, l_2\ so$

$$l_1;\ \ 2x + y = 10 \rightarrow (i)$$
$$l_2; x + 4y = 12 \rightarrow (ii)$$

By $2(ii) - (i) \quad \Rightarrow \quad 2x + 8y = 24$

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$$\underline{\pm 2x \pm y = -10}$$
$$7y = 14 \Rightarrow y = 2 \; put \; in \; (i)$$
$$\Rightarrow x = 4$$

So $(4,2)$ *is pt of intersection of lines* (i) *and* (ii) *thus corner pt. of feasible region are* $(0,0),(5,0),(4,2)$ *and* $(0,3)$

(iii) $\boldsymbol{2x + 3y \leq 18;\; x + 4y \leq 12; 3x + y \leq 12}$

$$\boldsymbol{x \geq 0, y \geq 0}$$

Solution:

$$2x + 3y \leq 18 \rightarrow (i);\;\; x + 4y \leq 12 \rightarrow (ii)$$
$$3x + y \leq 12 \rightarrow (iii)$$
$$; x + 2y \leq 0 \rightarrow (iii)$$

The associated *eqs. of* $(i),(ii)$ *and* (iii) *are*

$$l_1; 2x + 3y = 18 \rightarrow (iv) \quad , l_2; x + 4y = 12 \rightarrow (ii)$$

l_3 ; $\; 3x + y = 12 \rightarrow (vi)$

$$(iv) \Rightarrow put x = 0, y = 3 \; so \; that \; pt. (0,3)$$
$$put \; y = 0, x = 6 \; so \; the \; pt. (6,0)$$
$$(v) \Rightarrow put \; x = 0, y = 9 \; so \; the \; pt. (0,9)$$
$$put \; y = 0, x = 12 \; so \; the \; pt (12,0)$$
$$(vi) \Rightarrow put \; x = 0, y = 12 \; so \; the \; pt \; (0.12)$$
$$put \; y = 0, x = 4 \; so \; the \; pt. (4,0)$$

Test $pt(0,0)$: *we test* $(i),(ii)$ *and* (iii) *at* $(0,0)$

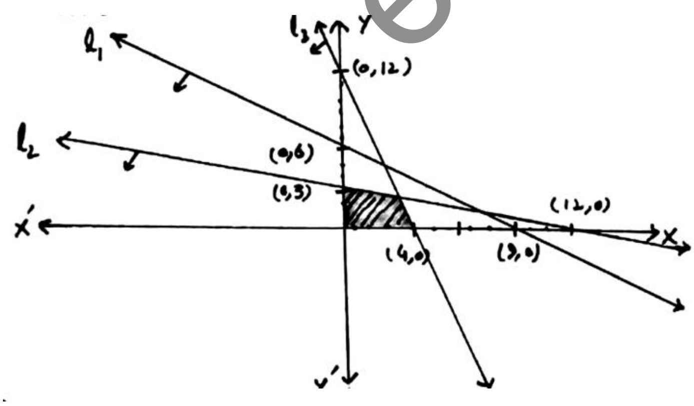
so $(i) \Rightarrow 0 \leq 18 \rightarrow true \quad (ii) \Rightarrow 0 \leq 12 \rightarrow True$

$$(iii) \Rightarrow 0 \leq 12 \rightarrow true$$

Feasible region:

The feasible region of the given system is the intersection of the graphs of (i) and (ii).

Also $x \geq 0, y \geq 0$ *indicates that graph of solution* Set in $1st$ *Quadrant as shown in fig*



Corner point.

We find pt. of intersection of lines l_1, l_2 *so*

$$l_1;\;\; x + 4y = 12 \rightarrow (i)$$
$$l_2; 3x + y = 12 \rightarrow (ii)$$

By $3(ii) - (i) \quad \Rightarrow \quad 3x + 12y = 36$

$$\underline{\pm 3x \pm y = -12}$$
$$11y = 24 \Rightarrow y = \frac{24}{11} \; put \; in \; (i)$$
$$\Rightarrow x + 4\left(\frac{24}{11}\right) = 12 \Rightarrow x12 - \frac{36}{11}$$
$$x = \frac{132 - 96}{11} = \frac{36}{11}$$

So $\left(\frac{36}{11}, \frac{24}{11}\right)$ *is pt of intersection of lines* (i) *and* (ii) *thus corner pt. of feasible region are* $(0,0),(4,0),\left(\frac{36}{11},\frac{24}{11}\right)$ *and* $(0,3)$

(iv) $\boldsymbol{x + 2y \leq 14; 3x + 4y \leq 36; 2x + y \leq 10}$

$$\boldsymbol{x \geq 0; y \geq o}$$

Solution:

$$x + 2y \leq 14 \rightarrow (i); 3x + 4y \leq 36 \rightarrow (ii)$$
$$; 2x + y \leq 10 \rightarrow (iii)$$

The associated *eqs. of* $(i),(ii)$ *and* (iii) *are*

$$l_1; x + 2y = 14 \rightarrow (iv) \quad , l_2; 3x + 4y = 36 \rightarrow (ii)$$

l_3 ; $\; 2x + y = 10 \rightarrow (vi)$

$$(iv) \Rightarrow put x = 0, y = 7 \; so \; that \; pt. (0,7)$$
$$put \; y = 0, x = 14 \; so \; the \; pt. (14,0)$$
$$(v) \Rightarrow put \; x = 0, y = 9 \; so \; the \; pt. (0,9)$$
$$put \; y = 0, x = 12 \; so \; the \; pt (12,0)$$
$$(vi) \Rightarrow put \; x = 0, y = 10 \; so \; the \; pt \; (0.10)$$
$$put \; y = 0, x = 5 \; so \; the \; pt. (5,0)$$

Test $pt(0,0)$: *we test* $(i),(ii)$ *and* (iii) *at* $(0,0)$

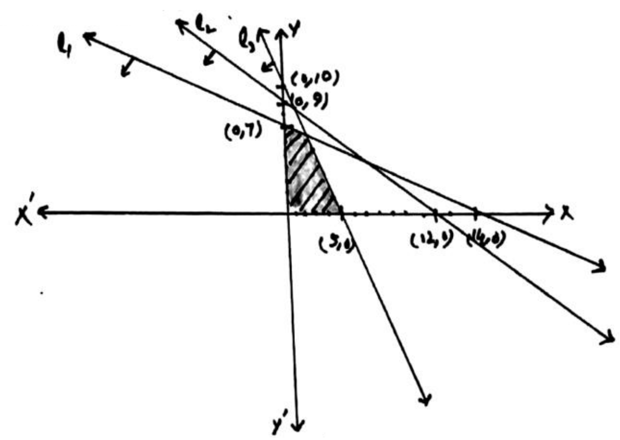
so $(i) \Rightarrow 0 \leq 14 \rightarrow true \quad (ii) \Rightarrow 0 \leq 36 \rightarrow True$

$$(iii) \Rightarrow 0 \leq 10 \rightarrow true$$

Feasible region:

The feasible region of the given system is the intersection of the graphs of (i) and (ii).

Also $x \geq 0, y \geq 0$ *indicates that graph of solution* Set in $1st$ *Quadrant as shown in fig.*



Corner point.

We find pt. of intersection of lines l_1, l_2 *so*

$$l_1;\;\; x + 2y = 14 \rightarrow (i)$$

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$l_2; 2x + y = 10 \rightarrow (ii)$

By$2(i) - (ii) \quad \Rightarrow \quad 2x + 4y = 28$

$\pm 2x \pm y = -10$

$3y = 18 \Rightarrow y = 6 \; put \; in \; (i)$

$\Rightarrow x + 12 = 14 \Rightarrow x = 2$

So$(2,6) is \; pt \; of \; intersection \; of \; lines \; (i)$ $and \; (ii) thus \; corner \; pt. of \; feasible \; region$ $are \; (0,0), (5,0), (2,6) and \; (0,7)$

(v) $\boldsymbol{x + 3y \le 15; \;\; 2x + y \le 12; 4x + 3y \le 24}$

$\boldsymbol{x \ge 0, y \ge 0}$

Solution:

$x + 3y \le 15 \rightarrow (i) \;\; 2x + y \le 12 \rightarrow (ii)$

$4x + 3y \le 24 \rightarrow (iii)$

The associated $eqs. of \; (i), (ii) and \; (iii) are$

$l_1; x + 3y = 15 \rightarrow (iv) \quad , l_2; 2x + y = 12 \rightarrow (ii)$

$l_3 ; \;\; 4x + 3y = 24 \rightarrow (vi)$

$(iv) \Rightarrow putx = 0, y = 5 \; so \; that \; pt. (0,5)$

$put \; y = 0, x = 15 so \; the \; pt. (15,0)$

$(v) \Rightarrow put \; x = 0, y = 12 \; so \; the \; pt. (0,12)$

$put \; y = 0, x = 6 \; so \; the \; pt(6,0)$

$(vi) \Rightarrow put \; x = 0, y = 8 \; so \; the \; pt \; (0.8)$

$put \; y = 0, x = 6 \; so \; the \; pt. (6,0)$

Test $pt(0,0)$: $we \; test \; (i), (ii) and \; (iii) at \; (0,0)$

$so \; (i) \Rightarrow 0 \le 15 \rightarrow true \quad (ii) \Rightarrow 0 \le 12 \rightarrow True$

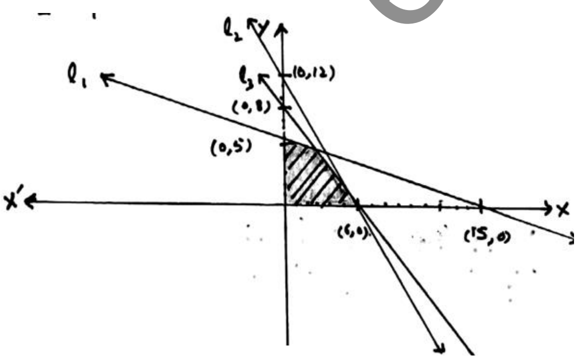
$(iii) \Rightarrow 0 \le 24 \rightarrow true$

Feasible region:

The feasible region of the given system is the intersection of the graphs of (i) and (ii).

Also $x \ge 0, y \ge$ $0 \; indicates \; that \; graph \; of \; solution$

Set in $1st \; Quadrant \; as \; shown \; in \; fig$



Corner point.

We find pt. of intersection of lines $l_1, l_2 \; so$

$l_1; \;\; x + 3y = 15 \rightarrow (i)$

$l_2; 4x + 3y = 24 \rightarrow (ii)$

By$(i) - (ii) \quad \Rightarrow -3x = -9 \Rightarrow x = 3 \; put \; in (i)$

$\Rightarrow 3y = 15 - 3 = 12 \Rightarrow y = 4$

So$(3,4) is \; pt. of \; intersection \; of \; lines \; (i)$ $and \; (ii) thus \; corner \; pt. of \; feasible \; region$ $are \; (0,0), (6,0), (3,4) and \; (0,5)$

(vi) $2x + y \le 20; 8x + 15y \le 20; x + y \le 11$

$x \ge 0, y \ge 0$

Solution:

$2x + y \le 20 \rightarrow (i); 8x + 15y \le 20 \rightarrow (ii)$

$; x + y \le 11 \rightarrow (iii)$

The associated $eqs. of \; (i), (ii) and \; (iii) are$

$l_1; 2x + y = 20 \rightarrow (iv) \quad , l_2; 8x + 15y = 20 \rightarrow (ii)$

$l_3 ; \;\; x + y = 11 \rightarrow (vi)$

$(iv) \Rightarrow putx = 0, y = 20 \; so \; that \; pt. (0,20)$

$put \; y = 0, x = 10 so \; the \; pt. (10,0)$

$(v) \Rightarrow put \; x = 0, y = 8 \; so \; the \; pt. (0,8)$

$put \; y = 0, x = 15 \; so \; the \; pt(15,0)$

$(vi) \Rightarrow put \; x = 0, y = 11 \; so \; the \; pt \; (0.11)$

$put \; y = 0, x = 11 \; so \; the \; pt. (11,0)$

Test $pt(0,0)$: $we \; test \; (i), (ii) and \; (iii) at \; (0,0)$

$so \; (i) \Rightarrow 0 \le 20 \rightarrow true \quad (ii) \Rightarrow 0 \le 120 \rightarrow True$

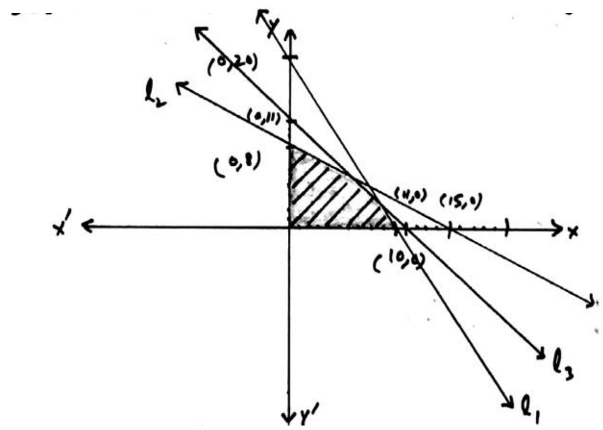
$(iii) \Rightarrow 0 \le 11 \rightarrow true$

Feasible region:

The feasible region of the given system is the intersection of the graphs of (i) and (ii).

Also $x \ge 0, y \ge$ $0 \; indicates \; that \; graph \; of \; solution$

Set in $1st \; Quadrant \; as \; shown \; in \; fig.$



Corner point.

We find pt. of intersection of lines

$l_1, l_3 \; also \; l_2 and \; l_3$

$l_1; \;\; 2x + y = 20 \rightarrow (i)$

$l_2; x + y = 11 \rightarrow (ii)$

By$2(ii) - (i) \quad \Rightarrow \quad 2x + 2y = 22$

$\pm 2x \pm y = \pm 20$

$y = 2 \;\; put \; in \; (ii) x = 9$

So

$(9,2) \; is \; the \; pt. of \; intersection \; of \; lines \; (i) and \; (ii)$

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Also l_2 ; $8x + 15y = 120 \rightarrow (iii)$

$l_3; x + y = 11 \rightarrow (iv)$

By $(8(iv) - (iii)$ $\Rightarrow 8x + 8y = 88$

$\pm 8x \pm 15y = \pm 120$

$-7y = -32 \Rightarrow y = \frac{32}{7}$ $put\ in (iv)$

$$\Rightarrow x + \frac{32}{7} = 11 = 11 - \frac{32}{7} = \frac{77-32}{7} = \frac{45}{7}$$

So $\left(\frac{45}{7}, \frac{32}{7}\right)$ $is\ pt\ of\ intersection\ of\ lines\ (i)$ $and\ (ii) thus\ corner\ pt.of\ feasible\ region$ $are\ (0,0), (9,2), \left(\frac{45}{7}, \frac{32}{7}\right)$ $and\ (0,8)$

Linear programing

Objective function:

A function which is to be maximized or minimized is called an objective function:

Optimal solution:

The feasible solution which maximizes or minimize the objective function is called optimal solution.

Procedure for finding optimal:

Solution:

(i) Graph the solution set of linear inequality constants to determine feasible region.

(ii) Find the corner points of the feasible region.

(iii) Evaluate the objective function at each corner point to find the optimal solution:

Exercise No.5.3

Question No.1

Maximize $f(x, y) = 2x + 5y$ *subject to the constranits* $2y - x \leq 8; x - y \leq 4;$ $x \geq 0, y => 0$

Solution:

$-x + 2y \leq 8 \rightarrow (i)$ $x - y \leq 4 \rightarrow (ii)$

$the\ associated\ eqs.of\ (i) and\ (ii) are$

$l_1; -x + 2y = 8 \rightarrow (iii),$ $l_2;\ x - y = 4 \rightarrow (iv)$

$(iii) \Rightarrow put\ x = 0, y = 4\ so\ the\ pt (0,4)$

$put\ x = 0, y = -8\ \ so\ the\ pt(-8,0)$

$(iv) \Rightarrow put\ x = 0, y = -4\ so\ the\ pt\ (0,2)$

$put\ y = 0\ , x = 4\ so\ the\ pt(4\ ,0)$

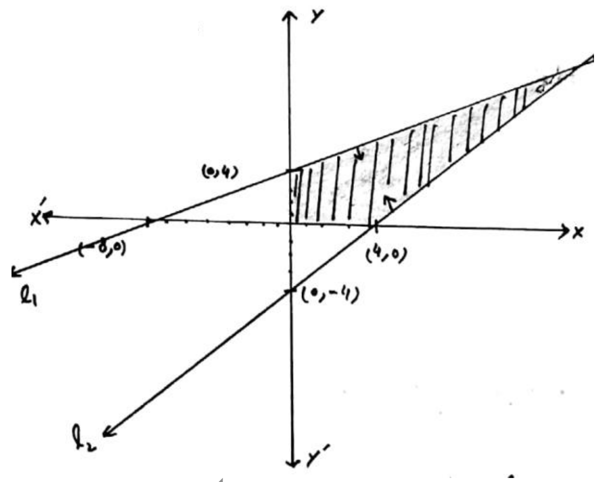
Test $pt(0,0)$: $we\ test\ (i) and\ (ii)\ at\ (0,0)\ so$

$(i) \Rightarrow 0 \leq 8 \rightarrow true\ \ (ii) \Rightarrow 0 \leq 4 \rightarrow true.$

Feasible region:

The feasible region of the given system is the intersection of the graphs of (i) and (ii).

Also $x \geq 0, y \geq$ $0\ indicates\ that\ graph\ of\ solution$ Set in $1st\ Quadrant\ as\ shown\ in\ fig.$



Corner point:

We find pt. of intersection of lines l_1, l_2 so

$l_1; -x + 2y = 8 \rightarrow (i)$

$x - y = 4 \rightarrow (iii)$

By $(i) + (ii) \Rightarrow y = 12\ put\ in\ (ii) \Rightarrow x = 16$

So

$(16,12) is\ the\ pt.of\ intersection\ of\ lines\ (i) and$ (ii) $hence\ corner\ pts.of\ feasible\ region\ are\ (o,o)$ $(4,0), (16,2) and\ (0,4).$

Optimal solution:

$we\ find\ valves\ of\ f(x,y)$

$= 2x + 5y\ at\ corner\ pts.$

$f(0,0) = 2(0) + 5(0) = 0, f(4,0)$

$= 2(4) + 5(0) = 8$

$f(16,12) = 2(16) + 5(12) = 92, f(0,4)$

$= 2(0) + 5(11) = 20$

Thus $f(x,y) hax\ maximum\ value$ 92 at (16,12)

Question No.2

Maximize $f(x, y) = x + 3y$ *subject to constraints*

$\mathbf{2x + 5y \leq 30, 5x + 4y \leq 20,}$ $\mathbf{x \geq 0, y \geq 0}$

Solution:

$2x + 5y \leq 30 \rightarrow (i)$ $5x + 4y \leq 20 \rightarrow (ii)$

$the\ associated\ eqs.of\ (i) and\ (ii) are$

$l_1; 2x + 5y = 30 \rightarrow (iii),$ $l_2;\ 5x + 4y = 20 \rightarrow (iv)$

$(iii) \Rightarrow put\ x = 0, y = 6\ so\ the\ pt (0,6)$

$put\ x = 0, y = 15\ \ so\ the\ pt(15,0)$

$(iv) \Rightarrow put\ x = 0, y = 5\ so\ the\ pt\ (0,5)$

$put\ y = 0\ , x = 4\ so\ the\ pt(4\ ,0)$

Test $pt(0,0)$: $we\ test\ (i) and\ (ii)\ at\ (0,0)\ so$

$(i) \Rightarrow 0 \leq 30 \rightarrow true\ \ (ii) \Rightarrow 0 \leq 20 \rightarrow true.$

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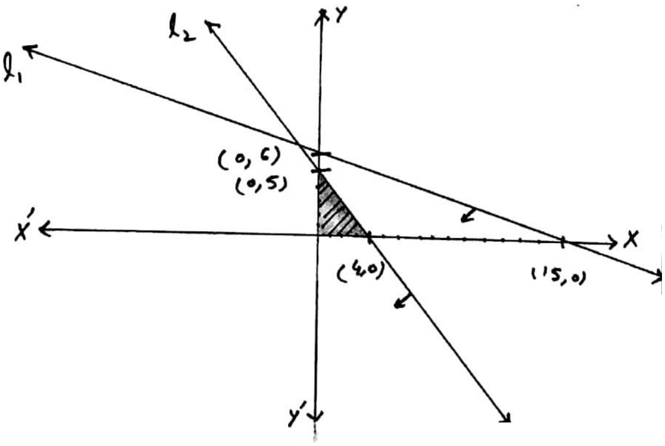
Feasible region:

The feasible region of the given system is the intersection of the graphs of (i) and (ii).

Also $x \geq 0, y \geq 0$ *indicates that graph of solution* Set in $1st$ *Quadrant as shown in fig.*



Corner point:

Corner points o feasible region are $(0,0), (4,0) and (0,5)$

Optimal solution:

We find valves of $f(x, y) = x + 3y$ *at corner pts.*

$f(0,0) = 0 + 3(0) = 0, f(4,0) = 4 + 3(0) = 4$

$f(0,5) = 0 + 3(5)$

$= 15$, *so* $f(x, y)$ *has maximum value* $a5$ *at* $(0,5)$

Question No.3

Maximize $z = 2x + 3y$ *subject to constraints*

$$\mathbf{2x + 4y \leq 12; 2x + y \leq 4; 2x - y \leq 4;}$$
$$\mathbf{x \geq 0, y \geq 0}$$

Solution:

$$3x + 4y \leq 12 \rightarrow (i), 2x + y \leq 4 \rightarrow (ii)$$
$$2x - y \leq 4 \rightarrow (iii)$$

the associated eqs. of (i) *and* (ii) *are*

$l_1; 3x + 4y = 12 \rightarrow (iv), \quad l_2;\ 2x + y = 4 \rightarrow (v)$

$l_3; 2x + y = 4 \rightarrow (vi)$

$(iv) \Rightarrow put\ x = 0, y = 3\ so\ the\ pt(0,3)$

$put\ x = 0, y = 4\ \ so\ the\ pt(4,0)$

$(v) \Rightarrow put\ x = 0, y = 4\ so\ the\ pt\ (0,4)$

$put\ y = 0\ , x = 2\ so\ the\ pt(2\ ,0)$

$(vi) \Rightarrow putx = 0y = -4\ so\ the\ pt. (0, -4)$

$put\ y = 0, x = 1\ so\ the\ pt. (2,0)$

Test

$pt(0,0)$: *we test* $(i), (ii)$ *and* (iii) *at* $(0,0)$ *so*

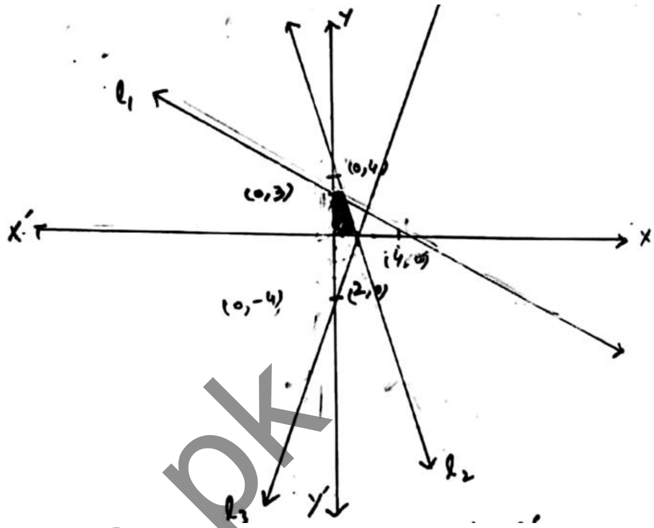
$(i) \Rightarrow 0 \leq 12 \rightarrow true\ \ (ii) \Rightarrow 0 \leq 4 \rightarrow true.$

$(iii) \Rightarrow 0 \leq 4 \rightarrow true$

Feasible region:

The feasible region of the given system is the intersection of the graphs of (i) and (ii).

Also $x \geq 0, y \geq 0$ *indicates that graph of solution* Set in $1st$ *Quadrant as shown in fig.*



Corner point:

We find $pt. of\ intersection\ of\ lines\ l_1, l_2. so$

$$l_1; 3x + 4y = 12 \rightarrow (i)$$
$$2x + y = 4 \rightarrow (ii)$$

By$4(ii) - (i) \Rightarrow 8x + 4y = 16$

$$\underline{\pm 3x \pm 4y = \pm 12}$$
$$5x = 4$$
$$\Rightarrow x = \frac{4}{5}\ put\ in\ (ii)$$
$$2\left(\frac{4}{5}\right) + y = 4 \Rightarrow y - \frac{8}{5} = \frac{12}{5}$$

so $\left(\frac{4}{5}, \frac{12}{5}\right)$ *is the pt. of intersection of lines* (i) *and . Hence corner pt. of festible region are*

$$(0,0), (2,0), \left(\frac{4}{5}, \frac{12}{5}\right)\ and\ (0,3).$$

Optimal solution:

We find valves of $z = 2x + 3y$ *at corner pts.*

$(0,0), Z = 2(0) + 3(0) = 0,\ (2,0),\ z = 2(2) + 3(0)$

$$\left(\frac{4}{3}, \frac{12}{3}\right), Z = 2\left(\frac{4}{5}\right) + 3\left(\frac{12}{5}\right) = \frac{8}{5} + \frac{36}{5} = \frac{44}{5}$$
$$= 8.8$$
$$(0,3), Z = 2(0) + 3(3) = 9$$

So $z = 2x + 3y$ *has maximum value* 9 *at* $(0,3)$

Q4.* Minimize $z = 2x + y$ *subject to the constraints

$$\mathbf{x + y \geq 3, 7x + 5y \leq 35\ x \geq 0, y \geq 0}$$

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Solution:

$x + y \geq 3 \to (i), 7x + 5y \leq 35 \to (ii)$

$the\ associated\ eqs.\ of\ (i) and\ (ii) are$

$l_1; x + y = 3 \to (iii),\ \ l_2;\ 7x + 5y = 35 \to (iv)$

$(iii) \Rightarrow put\ x = 0, y = 3\ so\ the\ pt(0,3)$

$put\ x = 0, y = 3\ \ so\ the\ pt(3,0)$

$(iv) \Rightarrow put\ x = 0, y = 7\ so\ the\ pt\ (0,7)$

$put\ y = 0\ , x = 5\ so\ the\ pt(5\ ,0)$

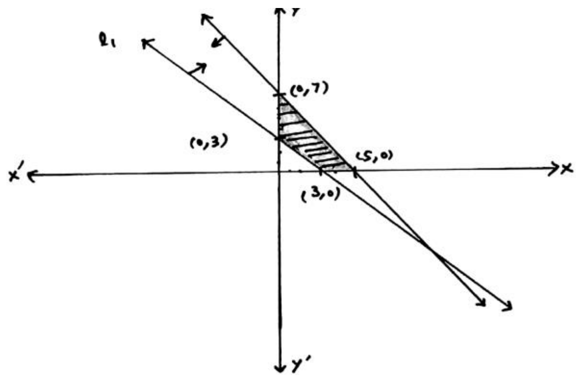
Test $pt(0,0): we\ test\ (i) and\ (ii)\ at\ (0,0)\ so$

$(i) \Rightarrow 0 \geq 3 \to false\ \ (ii) \Rightarrow 0 \leq 35 \to true.$

Feasible region:

The feasible region of the given system is the intersection of the graphs of (i) and (ii).

Also $x \geq 0, y \geq 0$ $indicates\ that\ graph\ of\ solution$ Set in $1st\ Quadrant\ as\ shown\ in\ fig.$



Corner point:

Corner point of feasible region are $(3,0)(5,0), (o,7)\ and (0,3)$

Optimal solution:

$we\ find\ valves\ of\ Z = 2x + y\ at\ corner\ pts.$

$(3,0), x = 2(3) + 0 = 6\ \ , (5,0), z = 2(5) + 0 = 10$

$(o,4), z = 2(0) + 7 = 7\ \ , (0,3), z = 2(0) + 3 = 3$

So,$z = 2x + y\ has\ minimum\ value\ 3\ at\ (0,3)?$

Question No.5 Maximize the function defined constraints $\boldsymbol{2x + y \leq 8; x + 2y \leq 14; x \geq 0, y \geq 0}$

Solution:

$2x + y \leq 8 \to (i), x + 2y \leq 14 \to (ii)$

$the\ associated\ eqs.\ of\ (i) and\ (ii) are$

$l_1; 2x + y = 8 \to (iii),\ \ l_2;\ x + 2y = 14 \to (iv)$

$(iii) \Rightarrow put\ x = 0, y = 8\ so\ the\ pt(0,8)$

$put\ x = 0, y = 4\ \ so\ the\ pt(4,0)$

$(iv) \Rightarrow put\ x = 0, y = 14\ so\ the\ pt\ (0,14)$

$put\ y = 0\ , x = 7\ so\ the\ pt(7\ ,0)$

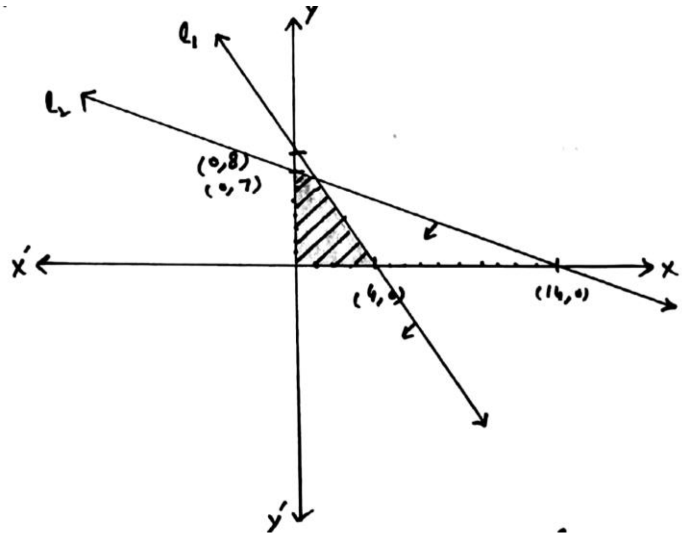
Test $pt(0,0): we\ test\ (i) and\ (ii)\ at\ (0,0)\ so$

$(i) \Rightarrow 0 \leq 8 \to true\ \ (ii) \Rightarrow 0 \leq 14 \to true.$

Feasible region:

The feasible region of the given system is the intersection of the graphs of (i) and (ii).

Also $x \geq 0, y \geq 0$ $indicates\ that\ graph\ of\ solution$ Set in $1st\ Quadrant\ as\ shown\ in\ fig.$



Corner point:

We find pt. of intersection of lines $l_1, l_2\ \ so$

$l_1; 2x + y = 8 \to (i)$

$x + 2y = 14 \to (ii)$

By $2(ii) + (i) \Rightarrow 2x + 4y = 28$

$$\frac{\pm 2x \pm 2y = 14}{3y = 20}$$

$$y = \frac{20}{3}\ put\ in\ (i)$$

$$2x + \frac{20}{3} = 8 \Rightarrow 2x = 8 - \frac{20}{3} \Rightarrow 2x = \frac{4}{3}$$

$$\Rightarrow x = \frac{4}{6} = \frac{2}{3}$$

$so\ \left(\frac{2}{3}, \frac{20}{3}\right)\ is\ the\ pt.\ of\ intersection\ of\ lines\ (i)\ and\ (ii). Hence\ coner\ pts.\ of\ feasible\ region\ are$

$$(0,0), (4,0), \left(\frac{2}{3}, \frac{20}{3}\right)\ and\ (0,8).$$

Optimal solution:

We find valves of $f(x, y) = 2x + 3y\ at\ corner\ pts.$

$$f(0,0) = 2(0) + 2(0) = 0, f(4,0) = 2(4) + 3(0) = 8$$

$$f\left(\frac{2}{3}, \frac{20}{3}\right) = 2\left(\frac{2}{3}\right) + 3\left(\frac{20}{3}\right) = \frac{64}{3} = 21.33$$

$$f(0,7) = 2(0) + 3(7) = 21$$

So $f(x, y) = 2x + 3y$

Has maximum value at $\left(\frac{2}{3}, \frac{20}{3}\right)$

Question No.6

Minimize $\boldsymbol{z = 3x + y}$; *subject to constraints*:

$$\boldsymbol{3x + 5y \geq 15; x + 6y \geq 9, x \geq 0, y \geq 0}$$

Solution:

$3x + 5y \geq 15 \to (i)$

$x + 6y \geq 9 \to (ii)$

$the\ associated\ eqs.\ of\ (i) and\ (ii) are$

$l_1; 3x + 5y = 15 \to (iii),\ \ l_2;\ x + 6y = 9 \to (iv)$

$(iii) \Rightarrow put\ x = 0, y = 3\ so\ the\ pt(0,3)$

$put\ x = 0, y = 5\ so\ the\ pt(5,0)$

$(iv) \Rightarrow put\ x = 0, y = \frac{3}{2}\ so\ the\ pt\ \left(0\ , \frac{3}{2}\right)$

$put\ y = 0\ , x = 9\ so\ the\ pt(9\ ,0)$

Test $pt. (0,0): we\ test\ (i) and\ (ii)\ at\ (0,0)\ so$

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$(i) \Rightarrow 0 \geq 15 \rightarrow false$ $(ii) \Rightarrow 0 \geq 9 \rightarrow false$

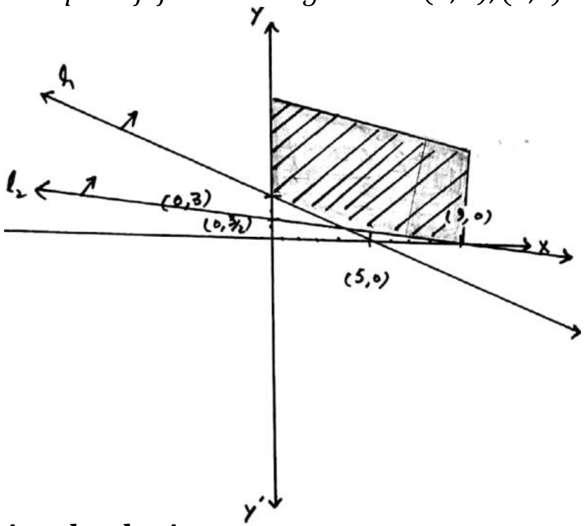
Feasible region:

The feasible region of the given system is the intersection of the graphs of (i) and (ii).

Also $x \geq 0, y \geq 0$ $indicates\ that\ graph\ of\ solution$ Set in $1st\ Quadrant\ as\ shown\ in\ fig.$

Corner point:

$Corner\ pts.\ of\ feasible\ region\ are\ (9, o), (0,3)$



Optimal solution:

$we\ find\ valves\ of\ z = 3x + y\ at\ corner\ pts.$

$(0,3), \quad z = 3(0) + 3 = 3$

$(9,0), z = 3(9) + 0 = 27$

So $z = 3x + y\ has\ minmum\ value\ 3\ at\ (0,3)$

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