Factorial:

The factorial of positive integer n is the product of n and all smaller positive n is the product of n and all smaller positive integer.

Factorial notation:

Let n be a positive integer. Then the product $n(n-1)(n-2)\ldots 3,2,1$ is denoted by $n!$ or $\angle n$ and read as n factorial.

That is

$$n! = n(n-1)(n-1)\ldots 3.2.1$$

Examples:

$$1! = 1 \qquad 2! = 2 \quad \Rightarrow 2.1!$$
$$3! = 3.2.1 = 6 \Rightarrow 3! = 3.2!$$
$$4! = 4.3.2.1 = 24 \Rightarrow 4! = 4.3!$$
$$5! = 5.4.3.2.1. = 120 \Rightarrow 5! = 5.4!$$
$$6! = 6.5.4.3.2.1 = 720 \Rightarrow 6! = 6.5!$$
$$n! = n(n-1)!$$

Prove that

$$0! = 1$$

Proof:

$$n! = n(n-1)$$

Put $n = 1$

$\Rightarrow 1! = 1$ *proved.*

Exercise 7.1

Question#1

Evaluate each of the following:

$(i).\ 4!$

Solution:

$4! = 4 \times 3 \times 2 \times 1$
$= 24$

$(ii).\ 6!$

Solution:

$6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1$
$= 710$

$(iii).\ \frac{8!}{7!}$

Solution:

$\frac{8!}{7!} = \frac{8\times 7!}{7!}$
$= 8$

$(iv).\ \frac{10!}{7!}$

Solution:

$\frac{10!}{7!} = \frac{10\times 9\times 8\times 7!}{7!}$
$= 10 \times 9 \times 8$
$= 720$

$(v).\ \frac{11!}{4!\,7!}$

Solution:

$\frac{11!}{4!\,7!} = \frac{10\times 9\times 8\times 7!}{7!}$
$= 10 \times 9 \times 8$
$= 720$

$(vi).\ \frac{6!}{3!3!}$

Solution:

$\frac{6!}{3!3!} = \frac{6\times 5\times 4\times 3!}{3!}$
$= \frac{120}{6}$
$= 20$

$(vii).\ \frac{8!}{4!2!}$

Solution:

$\frac{8!}{4!2!} = \frac{8\times 7\times 6\times 5\times 4!}{4!2!}$
$= 840$

$(viii).\ \frac{11!}{2!4!5!}$

Solution:

$\frac{11!}{2!4!5!} = \frac{11\times 10\times 9\times 8\times 7\times 6\times 5!}{2!4!5!}$
$= \frac{332640}{48}$
$= 6930$

$(ix).\ \frac{9!}{2!(9-2)!}$

Solution:

$\frac{9!}{2!(9-2)!} = \frac{9!}{2!7!}$
$= \frac{9\times 8\times 7!}{2!7!}$
$= \frac{72}{2}$
$= 36$

$(x).\ \frac{15!}{15!(4-4)!}$

Solution:

$\frac{15!}{15!(4-4)!} = \frac{15!}{15!(0!)}$
$= \frac{1}{0!} \qquad \because 0! = 1$
$= \frac{1}{1} = 1$

$(xi).\ \frac{3!}{0!}$

Solution:

$\frac{3!}{0!} = \frac{3\times 2\times 1}{1}$
$= 3 \times 2 \times 1$
$= 6$

$(xii).\ 4!\ 0!\ 1!$

Solution:

$4!\ 0!\ 1! = (4 \times 3 \times 2 \times 1)(1)(1)$
$= 24$

Question#2

Write each of the following in the factorial form:

$(i).\ 6.5.4.$

Solution:

Multiply and divide by $3!$

$\frac{6.5.4.3!}{3!}$
$= \frac{6!}{3!}$

$(ii).\ 12.11.10$

Solution:

Multiply and divide by $9!$

$\frac{12.11.10.9!}{9!}$

$=\frac{12!}{9!}$

$(iii).\ \mathbf{20.19.18.17}$

Solution:

Multiply and divide by $16!$

$\frac{20.19.18.17.16!}{16!}$

$=\frac{20!}{16!}$

$(iv).\ \frac{10.9}{2.1}$

Solution:

Multiply and divide by $8!$

$\frac{10.9.8!}{8!}$

$=\frac{10!}{2!8!}$

$(v).\ \frac{\mathbf{8.7.6}}{\mathbf{3.2.1}}$

Solution:

Multiply and divide by $5!$

$\frac{8.7.6}{3.2.1}=\frac{8.7.6.5!}{3!\,5!}$

$=\frac{8!}{3!5!}$

$(vi).\ \frac{\mathbf{52.51.50.49}}{\mathbf{4.3.2.1}}$

Solution:

Multiply and divide by $48!$

$\frac{52.51.50.49}{4.3.2.1}$

$=\frac{52.51.50.49.48!}{4.3.2.1.48!}$

$=\frac{52!}{4!\,48!}!$

$(vii).\ \boldsymbol{n(n-1)(n-2)}$

Solution:

Multiply and divide by $(n-3)!$

$\frac{n(n-1)(n-2)(n-3)!}{(n-3)!}$

$=\frac{n!}{(n-3)!}$

$(viii).\ \boldsymbol{(n+2)(n+1)(n)}$

Solution:

Multiply and divide by $(n-1)!$

$\frac{(n+2)(n+1)(n)(n-1)!}{(n-1)!}$

$=\frac{(n+2)!}{(n-1)!}$

$(ix).\ \frac{\boldsymbol{(n+1)(n)(n-1)}}{\mathbf{3.2.1}}$

Solution:

Multiply and divide by $(n-2)!$

$\frac{(n+1)(n)(n-1)(n-2)!}{3.2.1\,(n-2)!}$

$=\frac{(n+1)!}{3!(n-2)!}$

$(x).\ \boldsymbol{n(n-1)(n-2)....(n-r+1)}$

Solution:

As the numbers is decreasing by 1 so the next number would be

$(n-r+1)=n-r$

Multiply and divide by $(n-r)!$

$\frac{n(n-1)(n-2)....(n-r+1)(n-r)!}{(n-r)!}$

$=\frac{n!}{(n-r)!}$

Permutation:

A permutation of n different objects taken $r(\leq n)$ at a time is an arrangement of the r objects. Generally it is denoted as ${}^nP_r=\frac{n!}{(n-r)!}$

Fundamental principle of counting:

Suppose A and B are two events. The first event A can be occur in p different ways. After A has occupied B can occur in q different ways.

The number of ways that two events can occur is the product $p.q$

Prove that

$${}^nP_r=n(n-1)(n-2)\ldots(n-r+1)=\frac{n!}{(n-r)!}$$

Proof:

As there are n different objects to fill up r places. So, the first place can be filled in n ways.

$\because$ repetitions are not allowed, the second place can be filled in $(n-2)$ ways. The third place filled in $(n-2)$ ways. And so on.

The rth place has $n-(r-1)=n-r+1$ choices to be filled in. therefore by the fundamental principle of counting, r places can be filled by n different objects in $n(n-1)(n-1)\ldots(n-r+1)ways.$

$$\because\ {}^nP_r=n(n-1)(n-2)\ldots(n-r+1)$$

$$=\frac{n(n-1)(n-2)\ldots(n-r+1)(n-r)(n-r-1)\ldots3.2..1}{(n-r)(n-r-1)\ldots3.2.1}$$

$$\Rightarrow\ {}^nP_r=\frac{n!}{(n-r)!}$$

Hence proved.

Corollary:

If $r=n$ then

$${}^nP_n=\frac{n!}{(n-n)!}$$

$$=\frac{n!}{0!}=\frac{n!}{1}$$

$$\Rightarrow\ {}^nP_n=n!$$

Exercise 7.2

Question#1

Evaluate the following:

(i). $^{20}P_3$

Solution:

$^{20}P_3 = \frac{20!}{(20-3)!}$

$= \frac{20!}{17!}$

$= \frac{20.19.18.17!}{17!}$

$= 20.19.18$

$= 6840$

(ii). $^{16}P_4$

Solution:

$^{16}P_4 = \frac{16!}{(16-4)!} = \frac{16!}{12!}$

$= \frac{16.15.14.13.12!}{12!}$

$= 16.15.14.13$

$= 43680$

(iii). $^{12}P_5$

Solution:

$^{12}P_5 = \frac{12!}{(12-5)!} = \frac{12!}{7!} = \frac{12.11.10.9.8.7!}{7!}$

$= 95040$

(iv). $^{10}P_7$

Solution:

$^{10}P_7 = \frac{10!}{(10-7)!} = \frac{10!}{3!} = \frac{10.9.8.7.5.6.4.3!}{3!}$

$= 604800$

(v). $^{9}P_8$

Solution:

$^{9}P_8 = \frac{9!}{(9-8)!} = \frac{9!}{1!} = \frac{9.8.7.6.5.4.3.2.1}{1}$

$= 362880$

Question#2

Find the value of n when:

(i). $^{n}P_2 = 30$

Solution:

$^{n}P_2 = 30$

$\Rightarrow \frac{n!}{(n-2)!} = 30$

$\Rightarrow \frac{n(n-1)(n-2)!}{(n-2)!} = 30$

$\Rightarrow n(n-1) = 30$

$\Rightarrow n(n-1) = 6.5$

$\Rightarrow n = 6$

(ii). $^{11}P_n = 11.10.9$

Solution:

$^{11}P_n = 11.10.9$

$\Rightarrow \frac{11.10.9.8!}{(11-n)!} = 11.10.9$

$\Rightarrow \frac{8!}{(11-n)!} = 1$

$\Rightarrow 8! = (11-n)!$

$\Rightarrow 8 = 11 - n$

$\Rightarrow n = 11 - 8$

$\Rightarrow n = 3$

(iii). $^{n}P_4 : {}^{n-1}P_3 = 9 : 1$

Solution:

$\frac{^{n}P_4}{^{n-1}P_3} = \frac{9}{1}$

$\Rightarrow {}^{n}P_4 = 9\, {}^{n-1}P_3$

$\Rightarrow \frac{n!}{(n-4)!} = 9\frac{n\,(n-1)!}{(n-1-3)!}$

$\Rightarrow \frac{n(n-1)!}{(n-4)!} = 9\frac{(n-1)!}{(n-4)!}$

$\Rightarrow n = 9$

Question#3

Prove from the first principle that:

(i). $^{n}P_r = n.\,{}^{n-1}P_{r-1}$

Solution:

$R.H.S = n.\,{}^{n-1}P_{r-1}$

$= n.\frac{(n-1)!}{(n-1-(r-1))!}$

$= \frac{n\,(n-1)!}{(n-1-r+1)!}$

$= \frac{(n-1)!}{(n-r)!}$

$= \frac{n!}{(n-r)!}$

$= {}^{n}P_r = L.H.S$

(ii). $^{n}P_r = {}^{n-1}P_r + r.\,{}^{n-1}P_{r-1}$

Solution:

$R.H.S = {}^{n-1}P_{r-1} + r.\,{}^{n-1}P_{r-1}$

$= \frac{(n-1)!}{(n-1-r)!} + r.\frac{(n-1)!}{(n-1-r+1)!}$

$= \frac{(n-1)!}{(n-r-1)!} + r.\frac{(n-1)!}{(n-r)!}$

$= \frac{(n-1)!}{(n-r-1)!} + r.\frac{(n-1)!}{(n-r)(n-r-1)!}$

$= \frac{(n-1)!}{(n-r)(n-r-1)!}\left(1 + r.\frac{n!}{(n-r)}\right)$

$= \frac{(n-1)!}{(n-r)(n-r-1)!}\left(\frac{n-r+r}{(n-r)}\right)$

$= \frac{(n-1)!}{(n-r-1)!}\left(\frac{n}{(n-r)}\right)$

$= \frac{n\,(n-1)!}{(n-r)(n-r-1)!}$

$= \frac{n\,!}{(n-r)!}$

$= {}^{n}P_r = L.H.S$

Question#4

How many signals can be given by 5 lags of different colours, using 3 lags at a time?

Solution:

here $n = 5$, $r = 3$

Number of signals $= {}^{5}P_3$

$= \frac{5!}{(5-3)!} = \frac{5!}{2!}$

$= \frac{5.4.3.2!}{2!} = 60$

Question#5

How many signals can be given by 6 lags of different colours when any number of flags can be used at a time?

Solution:

Total number of flags $= n = 6$

Number of signals using one flag $= {}^6P_1 = 6$

Number of signals using two flags $= {}^6P_2 = 30$

Number of signals using three flags $= {}^6P_3 = 120$

Number of signals using four flags $= {}^6P_4 = 360$

Number of signals using five flags $= {}^6P_5 = 720$

Number of signals using six flags $= {}^6P_6 = 720$

Total number of signals $= 6 + 30 + 120 + 360 + 720 + 720 = 1956$

Question#6

How many words can be formed from the letters of the following words using all letters when no letter is to be repeated:

(i). **PLANE**

Solution:

Since number of letters in PLANE $= n = 5$

Therefore, total words form $= {}^5P_5 = 120$

(ii). **OBJECT**

Solution:

Since number of letters in OBJECT $= n = 6$

Therefore, total words forms $= {}^6P_6 = 720$

(iii). **FASTING**

Solution:

Since number of letters in FASTING $= n = 7$

Therefore, total words forms $= {}^7P_7 = 5040$

Question#7

How many 3-digit numbers can be formed by using each one of the digits 2, 3, 5, 7, 9 only once?

Solution:

Number of digits $= n = 5$

So, numbers forms taken 3 digits at a time $= {}^5P_3 = 60$

Question#8

Find the numbers greater than 23000 that can be formed from the digits 1, 2, 3, 5, 6 , without repeating any digit

HINT: The first two digits on L.H.S. will be 23 etc.

Solution:

Number greater than 23000 can be formed as

Number of numbers of the form $23***$ $= {}^3P_3 = 6$

Number of numbers of the form $25***$ $= {}^3P_3 = 6$

Number of numbers of the form $26***$ $= {}^3P_3 = 6$

Number of numbers of the form $3****$ $= {}^4P_4 = 24$

Number of numbers of the form $5****$ $= {}^4P_4 = 24$

Number of numbers of the form $6****$ $= {}^4P_4 = 24$

Thus, the total number formed $= 6 + 6 + 6 + 24 + 24 + 24 = 90$

Alternative Solution:

Permutation of 5 digits numbers $= {}^5P_5 = 120$

Numbers less than 23000 are of the form $1****$

Then permutations $= {}^4P_4 = 24$

If number less than 23000 are of the form $21***$

Then permutations $= {}^3P_3 = 6$

Thus, number greater than 23000 formed $= 120 - 24 - 6 = 90$

Question#9

Find the number of 5-digit numbers that can be formed from the digits 1, 2, 4, 6, 8 (when no digit is repeated), but

(i). **the digits 2 and 8 are next to each other.**

Solution:

Total number of digits = 5

If we take 28 as a single digit, then number of numbers $= {}^4P_4 = 24$

If we take 82 as a single digit, then number of numbers $= {}^4P_4 = 24$

So, the total numbers when 2 and 8 are next to each other $= 24 + 24 = 48$

(ii).**the digits 2 and 8 are not next to each other.**

Solution:

Number of total permutations $= {}^5P_5 = 120$

thus, number of numbers when 2 and 8 are not next to each other $= 120 - 48 = 72$

Question#10

How many 6-digit numbers can be formed, without repeating any digit from the digits 0, 1, 2, 3, 4, 5? In how many of them will 0 be at the tens place?

Solution:

Since number of permutations of 6 digits $= {}^6P_6 = 720$

But 0 at extreme left is meaning less

so, number of permutation when 0 is at extreme left $= {}^5P_5 = 120$

Thus, the number formed by 6 digits $= 720 - 120 = 600$

Now if we fix 0 at ten place then number formed $= {}^5P_5 = 120$

Question#11

How many 5-digit multiples of 5 can be formed from the digits 2, 3, 5, 7,9, when no digit is repeated.

Solution:

Number of digits = 5

For multiple of 5 we must have 5 at extreme right so number forms $= {}^4P_4 = 24$

Question#12

In how many ways can 8 books including 2 on English be arranged on a shelf in such a way that the English books are never together?

Solution:

Total numbers of books = 8

Total number of permutations $= {}^8P_8 = 40320$

Let E_1 *and* E_2 *denotes two English books then*

Number of permutation when E_1E_2 *place together* $= {}^7P_7 = 5040$

Number of permutation when E_1E_2 *place together* $= {}^7P_7 = 5040$

So total permutation when E_1 *and* E_2 *together* $= 5040 + 5040 = 10080$

Required permutation when English books are not together $= 40320 - 10080 = 30240$

Question#13

Find the number of arrangements of 3 books on English and 5 books on Urdu for placing them on a shelf such that the books on the same subject are together.

Solution:

Let E_1, E_2, E_3 *be the book on English and* U_1, U_2, U_3, U_4, U_5 *be the book on Urdu*

Then the permutation when books are arranged as $E_1, E_2, E_3, U_1, U_2, U_3, U_4, U_5$

$= {}^3P_3 \times {}^5P_5 = 6 \times 120 = 720$

Books are arranged As $U_1, U_2, U_3, U_4, U_5, E_1, E_2, E_3$

$= {}^5P_5 \times {}^3P_3 = 120 \times 6 = 720$

So total permutation when books of same subject are together $= 720 + 720 = 1440$

Question#14

In how many ways can 5 boys and 4 girls be seated on a bench so that the girls and the boys occupy alternate seats?

Solution:

Let the five boys be B_1, B_2, B_3, B_4, B_5 ***and the four girls are*** G_1, G_2, G_3, G_4, G_5

seats plane is $B_1, G_1, B_2, G_2, B_3, G_3, B_4, G_4, B_5$

Then the permutations

$= {}^5P_1 \times {}^4P_1 \times {}^4P_1 \times {}^3P_1 \times {}^3P_1 \times {}^2P_1 \times {}^2P_1 \times {}^1P_1 \times {}^1P_1$

$= 5 \times 4 \times 4 \times 3 \times 3 \times 2 \times 2 \times 1 \times 1 = 2880$

Theorem:

The number of permutations of n objects taken all at a time when n_1, of them are alike of one kind, n_2 are alike of second kind n_3 *are* alike of third are given by

$$\binom{n}{n_1 \; n_2 \; n_3} = \frac{n!}{n_1! \; n_2! \; n_3!}$$

Proof:

We know that arrangement of n_1 like objects

$$= {}^{n_1}P_{n_1} = n_1!$$

Arrangement of n_2 like objects $= {}^{n_2}P_{n_2} = n_2!$

Arrangement of n_3 like objects $= {}^{n_3}P_{n_3} = n_3!$

Let x be the required no. permutations of n objects.

Then total permutations $= x_n!\,n_2!\,n_3!$

But

$\boldsymbol{no. of\ permutations\ of\ n\ objects = n!}$

Therefore

$$x.n_1!\,n_2!\,n_3! = n!$$

$$\Rightarrow x = \frac{n!}{n_1!\,n_2!\,n_3!}$$

$$x = \binom{n}{n_1, \; n_2, \; n_3}$$

Circular Permutation:

The permutation of things which can be represented by the points on a circle is called circular permutation.

Exercise 7.3

Question#1

How many arrangements of the letters of the following words, taken all together, can be made:

(i). ***PAKPATTAN***

Solution:

Total number of letters $= 9$

P is repeated 2 times

A is repeated 3 times

T is repeated 2 times

K and N come only once.

Required number of permutations $= \binom{9}{2,3,2,1,1}$

$= \frac{9!}{2!\times3!\times2!\times1!\times1!}$

$= \frac{362880}{(2)(6)(2)}$

$= 15120$

(ii). ***PAKISTAN***

Solution:

Total number of letters $= 8$

A is repeated 2 times

P, K, I, S, T and N come only once.

Required number of permutations $= \binom{8}{2,1,1,1,1,1,1}$

$= \frac{8!}{2!\times 1!\times1!\times 1!\times1!\times1!\times1!}$

$= \frac{40320}{2}$

$= 20160$

(iii). ***MATHEMATICS***

Solution:

Total number of letters $= 11$

M is repeated 2 times

A is repeated 2 times

T is repeated 2 times

H, E, I, C and S come only once.

Required number of permutations $= \binom{11}{2,2,2,1,1,1,1,1}$

$= \frac{11!}{2!\times 2!\times2!\times 1!\times1!\times1!\times1!\times1!}$

$= \frac{39916800}{8}$

$= 4989600$

(iv). ***ASSASSINATION***

Solution:

Total number of letters =13

A is repeated 3 times

S is repeated 4 times

I is repeated 2 times

N is repeated 2 times

T and O come only once.

Required number of permutations $= \binom{13}{3,4,2,2,1,1}$

$= \frac{13!}{3!\times 4!\times2!\times 2!\times1!\times1!}$

$= \frac{6227020800}{(6)(24)(2)(2)}$

$= 10810800$

Question#2

How many permutations of the letters of the word PANAMA can be made, if P is to be the first letter in each arrangement?

Solution:

If P is the first letter, then words are of the form

$P*****$, where five $*$ can be replace with A,N,A,M,A.

So, number of letters $= 5$

A is repeated 3 times

M, N appears only once

So required permutations $= \binom{5}{3,4,1,1}$

$= \frac{13!}{3!\times1!\times1!}$

$= \frac{120}{6}$

$= 20$

Question#3

How many arrangements of the letters of the word ATTACKED can be made, if each arrangement begins with C and ends with K ?

Solution:

If C be the first letter and K is the last letter then words are of the form $C******K$. where each $*$ can be replaced with A, T, T, A, E, D.

So number of letters $= 6$

A is repeated 2 times

T is repeated 2 times

E and D come only once.

Required number of permutations $= \binom{6}{2,2,1,1}$

$= \frac{6!}{2!\times 2!\times1!\times1!}$

$= \frac{720}{4}$

$= 180$

Question#4

How many numbers greater than 1000,000 can be formed from the digits 0, $2,2,2,3,4,4$?

Solution:

The number greater than 1000000 are of the following forms.

If numbers are of the form $2******$,

where each $*$ can be filled with $0,2,2,2,3,4,4$

Then number of digits = 6

2 is repeated 2 times

4 is repeated 2 times

0 and 3 come only once.

So, number formed $= \binom{6}{2,2,1,1}$

$= \frac{6!}{2!\times 2!\times 1!\times 1!}$

$= \frac{720}{4}$

$= 180$

Now if numbers are of the form $3******$,

where each $*$ can be filled with 0,2,2,2,4,4

Then number of digits = 6

2 is repeated 3 times

4 is repeated 2 times

0 comes only once.

So number formed $= \binom{6}{3,2,1}$

$= \frac{6!}{3!\times 2!\times 1!}$

$= \frac{720}{12}$

$= 60$

Now if numbers are of the form $4******$,

where each $*$ can be filled with 0, 2,2,2,3,4

Then number of digits = 6

2 is repeated 3 times

0, 3 and 4 come only once.

So, number formed $= \binom{6}{3,1,1}$

$= \frac{6!}{3!\times 1!\times 1!}$

$= \frac{720}{6}$

$= 120$

So required numbers greater than 1000000 $=$ $180 + 60 + 120 = 360$.

Alternative

No. of digits $= 7$

No. of 2's $= 3$

No. of 4's $= 2$

0 and 3 come only once.

Permutations of 7 digits number $= \binom{7}{3,2,1,1}$

$= \frac{7!}{3!\times 2!\times 1!\times 1!}$

$= \frac{5040}{12}$

$= 420$

Number less than 1,000,000 are of the form $0******$, where each $*$ can be replaced with $2, 2, 3, 4, 4$.

No. of digits $= 6$

No. of 2's $= 3$

No. of 4's $= 2$

3 comes only once

So, permutations $= \binom{6}{3,2,1}$

$= \frac{6!}{3!\times 2!\times 1!}$

$= \frac{720}{12}$

$= 60$

Hence number greater than $1000000 = 420 - 60 = 360$

Question#5

How many 6-digit numbers can be formed from the digits 2, 2, 3, 3, 4, 4? How many of them will lie between 400,000 and 430,000?

Solution:

Total number of digits $= 6$

Number of 2's $= 2$

Number of 3's $= 2$

Number of 4's $= 2$

So, number formed by these 6 digits $= \binom{6}{2,2,2}$

$= \frac{6!}{(2!)(2!)(2!)}$

$= \frac{720}{(2)(2)(2)} = \frac{720}{8}$

$= 90$

The numbers lie between 400,000 and 430,000 are only of the form $42****$, where each $*$ can be filled by $2, 3, 3, 4$.

Here number of digits $= 4$.

Number of 2's $= 1$

Number of 3's $= 2$

Number of 4's $= 1$

So, number formed $= \binom{4}{1,2,1}$

$= \frac{6!}{(1!)(2!)(1!)}$

$= \frac{24}{(1)(2)(1)} = \frac{24}{2}$

$= 12$

Question#6

11 members of a club form 4 committees of 3, 4, 2, 2 members so that no member is a member of more than one committee. Find the number of committees.

Solution:

Total members $= 11$

Members in first committee $= 3$

Members in second committee $= 4$

Members in third committee $= 2$

Members in fourth committee $= 2$

So required number of committees $= \binom{11}{3,4,2,2}$

$= \frac{11!}{3!\times 4!\times 2!\times 2!}$

$= \frac{39916800}{(6)(24)(2)(2)}$

$= 69300$

Question#7

The D.C.Os of 11 districts meet to discuss the law and order situation in their districts.

In how many ways can they be seated at a round table, when two D.C.Os insist on sitting together?

Solution:

Number of D.C.O's $= 9$

Let D_1 and D_2 be the two D.C.O's insisting to sit together so consider them one.

If $D_1\ D_2$ sit together then permutations $= {}^9P_9 = 362880$

If $D_2\ D_1$ sit together then permutations $= {}^9P_9 = 362880$

So total permutations $= 362880 + 362880 = 725760$

Question#8

The Governor of the Punjab calls a meeting of 12 officers. In how many ways can they be seated at a round table?

Solution:

Fixing one officer on a particular seat,

we have permutations of remaining 11 officers $= {}^{11}P_{11} = 39916800$

Question#9

Fatima invites 14 people to a dinner. There are 9 males and 5 females who are seated at two different tables so that guests of one sex sit at one round table and the guests of the other sex at the second table. Find the number of ways in which all gests are seated.

Solution:

9 males can be seated on a round table

$= {}^8P_8 = 40320$

And 5 females can be seated on a round table

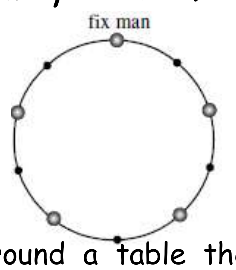
$= {}^4P_4 = 24$

So, permutations of both $= 40320 \times 24 = 967680$

Question#10

Find the number of ways in which 5 men and 5 women can be seated at a round table in such a way that no two persons of the same sex sit together.

Solution:



If we fix one man round a table then their permutations $= {}^4P_4 = 24$

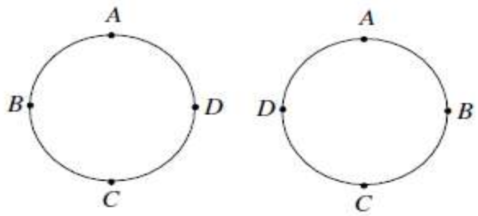
Now if women sit between the two men then their permutations $= {}^5P_5 = 120$

So total permutations $= 24 \times 120 = 2880$

Question#11

In how many ways can 4 keys be arranged on a circular key ring?

Solution:



Fixing one key we have permutation $= P_3 = 6$

Since above figures of arrangement are reflections of each other

Therefore permutations $= \frac{1}{2} \times 6 = 3$

Question#12

How many necklaces can be made from 6 beads of different colours?

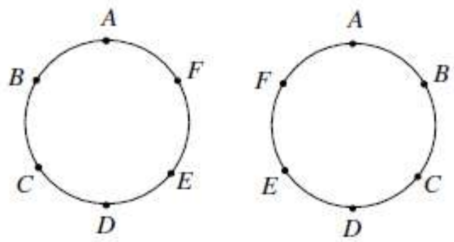
Solution:

Number of beads $= 6$

Fixing one bead, we have permutation $= {}^5P_5 = 120$

Since above figures of arrangement are reflections of each other

Therefore permutations $= \frac{1}{2} \times 120 = 60$



Combination:

When selection of objects is done neglecting its order, this is called combination.

- **The number of combinations of n different – objects taken $'r'$ at a time is denoted by ${}^nC_r$ or $\binom{n}{r}$ or $C(n,r)$ and defined as ${}^nC_r = \frac{{}^nP_r}{r!}$**

Or

$${}^nC_r = \frac{n!}{r!(n-r)!} \qquad \because \quad {}^nP_r = \frac{n!}{(n-r)!}$$

Prove that

$${}^nC_r = \frac{n!}{r!(n-r)!}$$

Proof:

There are ${}^nC_r$ **combinations of n different objects taken r at a time. Each combination consists of r different objects which can be permuted among t rise themselves ways. So each combination will give to r!**

permutation. Thus there will be ${}^{n}C_{r} \times r!$ Permutation of n different objects taken r at a time.

${}^{n}C_{r} \times r! = {}^{n}P_{r}$

${}^{n}C_{r} \times r! = \frac{n!}{(n-r)!}$

$\Rightarrow {}^{n}C_{r} = \frac{n!}{r!(n-r)!}$

Hence proved.

Corollary:

i **If $r = n$ then ${}^{n}C_{n} = \frac{n!}{n!(n-n)!}$**

$= \frac{n!}{n!\,o!} = 1$

ii If $r = 0$ then ${}^{n}C_{0} = \frac{n!}{0!(n-0)!}$

$= \frac{n!}{0!\,n!} = 1$

Complementary Combination:

Prove that

$${}^{n}C_{r} = {}^{n}C_{n-r}$$

proof:

$R.H.S = {}^{n}C_{n-r}$

$$= \frac{n!}{(n-r)!\,(n-(n-r))!}$$

$$= \frac{n!}{(n-r)!\,(n-n+r)1} = \frac{n!}{(n-r)!\,r!}$$

$$= \frac{n!}{r!(n-r)!} = {}^{n}C_{r} = L.H.S$$

Exercise 7.4

Question#1

Evaluate the following:

$(i).\ {}^{12}C_{3}$

Solution:

${}^{12}C_{3} = \frac{12!}{(12-3)!3!}$

$= \frac{12!}{9!3!}$

$= \frac{12.11.10.9!}{9!3!}$

$= \frac{12.11.10}{3!}$

$= \frac{1320}{6}$

$= 220$

$(ii).\ {}^{20}C_{17}$

Solution:

${}^{20}C_{17} = \frac{20!}{(20-17)!17!}$

$= \frac{20!}{3!17!}$

$= \frac{20.19.18!}{3!7!}$

$= \frac{20.19.18}{3!}$

$= \frac{6840}{6}$

$= 1140$

$(iii).\ {}^{n}C_{4}$

Solution:

${}^{n}C_{4} = \frac{n!}{(n-4)!4!}$

$= \frac{n(n-1)(n-2)(n-3)(n-4)!}{(n-4)!4!}$

$= \frac{n(n-1)(n-2)(n-3)}{4!}$

Question#2

Find the value of n, when

$(i).\ {}^{n}C_{5} = {}^{n}C_{4}$

Solution:

Since,

${}^{n}C_{5} = {}^{n}C_{4}$

$\Rightarrow {}^{n}C_{5} = {}^{n}C_{4}$

$\because {}^{n}C_{r} = {}^{n}C_{n-r}$

$\Rightarrow n - 5 = 4$

$\Rightarrow n = 4 + 5$

$\Rightarrow n = 9$

$(ii).\ {}^{n}C_{10} = \frac{12\times 11}{2!}$

Solution:

${}^{n}C_{10} = \frac{12\times 11}{2!}$

$\Rightarrow {}^{n}C_{10} = \frac{12.11.10!}{2!10!}$

$\Rightarrow {}^{n}C_{10} = \frac{12!}{(12-10)!10!}$

$\Rightarrow {}^{n}C_{10} = {}^{12}C_{10}$

$\Rightarrow n = 12$

$(iii).\ {}^{n}C_{12} = {}^{n}C_{6}$

Solution:

$\because\ {}^{n}C_{r} = {}^{n}C_{n-r}$

$\Rightarrow {}^{n}C_{12} = {}^{n}C_{n-12}$

$\Rightarrow {}^{n}C_{6} = {}^{n}C_{n-12}$

$\Rightarrow 6 = n - 12$

$\Rightarrow n = 18$

Question#3

Find the values of n and r, when

$(i).\ {}^{n}C_{r} = 35$ *and* ${}^{n}P_{r} = 210$

Solution:

${}^{n}C_{r} = 35$

Since,

${}^{n}C_{r} = \frac{12\times 11}{2!}$

$\Rightarrow \frac{n!}{(n-r)!r!} = 35$

$\Rightarrow \frac{n!}{(n-r)!} = 35.r! \quad \dots(i)$

Also,

${}^{n}P_{r} = 210$

$\Rightarrow \frac{n!}{(n-r)!} = 210 \quad \dots(ii)$

Comparing eq. (i) and eq. (ii)

$35\,.\,r! = 210$

$\Rightarrow \frac{n(n-1)(n-2)(n-3)!}{(n-3)!} = 210$

$\Rightarrow n(n-1)(n-2) = 210$

$\Rightarrow n(n-1)(n-2) = 7.6.5$

$\Rightarrow n = 7$

(ii). $^{n-1}C_{r-1} : {}^{n}C_{r} : {}^{n+1}C_{r+1} = 3 : 6 : 11$

Solution:

$\Rightarrow \frac{n(n-1)!}{(n-1-r+1)(r-1)!} : \frac{n!}{(n-r)!r!} = 3 : 6$

$\Rightarrow \frac{n(n-1)!}{(n-r)!(r-1)!} : \frac{n!}{(n-r)!r!} = 3 : 6$

$\Rightarrow \frac{\frac{(n-1)!}{(n-r)!(r-1)!}}{\frac{n!}{(n-r)!r!}} = \frac{1}{2}$

$\Rightarrow \frac{(n-1)!}{(n-r)!(r-1)!} : \frac{(n-r)!r!}{n!} = \frac{1}{2}$

$\Rightarrow \frac{r}{n} = \frac{1}{2}$

$\Rightarrow n = 2r \quad \ldots.(i)$

Now, consider

$^{n}C_{r} : {}^{n+1}C_{r+1} = 6 : 11$

$\Rightarrow \frac{n!}{(n-r)!r!} : \frac{(n+1)!}{(n+1-r-1)!(r+1)!} = 6 : 11$

$\Rightarrow \frac{n!}{(n-r)!r!} : \frac{(n+1)!}{(n-r)!(r+1)!} = 6 : 11$

$\Rightarrow \frac{\frac{n!}{(n-r)!r!}}{\frac{(n+1)!}{(n-r)!(r+1)!}} = \frac{6}{11}$

$\Rightarrow \frac{n!}{(n-r)!r!} \times \frac{(n-r)!(r+1)!}{(n+1)!} = \frac{6}{11}$

$\Rightarrow \frac{n!}{r!} \times \frac{(r+1)!}{(n+1)!} = \frac{6}{11}$

$\Rightarrow \frac{n!}{r!} \times \frac{(r+1)r!}{(n+1)n!} = \frac{6}{11}$

$\Rightarrow \frac{(r+1)}{(n+1)} = \frac{6}{11}$

$\Rightarrow 11(r+1) = 6(n+1)$

$\Rightarrow 11(r+1) = 6(2r+1)$

$\Rightarrow 11r + 11 = 12r + 6$

$\Rightarrow 11r - 12r = 6 - 11$

$\Rightarrow -r = -5$

$\Rightarrow r = 5$

Putting value of r in equation (ii)

$\Rightarrow n = 10$

Question#4

How many (a) diagonals and (b) triangles can be formed by joining the vertices of the polygon having:

(i). ***5 sides***

Solution:

Note:

For diagonal $^{n}C_2-n$

For triangle $={}^{n}C_3$

(a). 5 sides

$n = 5$

No. of diagonals= $^{5}C_2-5$

$= \frac{5!}{2!(5-2)!} - 5$

$= \frac{5!}{2!3!} - 5 = \frac{5.4.3!}{2!3!} - 5$

$10 - 5 = 5$

(b) No. of triangles $= {}^{5}C_3$

$= \frac{5!}{3!(5-3)!} = \frac{5!}{3!2!}$

$= \frac{5.4.3!}{3!2!} = 10$

(ii). ***8 sides***

Solution:

(a). 8 sides

$n = 8$

No. of diagonals= $^{8}C_2-8$

$= \frac{8!}{2!(8-2)!} - 8$

$= \frac{8.7.6!}{(2.1)!6!} - 8 = \frac{8.7.6!}{(2.1)!6!} - 8$

$28 - 8 = 20$

(b) No. of triangles $= {}^{8}C_3$

$= \frac{8!}{3!(8-3)!} = \frac{8.7.6.5!}{3!5!}$

$= \frac{8.7.6}{3.2.1} = 42$

(iii). ***12 sides***

Solution:

(a). 12 sides

$n = 12$

No. of diagonals= $^{12}C_2-12$

$= \frac{12!}{2!(12-2)!} - 12$

$= \frac{12.11.10!}{(2.1)10!} - 12$

$66 - 12 = 54$

(b) No. of triangles $= {}^{12}C_3$

$= \frac{12!}{3!(12-3)!}$

$\frac{12.11.10.9!}{(3.2.1)9!} = 220$

Question#5

The members of a club are 12 boys and 8 girls. In how many ways can a committee of 3 boys and 2 girls be formed?

Solution:

Number of boys $= 12$

So, committees formed taking 3 boys 12= $^{12}C_3 = 220$

Number of girls = 8

So, committees formed by taking 2 girls $= {}^{8}C_2 = 28$

Now total committees formed including 3 boys and 2 girls $= 220 \times 28 = 6160$

Question#6

How many committees of 5 members can be chosen from a group of 8 persons when each committee must include 2 particular persons?

Solution:

Number of persons $= 8$

Since two particular persons are included in every committee so we have to find combinations of 6 persons 3 at a time $= {}^{6}C_{3}$ $= 20$

Hence number of committees $= 20$

Question#7

In how many ways can a hockey team of 11 players be selected out of 15 players? How many of them will include a particular player?

Solution:

The number of players = 15

So, combination, taking 11 players at a time $= {}^{15}C_{11} = 1365$

Now if one particular player is in each collection then number of combinations $= {}^{14}C_{10} = 1001$

Question#8

Show that: ${}^{16}C_{11} + {}^{16}C_{10} = {}^{7}C_{11}$

Solution:

L.H.S $= {}^{16}C_{11} + {}^{16}C_{10}$

$= \frac{16!}{(16-11)!11!} + \frac{16!}{(16-10)!10!}$

$= \frac{16!}{5!11!} + \frac{16!}{6!10!}$

$= \frac{16!}{5!11.10!} + \frac{16!}{6.5!10!}$

$= \frac{16!}{10!5!}\left(\frac{1}{11} + \frac{1}{6}\right)$

$= \frac{16!}{10!5!}\left(\frac{6+11}{66}\right)$

$= \frac{16!}{10!5!}\left(\frac{17}{66}\right)$

$= \frac{16!}{10!5!}\left(\frac{17}{11.6}\right)$

$= \frac{17.16!}{11.10!6.5!}$

$= \frac{17!}{11!6!}$

$= \frac{17!}{11!\,(17-11)!}$

$= {}^{7}C_{11}$

Alternative

L.H.S $= {}^{16}C_{11} + {}^{16}C_{10} = 4368 + 8008 = 12376 \quad \ldots\ldots\ldots (i)$

R.H.S $= {}^{7}C_{11} = 12376 \quad \ldots\ldots\ldots (ii)$

From (i) ***and*** (ii)

L.H.S = R.H.S

Question#9

There are 8 men and 10 women members of a club. How many committees of can be formed, having:

(i). ***4 women***

Solution:

Number of men = 8

Number of women = 10

We have to form combination of 4 women out of 10 and 3 men out of 8

$= {}^{10}C_{4} + {}^{8}C_{3} = 210 \times 36 = 11760$

(ii). ***at the most 4 women***

Solution:

At the most 4 women means that women are less than or equal to 4, which implies the following possibilities (1W,6M),(2W,5M),(3W,4M),(4W,3M),(7M)

$= {}^{10}C_{1} \times {}^{8}C_{6} + {}^{10}C_{2} \times {}^{8}C_{5} + {}^{10}C_{3} \times {}^{8}C_{4} + {}^{10}C_{4} \times {}^{8}C_{3} + {}^{8}C_{7}$

$= (10)(28) + (45)(56) + (120)(70) + (210)(56) + (8)$

$= 280 + 2520 + 8400 + 117603 + 8$

$= 22968$

(iii). ***at least 4 women***

Solution:

At least 4 women means that women are greater than or equal to 4, which implies the following possibilities (4W,3M),(5W,2M),(6W,1M),(7W)

$= {}^{10}C_{4} \times {}^{8}C_{3} + {}^{10}C_{5} \times {}^{8}C_{2} + {}^{10}C_{6} \times {}^{8}C_{1} + {}^{10}C_{7}$

$= (210)(56) + (252)(28) + (210)(8) + 120$

$= 11760 + 7056 + 1680 + 120$

$= 20616$

Question#10

Prove that ${}^{n}C_{r} + {}^{n}C_{r-1} = {}^{n+1}C_{r}$

Solution:

L.H.S $= {}^{n}C_{r} + {}^{n}C_{r-1}$

$= \frac{n!}{(n-r)!r!} + \frac{n!}{(n-(r-1))!\,(r-1)!}$

$= \frac{n!}{(n-r)!r!} + \frac{n!}{(n-r+1)!\,(r-1)!}$

$= \frac{n!}{(n-r)!r(r-1)!} + \frac{n!}{(n-r+1)(n-r)!(r-1)!}$

$= \frac{n!}{(n-r)!(r-1)!}\left(\frac{1}{r} + \frac{1}{(n-r+1)}\right)$

$= \frac{n!}{(n-r)!(r-1)!}\left(\frac{n-r+1+r}{r\,(n-r+1)}\right)$

$= \frac{n!}{(n-r)!(r-1)!}\left(\frac{n+1}{r\,(n-r+1)}\right)$

$= \frac{(n+1)n!}{(n-r+1)\,(n-r)!r(r-1)!}$

$= \frac{(n+1)!}{(n-r+1)\,!r!}$

$= \frac{(n+1)!}{(n+1-r)\,!r!}$

$= {}^{n+1}C_{r}$

= R.H.S

Probability:

Probability is the numerical evaluation of a chance that a particular event would occur. OR Measurement of uncertainty.

Sample space:

The set S consisting of all possible outcome of a given experiment is called a sample space.

Event:

The particular outcome of an experiment is called an event.

- An event is a subset of the sample space.
- Sample space is denoted by S.
- Events are usually denoted by capital letters $A, B, C \dots$

Mutually Exclusive (disjoint) Events:

Two events $A \; and \; B$ are said to be mutually exclusive occur at the same time.

e.g;

in tossing a coin, the sample space $S = \{H, T\}$

Now if event

$A = \{H\}$ and event B= $\{T\}, then$

A and B are mutually exclusive events.

Equally likely Events:

Two events A and B are said to be equally likely if each one of them has equal number of chances of occurrence. e.g when a coin is tossed, we get either head H or tail T.

Chances of occurrence of head is $1/2$ while chances of occurrence of tail is also $1/2$

Thus the two events head and tail are equally likely events.

Note:

i Let E be an events than probability of E is denoted by P(E) and defined as

$$P(E) = \frac{n(E)}{n(S)}$$

$$= \frac{Number\ of\ favorable\ outcomes}{Number\ of\ possible\ outcomes}$$

ii Probability of an event must be a number lying between 0 and 1

$$i.e \quad 0 \leq P(E) \leq 1$$

iii If $P(E) = 0$ then E is called certain event (i.e) Event E will must occur.

iv If $P(E) = 0$ is called impossible event. (i.e.; event E could not occur)

Probability that an Evnt does not occur.

Suppose $n(S) = N \; and \; n(E) = R$

Then $P(E) = \frac{n(E)}{n(S)} = \frac{R}{N}$

Let $\bar{E}$ $denotes\ the$ non-occurrence of event E.

then $n(\bar{E}) = N - R$

$$\Rightarrow P(\bar{E}) = \frac{n(E)}{n(S)} \Rightarrow P(\bar{E}) = \frac{N - R}{N}$$

$$\Rightarrow P(\bar{E}) = \frac{N}{N} - \frac{R}{N}$$

$$\Rightarrow P(\bar{E}) = 1 - \frac{R}{N} \Rightarrow P(\bar{E}) = 1 - P(E)$$

Exercise 7.5

For the following experiments ,find the probability in each case:

Question#1

Experiment:

From a box containing orange-flavoured sweets, Bilal takes out one sweet without looking.

Events Happening:

(i). ***the sweet is orange-flavoured***

Solution:

Suppose A is the event that sweet is orange flavoured.

Since box only contained orange flavoured sweets

So favourable outcomes $= n(A) = 1$

Probability $= \frac{n(A)}{n(S)} = \frac{1}{1} = 1$

(ii). ***the sweet is lemon-flavoured***

Solution:

Let B be the event that the sweet is lemon flavoured.

Since box only contained orange-flavoured sweet

So favourable outcomes $= n(B) = 0$

Probability $= \frac{n(B)}{n(S)} = \frac{0}{1} = 0$

Question#2

Experiment:

Pakistan and India play a cricket match. The result is:

Events Happening:

(i). ***Pakistan wins***

Solution:

Since there are three possibilities that Pakistan wins, loses or the match tied.

Therefore, possible outcomes $= n(S) = 3$

Let A be the event that Pakistan wins

Favourable outcomes $= n(A) = 1$

Required probability $= \frac{n(A)}{n(S)} = \frac{1}{3}$

(ii). ***India does not lose.***

Solution:

Let B be the event that India does not lose.

If India does not lose then India may win, or the match tied

Therefore, favourable outcomes $= n(B) = 2$

Probability $= \frac{n(B)}{n(S)} = \frac{2}{3}$

Question#3

Experiment:

There are 5 green and 3 red balls in a box, one ball is taken out.

Events Happening:

Total number of balls $= 5 + 3 = 8$

Therefore, possible outcomes $= n(S) = 8$

(i). ***the ball is green***

Solution:

Let A be event that the ball is green

Then favourable outcomes $= n(A) = 5$

So, probability $= \frac{n(A)}{n(S)} = \frac{5}{8}$

(ii). ***the ball is red.***

Solution:

Let B be the event that the ball is red

Then favourable outcomes $= n(B) = 3$

So, probability $= \frac{n(B)}{n(S)} = \frac{3}{8}$

Question#4

Experiment:

A fair coin is tossed three times. It shows

Events Happening:

When a fair coin is tossed three times, the possible outcomes are HHH, HHT, HTH, THH, HTT, THT, TTH, TTT.

So total possible outcomes $= n(S) = 8$

(i). ***One tail***

Solution:

Let A be the event that the coin shows one tail then favourable outcomes are

HHT, HTH, THH,

i.e. $n(A) = 3$

So required probability $= \frac{n(A)}{n(S)} = \frac{3}{8}$

(ii). ***at least one head.***

Solution:

Let B be the event that coin shows at least one head then favourable outcomes

are

HHH, HHT, HTH, THH, HTT, THT, TTH.

i.e. $n(B) = 7$

So required probability $= \frac{n(B)}{n(S)} = \frac{7}{8}$

Question#5

Experiment:

A dice is rolled. The top shows

Events Happening:

The possible outcomes are that die show 1, 2, 3, 4, 5, 6.

So possible outcomes $= n(S) = 6$

(i). ***3 or 4 dots***

Solution:

Let A be the event that die show 3 or 4.

Then favorable outcomes $= n(A) = 2$

So required probability $= \frac{n(A)}{n(S)} = \frac{2}{6} = \frac{1}{3}$

(ii). ***dots less than 5.***

Solution:

Let B be the event that top of the die show dots less than 5 then

Favorable outcomes $= n(B) = 4$

So required probability $= \frac{n(B)}{n(S)} = \frac{4}{6} = \frac{2}{3}$

Question#6

Experiment:

From a box containing slips numbered $1, 2, 3, \ldots, 5$ ***one slip is picked up***

Events Happening:

Since the box contain 5 slips

So possible outcomes $= n(S) = 5$

(i). ***the number on the slip is a prime number***

Solution:

Let A be the event that the number on the slip are prime numbers 2, 3 or 5

Then favorable outcomes $= n(A) = 3$

So required probability $= \frac{n(A)}{n(S)} = \frac{3}{5}$

(ii). ***the number on the slip is a multiple of 3.***

Solution:

Let B be the event that number on the slips are multiple of 3 then

Favorable outcomes $= \mathrm{n(B)} = 1$

So probability $= \frac{\mathrm{n(B)}}{\mathrm{n(S)}} = \frac{1}{5}$

Question#7

Experiment:

Two die, one red and the other blue, are rolled simultaneously. The numbers of dots on the tops are added. The total of the two scores is:

Events Happening:

When two dice are rolled, the possible outcomes are

$(1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,5)\ (1,6)$
$(2,1)\ (2,2)\ (2,3)\ (2,4)\ (2,5)\ (2,6)$
$(3,1)\ (3,2)\ (3,3)\ (3,4)\ (3,5)\ (3,6)$
$(4,1)\ (4,2)\ (4,3)\ (4,4)\ (4,5)\ (4,6)$
$(5,1)\ (5,2)\ (5,3)\ (5,4)\ (5,5)\ (5,6)$
$(6,1)\ (6,2)\ (6,3)\ (6,4)\ (6,5)\ (6,6)$

This show possible outcomes $= n(S) = 36$

$(\boldsymbol{i})$. **5**

Solution:

Let A be the event that the total of two scores is 5 then favorable outcome are

$(1,4), (2,3), (3,2), (4,1)$

i.e. favorable outcomes $= n(A) = 4$

So required probability $= \frac{n(A)}{n(S)} = \frac{4}{36} = \frac{1}{9}$

$(\boldsymbol{ii})$. **7**

Solution:

Let B be the event that the total of two scores is 7 then favorable outcomes are $(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)$

i.e. favorable outcomes $= n(B) = 6$

So, probability $= \frac{n(B)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

$(\boldsymbol{iii})$. **11**

Solution:

Let C be the event that the total of two score is 11 then

favorable outcomes are $(5,6), (6,5)$ i.e. $\mathrm{n(C)} = 2$

So, probability $= \frac{\mathrm{n(B)}}{\mathrm{n(S)}} = \frac{2}{36} = \frac{1}{18}$

Question#8

Experiment:

Two die, one red and the other blue, are rolled simultaneously. The numbers of dots on the tops are added. The total of the two scores is:

Events Happening:

Total number of balls $= 40$ i.e. $n(S) = 40$

Black balls $= 15$

Green balls $= 5$

Yellow balls $= 40 - (15 + 5) = 20$

$(\boldsymbol{i})$. ***The ball is black***

Solution:

Let A be the event that the ball is black then $n(A) = 15$

So required probability $= \frac{n(A)}{n(S)} = \frac{15}{40} = \frac{3}{8}$

$(\boldsymbol{ii})$. ***The ball is green***

Solution:

Let B denotes the event that the ball is green then $n(B) = 5$

So required probability $= \frac{n(B)}{n(S)} = \frac{5}{40} = \frac{1}{8}$

$(\boldsymbol{iii})$. ***The ball is not green.***

Solution:

Let C denotes the event that the ball is not green then ball is either black or yellow therefore favorable outcomes $= n(C) = 15 + 20 = 35$

So required probability $= \frac{n(C)}{n(S)} = \frac{35}{40} = \frac{7}{8}$

Question#9

Experiment:

One chit out of 30 containing the names of 30 students of a class of 18 boys and 12 girls is taken out at random, for nomination as the monitor of the class.

Events Happening:

$(\boldsymbol{i})$.***the monitor is a boy***

Number of students $= 30$

Then possible outcomes $= n(S) = 30$

Solution:

Now if A be the event that the monitor is the boy then

Favorable outcomes $= n(A) = 18$

So, probability $= \frac{n(A)}{n(S)} = \frac{18}{30} = \frac{3}{5}$

$(\boldsymbol{ii})$. ***the monitor is a girl.***

Solution:

Now if B be the event that the monitor is the girl then

Favorable outcomes $= n(B) = 12$

So, probability $= \frac{n(B)}{n(S)} = \frac{12}{30} = \frac{2}{5}$

Question#10

Experiment:

A coin is tossed four times. The tops show

Events Happening:

When the coin is tossed four times the possible outcomes are

HHHT HHTH HTHH THHH
HHTT HTTH TTHH THHT

HTTT TTTH TTHT THTT
TTTT HHHH THTH HTHT

i.e. $n(S) = 16$

(i). ***all heads***

Solution:

Let A be the event that the top shows all head then
favorable outcome is HHHH i.e. $n(A) = 1$

Now probability $= \frac{n(A)}{n(S)} = \frac{1}{16}$

(ii). ***2 heads and 2 tails.***

Solution:

Let B be the event that the top shows 2 head and two tails the favorable
outcomes are HHTT, HTTH, TTHH, THHT, THTH, HTHT

i.e. $n(B) = 6$

Now probability $= \frac{n(B)}{n(S)} = \frac{6}{16} = \frac{3}{8}$

Estimating probability and Tally Marks:

Exercise 7.6

Question#1

A fair coin is tossed 30 times, the result of which is tabulated below. Study the table and answer the questions given below the table:

Event	Tally Marks	Frequency
Head	𝍸 𝍸 ///	14
Tail	𝍸 𝍸 𝍸 //	16

Solution:

From the table, total outcomes $= 30 \Rightarrow n(S) = 30$

From the table, we see that

(i). ***How many times does 'head' appear?***

Solution:

Let A = event the times 'head' appears $\Rightarrow n(A) = 14$

(ii). ***How many times does 'tail' appear?***

Solution:

Let B = event the times 'tail' appears $\Rightarrow n(B) = 16$

(iii). ***Estimate the probability of the appearance of head?***

Solution:

Probability that head appears $= P(A) = \frac{n(A)}{n(S)} = \frac{14}{30} = \frac{7}{15}$

(iv). ***Estimate the probability of the appearance of tail?***

Solution:

Probability that tail appears $= P(B) = \frac{n(B)}{n(S)} = \frac{16}{30} = \frac{8}{15}$

Question#2

A die is tossed 100 times. The result is tabulated below. Study the table and answer the questions given below the table:

Event	Tally Marks	Frequency
1	𝍸 𝍸 ////	14
2	𝍸 𝍸 𝍸 //	17
3	𝍸 𝍸 𝍸 𝍸	20
4	𝍸 𝍸 𝍸 ///	18
5	𝍸 𝍸 𝍸	15
6	𝍸 𝍸 𝍸 /	16

Solution:

From the table, total outcomes $= 100 \Rightarrow n(S) = 100$

From the table, we see that

(i). ***How many times do 3 dots appear?***

Solution:

Let A = event the number of times ***3 dots*** appears $\Rightarrow n(A) = 20$

(ii). ***How many times do 5 dots appear?***

Solution:

Let B = event the number of times ***5 dots*** appears $\Rightarrow n(B) = 15$

(iii). ***How many times does an even number of dots appear?***

Solution:

Let C = event the number of times, ***even number of dots*** appears

$\Rightarrow n(C) = 17 + 18 + 16 = 51$

(iv). ***How many times does a prime number of dots appear?***

Solution:

Let C = event the number of times, ***even number of dots*** appears

$\Rightarrow n(D) = 17 + 15 + 20 = 52$

(v). ***Find the probability of each one of the above cases.***

Solution:

Required probabilities are as :

$\mathbf{P(A)} = \frac{n(A)}{n(S)} = \frac{20}{100} = \frac{1}{5}$

$\mathbf{P(B)} = \frac{n(B)}{n(S)} = \frac{15}{100} = \frac{3}{20}$

$\mathbf{P(C)} = \frac{n(C)}{n(S)} = \frac{51}{100}$

$\mathbf{P(D)} = \frac{n(D)}{n(S)} = \frac{52}{100} = \frac{13}{25}$

Question#3

The eggs supplied by a poultry farm during a week broke during transit as follows:

$\mathbf{1\%}, \mathbf{2\%}, \mathbf{1\frac{1}{2}\%}, \mathbf{\frac{1}{2}\%}, \mathbf{1\%}, \mathbf{2\%}, \mathbf{1\%}$

Find the probability of the eggs that broke in a day. Calculate the number of eggs that will be broken in transiting the following number of eggs:

Solution:

total eggs $= 70 \Rightarrow n(S) = 70$

Let $A =$ event the egg broke $\Rightarrow n(A) = 1 + 2 + 1.5 + 0.5 + 1 + 2 + 1 = 9$

$\mathbf{P(A)} = \frac{n(A)}{n(S)} = \frac{9}{700}$

$(\boldsymbol{i})$. 7,000

Solution:

Number of eggs broke in $7{,}000 = 7{,}000 \times \frac{9}{700} = 90$

$(\boldsymbol{ii})$. 8,400

Solution:

Number of eggs broke in $8{,}400 = 8{,}400 \times \frac{9}{700} = 108$

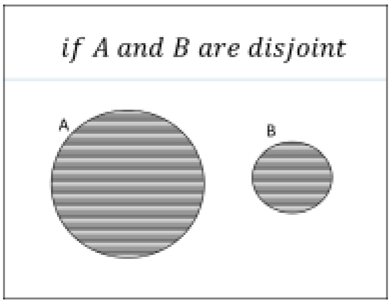
iii) 10,500

Eggs are 10500 then broken eggs $= 10500 \times \frac{9}{100}$

$= 135$

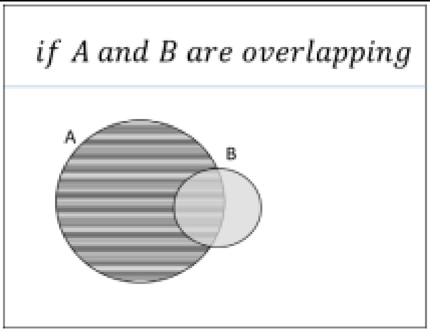
Addition of probabilities:

Suppose A and B be two events.



A ∪ B

then $P(A \text{ or } B) = P(A \cup B) = P(A) + P(B)$

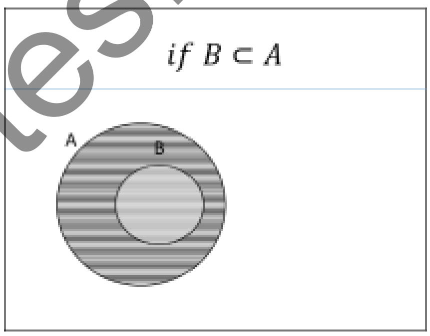


A ∪ B

then $P(A or B) = P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$

Event	Broken Eggs	Number of eggs
1	1	100
2	2	100
3	$1\frac{1}{2} = 1.5$	100
4	$\frac{1}{2} = 0.5$	100
5	1	100
6	2	100
7	1	100



A ∪ B

then $P(A or B) = P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$

Exercise 7.7

Question#1

If sample space $= \{\mathbf{1, 2, 3, 9}\}$ ***, Event*** $\boldsymbol{A} = \{\mathbf{2, 4, 6, 8}\}$ ***and Event*** $\boldsymbol{B} = \{\mathbf{1, 3, 5}\}$***, find*** $\mathbf{P(A \cup B)}$ ***.***

Solution:

Sample space $= \{1,2,3, \ldots\ldots\ldots, 9\}$

then $n(S) = 9$

Since event $A = \{2,4,6,8\}$

then $n(A) = 4$

Also, event $B = \{1,3,5\}$

then $n(B) = 3$

Now

$P(A \cup B) = P(A) + P(B)$

$= \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)}$

$= \frac{4}{9} + \frac{3}{9} = \frac{7}{9}$

Question#2

A box contains 10 red, 30 white and 20 black marbles. A marble is drawn at random. Find the probability that it is either red or white.

Solution:

Red marble $= 10$

White marble $= 30$

Black marble $= 20$

Total marble $= 10 + 30 + 20 = 60$

Therefore

$n(S) = 60$

Let A be the event that the marble is red then $n(A) = 10$

And let B be the event that the marble is white then $n(B) = 30$

Since A and B are mutually exclusive event therefore,

Probability $= P(A \cup B) = P(A) + P(B)$

$= \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)}$

$= \frac{10}{60} + \frac{30}{60}$

$= \frac{40}{60} = \frac{2}{3}$

Question#3

A natural number is chosen out of the first fifty natural numbers. What is the probability that the chosen number is a multiple of 3 or of 5?

Solution:

Since sample space is first fifty natural number so $S = \{1,2,3,\dots\dots\dots,50\}$

Then $n(S) = 50$

Let A be the event that the chosen number is a multiple of 3 then

$A = \{3,6,9,\dots\dots,48\}$

so, $n(A) = 16$

If B be the event that the chosen number is multiple of 5 then

$B = \{5,10,15,\dots\dots,50\}$

so, $n(B) = 10$

Now $A \cap B = \{15,30,45\}$

so, $n(A \cap B) = 3$

Since A and B are not mutually exclusive event therefore

Probability $= \boldsymbol{P(A \cup B)} = P(A) + P(B) - P(A \cap B)$

$= \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)} - \frac{n(A \cap B)}{n(S)}$

$= \frac{16}{50} + \frac{10}{50} - \frac{3}{50}$

$= \frac{16+10-3}{50}$

$= \frac{23}{50}$

Question#4

A card is drawn from a deck of 52 playing cards. What is the probability that it is a diamond card or an ace?

Solution:

Total number of cards $= 52$,

therefore, possible outcomes $= n(S) = 52$

Let A be the event that the card is a diamond card.

Since there are 13 diamond cards in the deck therefore $n(A) = 13$

Now let B the event that the card is an ace card.

Since there are 4 ace cards in the deck therefore $n(B) = 4$

Since one diamond card is also an ace card therefore A and B are not mutually exclusive event and $n(A \cap B) = 1$

Now probability $= P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)} - \frac{n(A \cap B)}{n(S)}$

$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52}$

$= \frac{13+4-1}{52}$

$= \frac{16}{52}$

$= \frac{4}{13}$

Question#5

A die is thrown twice. What is the probability that the sum of the number of dots shown is 3 or 11?

Solution:

When die is thrown twice the possible outcomes are

$(1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,5)\ (1,6)$
$(2,1)(2,2)(2,3)(2,4)(2,5)(2,6)$
$(3,1)\ (3,2)\ (3,3)\ (3,4)\ (3,5)\ (3,6)$
$(4,1)\ (4,2)\ (4,3)\ (4,4)\ (4,5)\ (4,6)$
$(5,1)\ (5,2)\ (5,3)\ (5,4)\ (5,5)\ (5,6)$
$(6,1)\ (6,2)\ (6,3)\ (6,4)\ (6,5)\ (6,6)$

This shows possible outcomes $= n(S) = 36$

Let A be the event that the sum is 3

Then the favourable outcomes are $(1,2)$ and $(2,1)$, i.e. $n(A) = 2$

Now let B the event that the sum is 11

Then the favourable outcomes are $(5,6)$ and $(6,5)$ i.e. $n(B) = 2$

Since A and B are mutually exclusive events therefore

Probability $= P(A \cup B) = P(A) + P(B)$

$= \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)}$

$= \frac{2}{36} + \frac{2}{36}$

$= \frac{4}{36}$

$= \frac{1}{9}$

Question#6

Two dice are thrown. What is the probability that the sum of the numbers of dots appearing on them is 4 or 6 ?

Solution:

$$n(S) = 36$$

$$A \text{ represent sum is } 4$$

$$B \text{ represents sum is } 6$$

$$A = \{(1,3)(2,2),(3,1)\}$$

$$,n(A) = 3$$

$$B = \{(1,5),(2,4),(3,3),(4,2),(5,1)\}$$

$$,n(B) = 5$$

$$P(A) = \frac{n(A)}{n(B)} = \frac{3}{36}$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{5}{36}$$

$$\because A \cap B = \emptyset (A \text{ and } B \text{ are disjoint events})$$

$$\Rightarrow P(A \cup B) = P(A) + P(B)$$

$$= \frac{3}{36} + \frac{5}{36} = \frac{8}{36}$$

$$P(A \cup B) = \frac{2}{9}$$

Question#7

Two dice are thrown simultaneously. If the event A is that the sum of the numbers of dots shown is an odd number and the event B is that the number of dots shown on at least one die is 3. Find $P(A \cup B)$.

Solution:

When two dice are thrown the possible outcomes are

$(1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,5)\ (1,6)$
$(2,1)\ (2,2)\ (2,3)\ (2,4)\ (2,5)\ (2,6)$
$(3,1)\ (3,2)\ (3,3)\ (3,4)\ (3,5)\ (3,6)$
$(4,1)\ (4,2)\ (4,3)\ (4,4)\ (4,5)\ (4,6)$
$(5,1)\ (5,2)\ (5,3)\ (5,4)\ (5,5)\ (5,6)$
$(6,1)(6,2)(6,3)(6,4)(6,5)(6,6)$

This shows possible outcomes $= n(S) = 36$

Since A be the event that the sum of dots is and odd number

Then favourable outcomes are

$(1,2),(1,4),(1,6),(2,1),(2,3),(2,5),(3,2),(3,4),(3,6),$
$(4,1),(4,3),(4,5),(5,2),(5,4),(5,6),(6,1),(6,3),(6,5)$

i.e. favourable outcomes $= n(A) = 18$

Sine B is the event that the least one die has 3 dot on it therefore Favourable outcomes are $(1,3),(2,3),(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),$ $(4,3),(5,3),(6,3)$ i.e. favourable outcomes

$$= n(B) = 11$$

Since A and B have common outcome $(2,3),(3,2),(3,4),(3,6),(4,3),(6,3)$

i.e. $n(A \cap B) = 6$

Now probability $= P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)} - \frac{n(A \cap B)}{n(S)}$

$= \frac{18}{36} + \frac{11}{36} - \frac{6}{36}$

$= \frac{18+11-6}{36}$

$= \frac{23}{36}$

Question#8

There are 10 girls and 20 boys in a class. Half of the boys and half of the girls have blue eyes. Find the probability that one student chosen as monitor is either a girl or has blue eyes.

Solution:

Number of girls $= 10$

Number of boys $= 20$

Total number of students $= 10 + 20 = 30$

Since half of the girls and half of the boys have blue eyes

Therefore, students having blue eyes $= 5 + 10 = 15$

Let A be event that monitor of the class is a student of blue eyes then $n(A) = 15$

Now Let B be the event that the monitor of the class is girl then $n(B) = 10$

Since 5 girls have blue eyes therefore A and B are not mutually exclusive

Therefore,

$n(A \cap B) = 5$

Now probability $= P(A \cup B)$

$$= P(A) + P(B) - P(A \cap B)$$

$= \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)} - \frac{n(A \cap B)}{n(S)}$

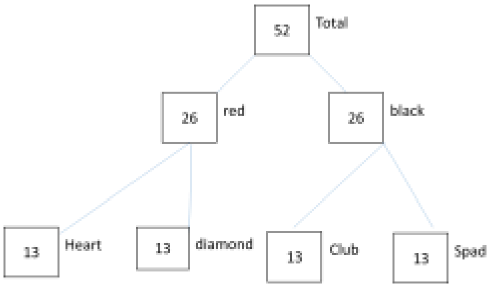
$= \frac{15}{10} + \frac{10}{30} - \frac{5}{30}$

$= \frac{15+10-5}{30}$

$= \frac{20}{30}$

$= \frac{2}{3}$

Note "Plying card"



Each type (Heart, Diamond, Club ,Sped)
Consists of $One - Ace$

$One - king$

$One - Queen$

$one - jack$

Multiplication of Probabilities

Dependent Event:

Two events are said to be dependent, if the occurrence of any one of them affect the occurrence of the other.

Independent Events:

two events are said to be independent , if the occurrence of the other.

Theorem:

If A and , B are independent events, the probability events that both of them occur is equal to the probability of the occurrence of B. Symbolically, it is denoted by

$$\boldsymbol{P(A \cap B) = P(A).P(B)}$$

Proof:

Let event A belong to the sample space S, such that

$$n(S_1) = n_1 \ \ and \ n(A) = m_1$$

$\Rightarrow$ *let event B belong to the sample sapce* S_2

Such that $n(S_2) = n_2$ and $n(B) = m_2$

$\Rightarrow P(B) = \frac{m_2}{n_2}$

Favorable case of $A \cap B = m_1 m_2$

Possible cases of $A \cap B = n_! n_2$

$$P(A \cap B) = \frac{favorable\ case}{possible\ case}$$

$$= \frac{m_1 m_2}{n_1 n_2}$$

$$= \frac{m_1}{n_1} . \frac{m_2}{n_2}$$

$$\Rightarrow P(A \cap B) = P(A).P(B)$$

Exercise 7.8

Question#1

The probability that a person A will be alive 15 years hence is $\frac{5}{7}$ and the probability that another person B will be alive 15 years hence is $\frac{7}{9}$. Find the probability that both will be alive 15 years hence.

Solution:

Since

$P(A) = \frac{5}{7}$

And

$P(B) = \frac{7}{9}$

Then the probability that both will alive 15 year is

$P(A \cap B) = P(A).P(B) = \frac{5}{7} . \frac{7}{9} = \frac{5}{9}$

Question#2

A die is rolled twice: Event E_1 is the appearance of even number of dots and event E_2 is the appearance of more than 4 dots. Prove that: $P(E_1 \cap E_2) = P(E_1) . P(E_2)$

Solution:

When a die is rolled then possible outcomes are $1, 2 ,3 , 4, 5, 6$

This shows that possible outcomes $= n(S) = 6$

Since 1 E is the event that the dots on the die are even then favourable

outcomes are $2, 4, 6$

this shows $n(E_1) = 3$

So, probability $= P(E_1) = \frac{n(E_1)}{n(S)}$

$= \frac{3}{6}$

$= \frac{1}{2}$

Now since E_2 is the event that the dot appear are more than four then

favourable outcomes are 5 and 6. This show $n(E_2) = 2$

So, probability $= P(E_2) = \frac{n(E_2)}{n(S)}$

$= \frac{2}{6}$

$= \frac{1}{3}$

Since E_1 and E_1 are not mutually exclusive

And the possible common outcome is 6 i.e.

$n(E_1 \cap E_2) = 1$

So, probability $P(E_1 \cap E_2) = \frac{n(E_1 \cap E_2)}{n(S)} = \frac{1}{6}$ $\dots\dots\dots..(i)$

Now

$P(E_1).P(E_2) = \frac{1}{2}.\frac{1}{3} = \frac{1}{6}$ ………….(ii)

Form (i) and (ii)

$P(E_1 \cap E_2) = P(E_1).P(E_2)$

Proved

Question#3

Determine the probability of getting 2 heads in two successive tosses of a balanced coin.

Solution:

When two coins are tossed then possible outcomes are

HH, HT, TH, TT

i.e. $n(S) = 4$

Let A be the event of getting two heads then favourable outcome is HH.

so, $n(A) = 1$

Now probability $= P(A) = \frac{n(A)}{n(S)} = \frac{1}{4}$

Question#4

Two coins are tossed twice each. Find the probability that the head appears on the first toss and the same faces appear in the two tosses.

Solution:

When the two coins are tossed then possible outcomes are

HH, HT, TH, TT

This shows $n(S) = 4$

Let A be the event that head appear in the first toss then

favourable outcomes are HT, HH, i.e. $n(A) = 2$

Let B be the event that same face appear on the second toss then

favourable outcomes are HH, TT. i.e. $n(B) = 2$

Now probability $= P(A \cap B) = P(A).P(B)$

$= \frac{n(A)}{n(S)}.\frac{n(B)}{n(S)}$

$= \frac{2}{4}.\frac{2}{4}$

$= \frac{1}{2}.\frac{1}{2}$

$= \frac{1}{4}$

Question#5

Two cards are drawn from a deck of 52 playing cards. If one card is drawn and replaced before drawing the second card, find the probability that both the cards are aces.

Solution:

Since there are 52 cards in the deck therefore $n(S) = 52$

Let A be the event that first card is an ace then $n(A) = 4$

And let B be the event that the second card is also an ace then $n(B) = 4$

Now probability $= P(A \cap B) = P(A).P(B)$

$= \frac{n(A)}{n(S)}.\frac{n(B)}{n(S)}$

$= \frac{4}{52}.\frac{4}{52}$

$= \frac{1}{169}$

Question#6

Two cards from a deck of 52 playing cards are drawn in such a way that the card is replaced after the first draw. Find the probabilities in the following cases:

(i). ***first card is king and the second is a queen***

Solution:

Let A be the event that the first card is king then $n(A) = 4$

and let B be the event that the second card is queen then $n(B) = 4$

Now probability $= P(A \cap B) = P(A).P(B)$

$= \frac{n(A)}{n(S)}.\frac{n(B)}{n(S)}$

$= \frac{4}{52}.\frac{4}{52}$

$= \frac{1}{169}$

(ii). ***both the cards are faced cards i.e. king, queen, jack.***

Solution:

Let C be the event that first card is faced card. Since there are 12 faced cards in the deck therefore $n(C) = 12$

and let D be the event that the second card is also faced card then $n(D) = 12$

Now probability $= P(A \cap B) = P(A).P(B)$

$= \frac{n(A)}{n(S)}.\frac{n(B)}{n(S)}$

$= \frac{12}{52}.\frac{12}{52}$

$= \frac{3}{13}.\frac{3}{13}$

$= \frac{9}{169}$

Question#7

Two dice are thrown twice. What is probability that sum of the dots shown in the first throw is 7 and that of the second throw is 11 ?

Solution:

When the two dice are thrown the possible outcomes are

$(1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,5)\ (1,6)$

$(2,1)\ (2,2)\ (2,3)\ (2,4)\ (2,5)\ (2,6)$

(3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (3, 6)
(4, 1) (4, 2) (4, 3) (4, 4) (4, 5) (4, 6)
(5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6)
(6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6)

Which shows that $n(S) = 36$

Let A be the event that the sum of dots in first throw is 7 then favourable outcomes are $(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)$ i.e. $n(A) = 6$

Let B be the event that the sum of dots in second throw is 11 then

favourable outcomes are $(5, 6), (6, 5)$ i.e. $n(B) = 2$

Now probability $= P(A \cap B) = P(A).P(B)$

$= \frac{n(A)}{n(S)} \cdot \frac{n(B)}{n(S)}$

$= \frac{6}{36} \cdot \frac{2}{36}$

$= \frac{1}{6} \cdot \frac{1}{18}$

$= \frac{1}{108}$

Question#8

Find the probability that the sum of dots appearing in two successive throws of two dice is every time 7.

Solution:

When the two dice are thrown the possible outcomes are

(1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6)
(2, 1) (2, 2) (2, 3) (2, 4) (2, 5) (2, 6)
(3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (3, 6)
(4, 1) (4, 2) (4, 3) (4, 4) (4, 5) (4, 6)
(5, 1)(5, 2)(5, 3)(5, 4)(5, 5)(5, 6)
(6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6)

Which shows that $n(S) = 36$

Let A be the event that the sum of dots in first throw is 7 then

favourable outcomes are $(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)$ i.e. $n(A) = 6$

Let B be the event that the sum of dots in second throw is also 7 then

similarly, favourable outcomes $= n(B) = 6$

Now probability $= P(A \cap B) = P(A).P(B)$

$= \frac{n(A)}{n(S)} \cdot \frac{n(B)}{n(S)}$

$= \frac{6}{36} \cdot \frac{6}{36}$

$= \frac{1}{6} \cdot \frac{1}{6}$

$= \frac{1}{36}$

Question#9

A fair die is thrown twice. Find the probability that a prime number of dots appear in the first throw and the number of dots in the second throw is less than 5.

Solution:

When the die is thrown twice then the top may show $1, 2, 3, 4, 5, 6$

This shows possible outcomes $= n(S) = 6$

Let A be the event that the number of the dots is prime then

favourable outcomes are $2, 3, 5$, i.e. $n(A) = 3$

Let B be the event that the number of dots in second throw is less than 5

then favourable outcomes are $1, 2, 3, 4$ i.e. $n(B) = 4$

Now probability $= P(A \cap B) = P(A).P(B)$

$= \frac{n(A)}{n(S)} \cdot \frac{n(B)}{n(S)}$

$= \frac{3}{6} \cdot \frac{4}{6}$

$= \frac{1}{2} \cdot \frac{2}{3}$

$= \frac{1}{3}$

Question#10

A bag contains 8 red, 5 white and 7 black balls, 3 balls are drawn from the bag. What is the probability that the first ball is red, the second ball is white, and the third ball is black, when every time the ball is replaced?

Hint: $\left(\frac{8}{20}\right), \left(\frac{5}{20}\right), \left(\frac{7}{20}\right)$ is the probability.

Solution:

Since number of red balls $= 8$

Number of white balls $= 5$

Number of black balls $= 7$