Set

"A well- defined collection of distinct objects is called a set."

*Set are usually denoted by capital English alphabets such as A,B,C,…,X,Y,Z

*the objects of a set are called its elements denoted by small letters such as $a, b, c, \dots x, y, z$.

$e.g\ A = \{a, b, c\}$

There are three different ways of describing a set.

1. Descriptive method.

A method of describing a set in words is known as "Descriptive Method" e.g.,

A set of all vowels of English alphabets.

2.**Tabular Method** of describing a set by writing the elements of the set within brackets is known as "Tabular *method" e.g,*

$A = \{a, e, I, o, u\}$

$O = \{\pm 1, \pm 3, \pm 5, \pm 7..\}$

3.set-builder Method

A method of describing a set in which the element of a set are denoted by an arbitrary variable (say x) starting common property or properties possessed by all elements of the set is known as set- builder Method

$A = \{x | x \text{ is vowels of English alphabets}\}$

$O = \{x | x \text{ is an odd number}\}$

The symbol used for membership of a set is $\in$

and $\notin$ *means member not belong to a set.*

Some important sets.

$N = \text{the set all natural numbers}$

$N = \{1,2,3, \dots\}$

$W = \text{the set of all whole numbers}$

$W = \{0,1,2,3, \dots\}$

$Z = \text{the set of all integers}$

$Z = \{0, \pm 1, \pm 2, \pm 3 \dots\}$

$z' = \text{the set of all negative integers}$

$z' = \{-1, -2, , -3 \dots .\}$

$O = \text{the set of all odd integers}$

$O = \{\pm 1, \pm 3, \pm 5 \dots .\}$

$E = \text{the set of all even integers}$

$E = \{0, \pm 2, \pm 4, \pm 6, \dots\}$

$Q = \text{the set of all rational numbers}$

$Q = \{x | x = \frac{p}{q} \text{ where } p, q \in Z \text{ and } q \neq 0$

$Q' = \text{the set of all irrational numbers}$

$Q' = \left\{x \middle| x \neq \frac{p}{q} \text{ where } p, q \in Z\right\}$

$\mathbb{R} = \text{the set of real numbers}$

$\mathbb{R} = QUQ'$

KINDS OF SETS

Null set:

A set which contains no element is called null or empty set. It is denoted by$\emptyset$ *or* { }

Order of a Set:

The number of element present in a set is called order of a set.

Number or element in a set A is denoted by n(A) e.g.,

If A{1,2,3}, n(A)=3

So order of set A=3

Singleton Set:

A set having only one element is called singleton set e.g.,

$A = \{4\}, B = \{x | x \in N \ \wedge 1 < x < 3\}$

Finite Set:

A set in which element are countable or finite is called finite Set.

e.g., $A = \{1,2,3,4\}, B = \{x | x \in N \ \wedge 6 < x < 9\}$

Infinite Set:

A set in which element are uncountable or infinite set.

e.g.,

$A = \{x | x \in R \wedge 0 < x < 1\}$

$B = \{1,2,3\}$

Equal Sets:

Two sets A and B are equal i.e A=B if and only if they have the same elements ,that is, if and only if every element of each set is an element of the other set.

Mathematically if A=B

$$iff A \subseteq B \text{ and } B \subseteq A$$

e.g.,

$$A = \{1,2,3\}, B = \{2,1,3\} so\ A = B$$

Note;

Two sets A and B are said to b equal if they have i)same order ii)same elements

One –To-One correspondence.

One –to one correspondence between two sets A and B is defined as

Each element of A can be paired with one and only one element of B and each element of B can be paired with one and only one element of A.

e.g.,

let A={a,b,c,d}

B={1,2,3,4}

Example:

If A{1,2,3} subset of A are

{},{1},{2},{3},{1,2},{1,3},{2,3}{1,2,3}

Proper subset of A are

{},{1},{2},{3},{1,2},{1,3},{2,3}

*if A is proper subset of a finite set B i.e,

A⊏ B then $n(A) < n(B)$

$\left\{\begin{matrix} a & b & c & d \\ \updownarrow & \updownarrow & \updownarrow & \updownarrow \\ 1 & 2 & 3 & 4 \end{matrix}\right\}$ there is (1-1) correspondence between A and B.

*(1-1) correspondence cannot be established between a finite and infinite set.

Equivalent Sets:

Two sets A and B to be equivalent if(1-1) correspondence can be made between them e.g.,

A{2,4,6} , B={ a,b,c} are equivalent Sets.

And A={1,2,3..} and B={2,4,6….100}

Are not equivalent sets.

Note:

If A is equivalent to B it is written as $A{\sim}B$ or $A \cong B$

*Two equal sets are necessary equivalent.

*Two equivalent set may or may not equal.

Subset:

Any set A is said to be subset of a set B if every element of the set A is also an element of B mathematically

$$A \subseteq B \ iff\ x \in A \rightarrow x \in B$$

*if A is a subset of B then it is denoted by

$A \subseteq B$ and read as "A is a subset of B"

*if A is a subset of B then $n(A) \leq n(B)$

*every set is subset of itself and empty set is subset of every set.

If A⊆B then we may read it in two ways.

i)A⊆ B i.e., A is a subset of B

ii)B⊆ A i.e., B is a subset of A

Note:

Possible subset of a set can be calculated by the formula 2^n ($2^{\text{no. of elements present in the set}}$)

e.g., if A{1,2,3} then there will be 8 subsets of A are

{},{1},{2},{3},{1,2},{1,3},{2,3}{1,2,3}

$$\therefore 2^3 = 8$$

Types of Subset:

There are two types of subset

i)**proper subset**

ii)Improper subset

i) Proper subset

If A is a subset of set B such that at least one element of B does not belong to A, then A is called proper subset of B denoted by A⊏ B

*all the subsets of any set (except the set its self) and its proper subsets.

*there is no of an empty and singleton set.

Improper subset:

If A is subset of set B such that A=B then A is called improper subset of B denoted by A⊑ B

e.g,

if A={1,2,3,4,5}and B={5,3,4,1,2}

then A=B

*we may say that two equal sets are also improper subset of each other.

*every set has one and only one improper subset

Power set:

The collection of all possible subset of any set A is called power set of A denoted by P(A)

Example:

B={1,2,3}

Then

P(b)={{},{1},{2},{3},{1,2},{1,3},{2,3}{1,2,3}}

Universal Set:

"The supper set of all the sets under discussion is called universal set and is denoted by U"

*universal set is also called universe of discourse.

Exercise 2.1

$\boldsymbol{Q.1: Write\ the\ following\ sets\ in\ the\ set\ builder\ notation}$:

$i)\ \{1,2,3, ...\ ,1000\}$
$= \{x\ |x \in \mathbb{N} \wedge x\ \leq 1000\}$

$ii)\ \{0,1,2,3, ...\ ,100\}$
$= \{x\ |x \in \mathbb{W} \wedge x\ \leq 100\}$

$iii)\ \{0, \pm1, \pm2, \pm3, ...\ , \pm1000\}$
$= \{x\ |x \in \mathbb{Z}\ \wedge \leq x \leq 1000\}$

$iv)\ \{0, -1, -2, -3, ...\ , -500\}$
$= \{x\ |x \in \mathbb{Z}\ \wedge -500 \leq x \leq 0\}$

$v)\ \{100,101,102, ...\ ,400\}$
$= \{x\ |x \in \mathbb{N} \wedge\ 100 \leq x \leq 400\}$

$vi)\ \{-100, -101, -102, ...\ , -500\}$
$= \{x\ |x \in \mathbb{Z} \wedge -500 \leq x \leq -100\}$

$vii)\ \{Peshawar, Lahore, Karachi, Quetta\}$
$= \{x\ |x\ \ is\ a\ capital\ of\ a\ province\ of\ pakistan\}$

$viii)\ \{January, june, july\}$
$= \{x\ |x\ is\ a\ month\ of\ the\ calendar\ year\ beings\ with\ j\}$

$ix)\ The\ set\ of\ all\ odd\ natural\ numbers$
$= \{x\ |x\ is\ an\ odd\ natural\ number\}$

$x)\ The\ set\ of\ all\ rational\ numbers$
$= \{x\ |x \in \mathbb{Q}\}$
$xi)\ The\ set\ of\ all\ real\ numbers\ between\ 1\ and\ 2$
$= \{x\ |x \in \mathbb{R} \wedge 1 > x > 2\ \}$

$xii)\ The\ set\ of\ all\ integers\ between\ -100\ and\ 1000$
$= \{x\ |x \in \mathbb{Z}\ \wedge\ -100 < x \leq 1000\}$

Q.2
$\boldsymbol{Write\ each\ of\ the\ following\ sets\ in\ descriptive\ and\ tabular\ forms}$:

$i)\ \{\boldsymbol{x} \mid \boldsymbol{x} \in \mathbb{N}\ \ \boldsymbol{\wedge\, x \leq 10}\}$

$Descriptive\ form:\ The\ set\ of\ first\ ten\ natural\ number$
$Tabular\ form: \{\ 1,2,3,4,5,6,7,8,9,10\}$

$\boldsymbol{ii})\ \{\boldsymbol{x} \mid \boldsymbol{x} \in \mathbb{N}\ \ \boldsymbol{\wedge\ 4 < x < 12}\}$
$Descriptive\ form: The\ set\ of\ natural\ num.\ between\ 4\ \&\ 12$
$Tabular\ form: \{5,6,7,8,9,10,11\ \}$

$\boldsymbol{iii})\ \{\boldsymbol{x} \mid \boldsymbol{x} \in \mathbb{Z}\ \ \boldsymbol{\wedge -5 < x < 5}\}$
$Descriptive\ form: The\ set\ of\ integers\ between\ -5\ and\ 5.$
$Tabular\ form: \{\ -4, -3, -2, -1, 0, 1, 2, 3, 4\}$

$\boldsymbol{iv})\ \{\boldsymbol{x} \mid \boldsymbol{x} \in \mathbb{E}\ \ \boldsymbol{\wedge\ 2 < x \leq 4}\}$
$Descriptive\ form: The\ set\ of\ even\ numbers\ between\ 2\ and\ 5.$
$Tabular\ form: \{4\ \}$

$\boldsymbol{v})\ \{\boldsymbol{x} \mid \boldsymbol{x} \in \mathbb{P}\ \ \boldsymbol{\wedge\ x < 12}\}$

$Descriptive\ form: The\ set\ of\ prime\ numbers\ less\ than\ 12.$
$Tabular\ form: \{2,3,5,7,11\ \}$

$\boldsymbol{vi})\ \{\boldsymbol{x} \mid \boldsymbol{x} \in \mathbb{O}\ \ \boldsymbol{\wedge\ 3 < x < 12}\}$

$Descriptive\ form: The\ set\ of\ \ odd\ integers\ between\ 3\ and\ 12.$
$Tabular\ form: \{5,7,9,11\ \}$

$\boldsymbol{vii})\ \{\boldsymbol{x} \mid \boldsymbol{x} \in \mathbb{E}\ \ \boldsymbol{\wedge\ 4 \leq x \leq 10}\}$

$Descriptive\ form: The\ set\ of\ Even\ integers\ from\ 4\ to\ 10.$
$\boldsymbol{Tabular\ form}: \{\boldsymbol{4, 6, 8, 10}\ \}$

$\boldsymbol{viii})\ \{\boldsymbol{x} \mid \boldsymbol{x} \in \mathbb{E}\ \ \boldsymbol{\wedge\ 4 < x < 6}\}$
$Descriptive\ form: The\ set\ of\ even\ integers\ between\ 4\ and\ 6.$
$Tabular\ form: \{\ \}$

$\boldsymbol{ix})\ \{\boldsymbol{x} \mid \boldsymbol{x} \in \mathbb{O}\ \ \boldsymbol{\wedge\ 5 \leq x \leq 7}\}$
$Descriptive\ form: The\ set\ of\ odd\ integers\ from\ 5\ upto\ 7.$
$Tabular\ form: \{5,7\ \}$

$\boldsymbol{x})\ \{\boldsymbol{x} \mid \boldsymbol{x} \in \mathbb{O}\ \ \boldsymbol{\wedge\ 5 < x < 7}\}$
$Tabular\ form: \{\ \}$

$\boldsymbol{xi})\ \{\boldsymbol{x} \mid \boldsymbol{x} \in \mathbb{N}\ \ \boldsymbol{\wedge\ x + 4 = 0}\}$
$D.F: The\ set\ of\ natural\ numbers\ x\ satisfying\ x + 4 = 0$
$Tabular\ form: \{\ \}$

$\boldsymbol{xii})\ \{\boldsymbol{x} \mid \boldsymbol{x} \in \mathbb{Q}\ \ \boldsymbol{\wedge\ \ x^2 = 2}\}$
$D.F: The\ set\ rational\ numbers\ x\ satisfying\ x^2 = 2$
$Tabular\ form: \{\ \ \}$

$xiii)\ \{x \mid x \in \mathbb{R}\ \Lambda\ x = x\}$

$D.F$: The set of real numbers x satisfying $x = x$

Tabular form: $\mathbb{R}$

$xiv)\ \{x \mid x \in \mathbb{Q}\ \Lambda\ x = -x\}$

$D.F$: The set of rational numbers x satisfying $x = -x$

Tabular form: $\{0\}$

$xv)\ \{x \mid x \in \mathbb{R}\ \Lambda\ x \neq 2\}$

$D.F$: The set of real numbers x satisfying $x \neq 2$

Tabular form: $\mathbb{R} - \{2\}$

$xvi)\ \{x \mid x \in \mathbb{R}\ \Lambda\ x \neq \mathbb{Q}\}$

$D.F$: The set of real numbers x which are not rational.

Tabular form: $\mathbb{Q}'$

Q.3: Which of the following sets are finite or infinite.

i) The set of your students in class.
Finite

ii) The set of all schools in pakistan.
Finite

iii) The set of natural numbers between 3 and 10.
Finite

iv) The set of rational numbers between 3 and 10.
Infinite

v) The set of real numbers between 0 and 1.
Infinite

vi) The set of rational numbers between 0 and 1.
Infinite

vii) The set of whole numbers between 0 and 1.
Finite

viii) The set of all leaves of trees in pakistan.
infinite

ix) $P(N)$ Infinite

x) $P\{a, b, c\}$. Finite

xi) $\{1, 2, 3, \ldots\}$ Infinite

xii) $\{1, 2, 3, \ldots 100000000\}$ finite

xiii) $\{x \mid x \in \mathbb{R}\ \Lambda\ x \neq x\}$ Finite

xiv) $\{x \mid x \in \mathbb{R}\ \Lambda\ x^2 = -16\}$ Finite

xv) $\{x \mid x \in \mathbb{Q}\ \Lambda\ x^2 = 5\}$ Finite

xvi) $\{x \mid x \in \mathbb{Q}\ \Lambda\ 0 \leq x \leq 1\}$ Infinite

Q.4: Write two proper subsets of each of the following sets.

i) $\{a, b, c\}$

SOLUTION:

$\{a, b\}, \{c\}$

ii) $\{0, 1\}$

SOLUTION:

$\{0\}, \{1\}$

iii) $\mathbb{N}$

SOLUTION:

$\{1\}, \{2\}$

iv) $\mathbb{Z}$

SOLUTION:

$\{1\}, \{2\}$

v) $\mathbb{Q}$

SOLUTION:

$\{1\}, \{2\}$

vi) $\mathbb{R}$

SOLUTION:

$\{1\}, \{2\}$

vii) $\mathbb{W}$

SOLUTION:

$\{1\}, \{2\}$

viii) $\{x \mid x \in \mathbb{Q}\ \Lambda\ 0 \leq x \leq 2\}$

SOLUTION:

$\{1\}, \{2\}$

Q.5: Is any set which have no proper set.

SOLUTION:

Yes, $\emptyset$ is a set has no proper subset.

Q.6: what is the difference between $\{a, b\}$ and $\{\{a, b\}\}$?

SOLUTION:

$\{a, b\}$ is a set containing two elements a and b.

$\{\{a, b\}\}$ is a set containing only one element $\{a, b\}$

Q.7: Which of the following sentences are true

and which of them are false ?

SOLUTION:

$i)\ \ \{1,2\} = \{2,1\}$ ***True***

$ii)\ \ \emptyset \subseteq \{\{a\}\}$ ***True***

$iii)\ \ \{a\} \supseteq \{\{a\}\}$ ***False***

$iv)\ \ \{a\} \in \{\{a\}\}$ ***True***

$v)\ \ a \in \{\{a\}\}$ ***False***

$vi)\ \ \emptyset \in \{\{a\}\}$ ***False***

Q.8: Which is the number of elements of the power set of the given sets ?

SOLUTION:

FORMULA: No. of elements in power set = 2^n ***, where n is the no. of elements in a set.***

$i)\ A = \{\ \}$

Power set of { } has element =$2^n = 2^0 = 1$

$ii)\ A = \{0, 1\}$

Power set of {0,1} has element $2^n = 2^2 = 4$

$iii)\ A = \{1, 2, 3, 4, 5, 6, 7\}$

Power set of $A = \{1, 2, 3, 4, 5, 6, 7\}$ **has element**

$2^n = 2^7 = 128$

$iv)\ A = \{0, 1, 2, 3, 4, 5, 6, 7\}$

Power set of $A = \{0, 1, 2, 3, 4, 5, 6, 7\}$ **has element**

$= 2^n = 2^8 = 256$

$)A = \{a, \{a, b\}\}$

Power set of $A = \{a, \{a, b\}\}$ **has element** $= 2^n = 2^2 = 4$

$vi)\ A = \{\{a, b\}, \{b, c\}, \{d, e\}\}$

Power set of $A = \{\{a, b\}, \{b, c\}, \{d, e\}\}$ **has element** $= 2^n = 2^3 = 8$

Q.9: Write down the power set of each of the following sets:

$i)\ \{9, 11\}$

$P(A) = \{\emptyset, \{9\}, \{11\}, \{9,11\}\}$

$ii)\ \{+, -, \times, \div\}$

$P(A) = \{\emptyset, \{+\}, \{-\}, \{\times\}, \{\div\}, \{+,-\}, \{+,\times\}, \{+,\div\},$
$\{-,\times\}, \{-,\div\}, \{\times,\div\}, \{+,-,\times\}, \{+,-,\div\}, \{+,\times,\div\},$
$\{-,\times,\div\}, \{+,-,\times,\div\}$

$iii)\ \{\emptyset\}$

$P(A) = \{\emptyset, \{\emptyset\}\}$

$iv)\ \{a, \{b, c\}\}$

$P(A) = \{\emptyset, \{a\}, \{\{b, c\}\}, \{a, \{b, c\}\}\}$

Q.10: Which pairs of sets are equilent ? Which of them are also equal ?

$i)\ \{a, b, c\}, \{1, 2, 3\}$

These two sets are equilent becuase each set has three elememnts.

ii) The set of fiest 10 whole numbers ,
$\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

These two sets are equal becuase each set has same ten elememnts.

iii) Set of angles of quadrilateral ABCD , Set of sides of the same quadrilateral.

These two sets are equilent becuase each set has same number of elememnts.

iv) Set of sides of hexagon ABCDEF , Set of angles of the same heagon.

These two sets are equilent becuase each set has same number of elememnts.

$v)\ \{1, 2, 3, 4, \ldots\},\ \{2, 4, 6, 8, \ldots\}$

These two sets are equilent becuase there is $1-1$ *correspondance between the elements of the sets.*

$vi)\ \{1, 2, 3, 4, \ldots\},\ \{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots\}$

$These\ two\ sets\ are\ equilent\ becuase\ there\ is\ 1-1\ correspondance\ between\ the\ elements\ of\ the\ sets.$

$\boldsymbol{vii})$ $\{\mathbf{5,10,15,20,\ ...55555}\},\ \{\mathbf{5,10,15,20,\ ...}\}$

$These\ two\ sets\ are\ not\ equilent\ becuase\ the$

$first\ set\ is\ finite\ while\ the\ second\ is\ infinite.$

2.2 Operation on Sets

Union of the two sets:

Union of two sets A and B, denoted by A∪ B is the set of all elements, which belong to A or B

Symbolically,

$A \cup B = \{x|x \in A \vee x \in B\}$

Example:

If A={1,2,3} and B={2,3,4,5}

Then $A \cup B = \{1,2,3,4,5\}$

Intersection of two sets:

Intersection of two sets A and B, denoted by A⋂B is the set of all elements, which belong to A and B

Symbolically,

$A \cap B = \{x|x \in A \wedge x \in B\}$

Example:

If A={1,2,3} and B={2,3,4,5}

Then $A \sqcap B = \{12,3\}$

Disjoint Sets:

If intersection of two sets A and B is empty set then seta and A and B are called disjoint sets.

Example:

$one = \emptyset\ wher\ O\ is\ the\ set\ of\ odd\ and\ E$

$is\ the\ set\ of\ even\ intergers.$

Overlapping Set:

If the intersection of two sets A and B is non empty set but neither is subset of the other, the sets are called overlapping sets.

Example:

If A={1,2,3,4} and B={,3,4,5,6}

Then $A \sqcap B = \{3,4\}$ overlapping set.

Complement of a set:

If U is universal Set then U\A or U-A is called complement of A, denoted by $A'\ or\ A^c\ thus$

$A^c = U - A$

Symbolically, $A' = \{x|x \in U\ \wedge x \notin A\}$

Example:

If U=N then $E' = O\ and\ O'$ =E

Example1.

If U= set of alphabets of English language

C= set of consonants, we set of vowels then

$C' = W\ and\ W' = C$

Difference of two sets:

The difference A-B or A/B of two sets A and B is the set of elements which belong to A, but not belong to B.

Symbolically,

$A - B = \{x|x \in A\ \wedge x \notin B\}$

Example2:

$if\ A = \{1,2,3,4,5\} and\ B = \{4,5,6,7,8,9,10\}$

$A - B = \{1,2,3\}\ and\ B - A = \{6,7,8,9,10\}$

Notice that $A - B \neq B - A$

2.3 Venn Diagrams:

(named by JOHN VENN" The English logician and mathematician(1834- 1883 A.D)

"if the picture representation of given sets in the form of rectangle and circles"

*in Venn Diagram, rectangular region represent given sets

Venn diagram of given sets.

When A and B are Disjoint sets $A \cap B = \emptyset$

Results

$n(A \cup b) = n(A) + n(B)$

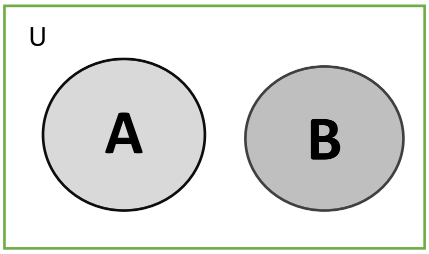
$n(A \cap B) =$

$n(A - B) = n(A)$

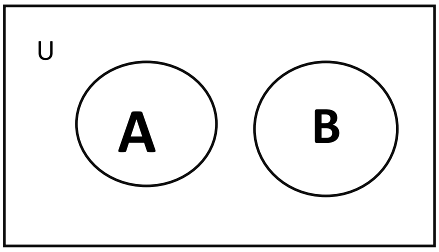
$n(B - A) = n(B)$

$n(B) \leq n(A \cup B)$

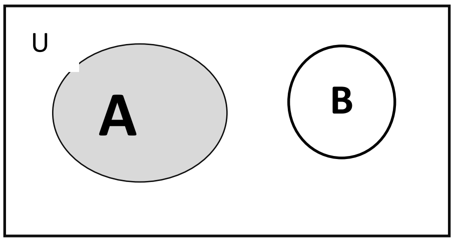
$n(A) \leq n(A \cup B)$



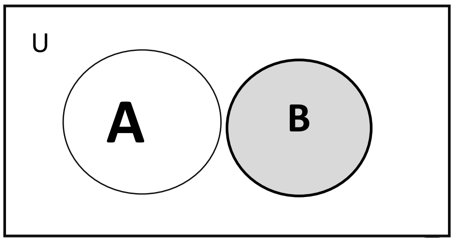
AUB



A⋂B



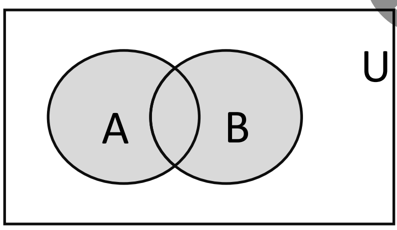
A−B



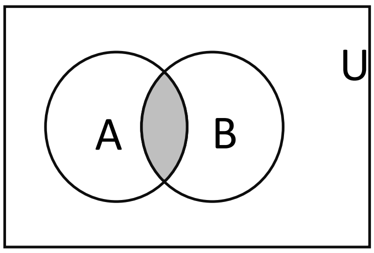
B−A

When A and B are overlapping ($A \cap B \neq \emptyset$)

$A \cup B$



$A \cap B$



Results

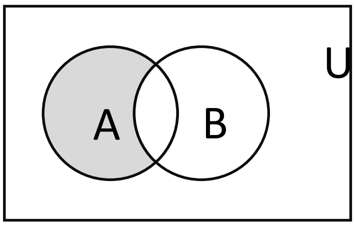
$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$n(A \cap B) \leq n(A)$

$n(A \cap B) \leq n(B)$

$n(A - B) = n(A) + n(B) - n(A \cap B)$

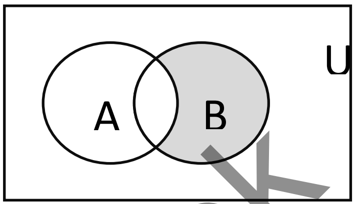
$A - B$



$n(B - A) = n(B) - n(A \cap B)$

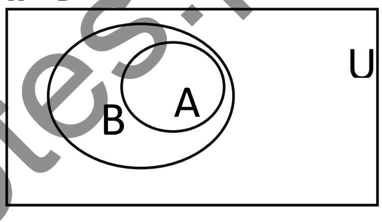
$n(A \cap B) = n(A) + n(B) - n(A \cup B)$

$B - A$



When A is subset of B($A \subseteq B$)

$A - B$



Result:

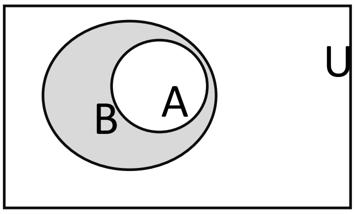
$n(\cup B) = n(B)$

$n(A \cap B) = n(A)$

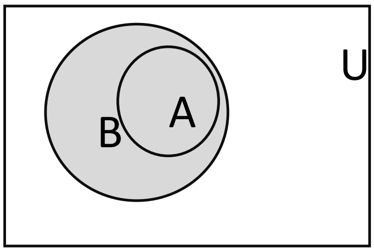
$n(A) \leq n(B)$

$n(A - B) = 0$

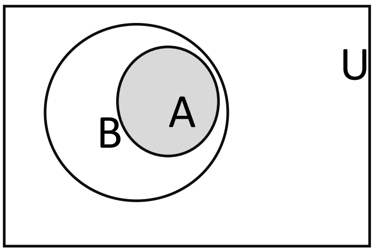
$n(B - A) = n(B) - n(A)$



$A \cup B$



$A \cap B$



when Bis subset of $A(B \subseteq A)$

Result:

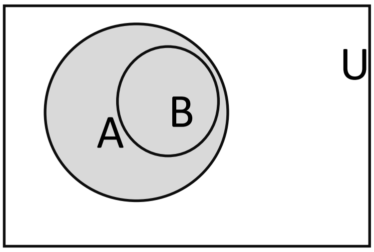
$n(B \cup A) = n(A)$

$n(A \cap B) = n(B)$

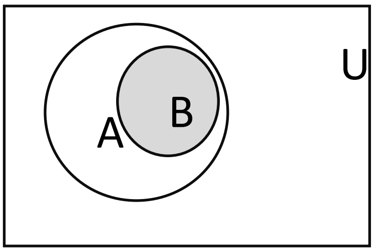
$n(B) \leq n(A)$

$n(B - A) = 0$

$A \cup B$

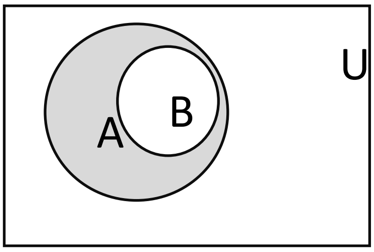


$A \cap B$

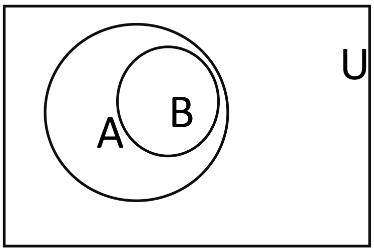


$n(A - B) = n(A) - n(B)$

$A - B)$



$B - A$



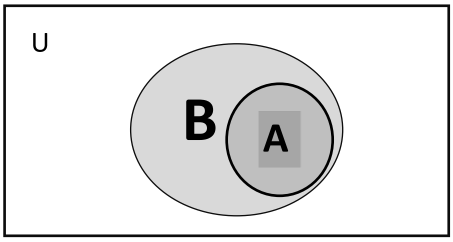
Note Shaded area gives required region or required result

Exercise 2.2

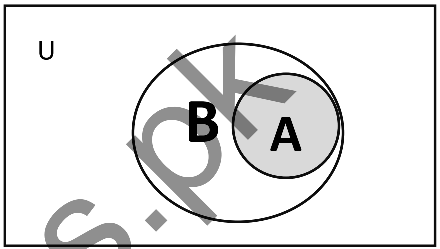
Exhibit $\boldsymbol{A \cup B}$ ***and*** $\boldsymbol{A \cap B}$ ***by venn diagrams in the following cases:***

i) $\boldsymbol{A \subseteq B}$

SOLUTION:



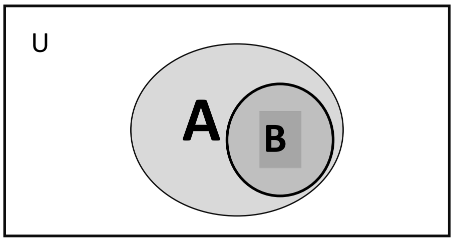
A⊔B



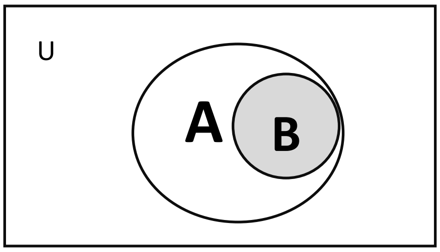
A⋂B

ii) $\boldsymbol{B \subseteq A}$

SOLUTION



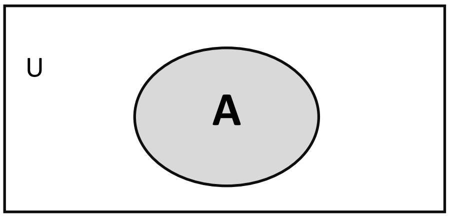
A⊔B



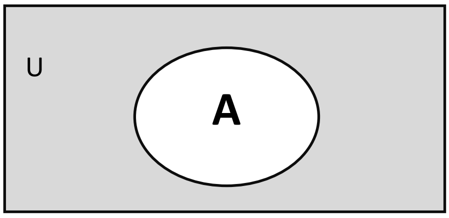
A⋂B

iii) $\boldsymbol{A \cup A'}$

SOLUTION:



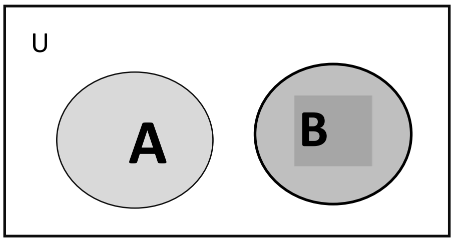
A



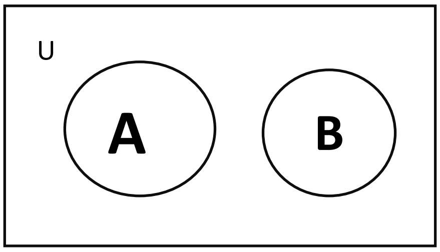
A’

iv)A and B are disjoint sets

***SOLUTION*:**



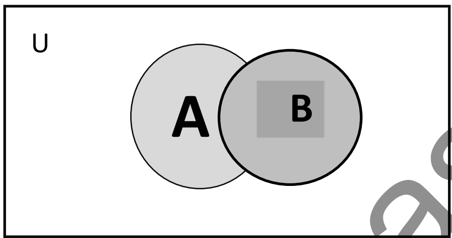
A⊔B



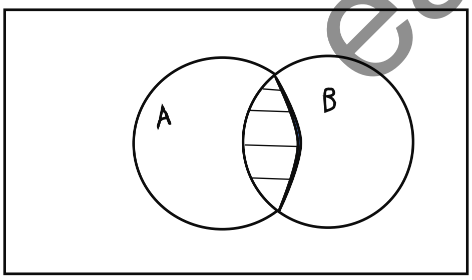
A⋂B

v)A and B are overlapping sets

***SOLUTION*:**



A⊔B

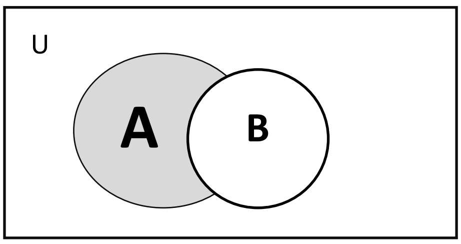


A⋂B

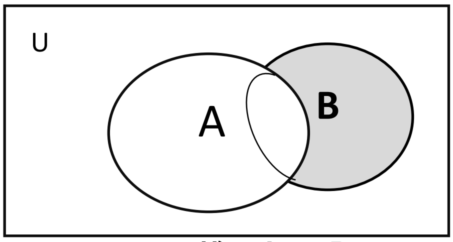
Q.2: Show that A − b and

***B − A by venn diagram when*:**

i)A and B are overlapping sets

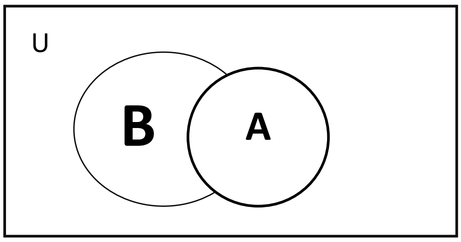


A−B

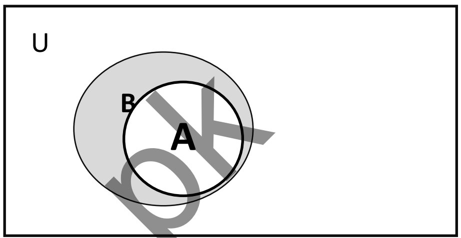


B−A

ii) A ⊆ B

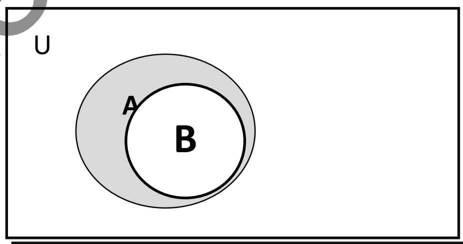


A−B

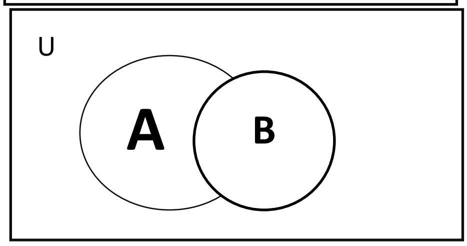


B−A

iii) B ⊆ A



A−B



B−A

Q,3:

***Under what conditions on A and B are the following statements are true*?**

i) $A \cup B = A$ if $B \subseteq A$

ii) $A \cup B = B$ if $A \subseteq B$

iii) $A - \emptyset = \emptyset$ if $A = \emptyset$

iv) $A \cap B = B$ if $B \subseteq A$

v) $n(A \cup B) = n(A) + n(B)$ if $A \cap B = \emptyset$

vi) $n(A \cup B) = n(A) + n(B)$ if $A \subseteq B$

vii) $A - B = A$ if $A \cap B = \emptyset$ or $B = \emptyset$

viii) $n(A \cap B) = 0$ if $A \cap B = \emptyset$

ix) $A \cup B = U$ if $A = B'$ or $B = A'$

x) $A \cup B = B \cup A$ *it is always true*

xi) $n(A \cup B) = n(B)$ When $B \subseteq A$

xii) $U - A = \emptyset$ if $U = A$

Q.4: Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$,

$A = \{2, 4, 6, 8, 10\}$,

$B = \{1, 2, 3, 4, 5\}$, $C = \{1, 3, 5, 7, 9\}$

List the members of each of the following sets:

i) A^c

$A^c = U - A = \{1,2,3,4,5,6,7,8,9,10\} - \{2,4,6,8,10\}$

$= \{1,3,5,7,9\}$

ii) B^c

$B^c = U - B = \{1,2,3,4,5,6,7,8,9,10\} - \{1,2,3,4,5\}$

$= \{6,7,8,9,10\}$

iii) $A \cup B$

$A \cup B = \{2,4,6,8,10\} \cup \{1,2,3,4,5\}$

$= \{1,2,3,4,5,6,7,8,9,10\}$

iv) $A - B$

$A - B = \{2,4,6,8,10\} - \{1,2,3,4,5\}$

$= \{6,8,10\}$

v) $A \cap C$

$A \cap C = \{2,4,6,8,10\} \cap \{1,3,5,7,9\} = \{\ \}$

vi) $A^c \cup C^c$

$A^c \cup C^c = (U - A) \cup (U - C)$

$= (\{1,2,3,4,5,6,7,8,9,10\} - \{2,4,6,8,10\})$

$\cup (\{1,2,3,4,5,6,7,8,9,10\} - \{1,3,5,7,9\})$

$= \{1,3,5,7,9\} \cup \{2,4,6,8,10\} = \{1,2,3,4,5,6,7,8,9,10\}$

vii) $A^c \cup C$

$A^c \cup C = (U - A) \cup C$

$= (\{1,2,3,4,5,6,7,8,9,10\} - \{2,4,6,8,10\}) \cup \{1,3,5,7,9\}$

$= \{1,2,3,4,5\}$

viii) U^c

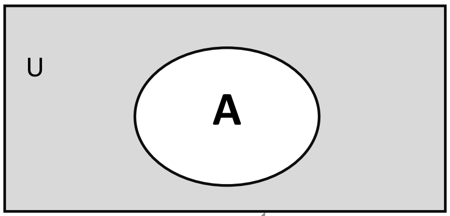
$U^c = U - U$

$= \{1,2,3,4,5,6,7,8,9,10\} - \{1,2,3,4,5,6,7,8,9,10\}$

$= \{\ \} = \emptyset$

Q.No.5 using the Venn Diagram if necessary find the singleton set equal to the following.

i) $A^c = U - A$

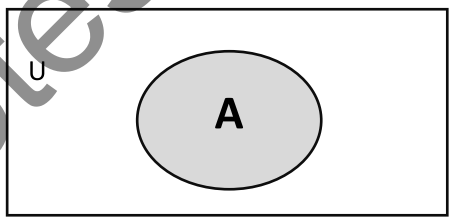


A^c

Shaded area shows A^c

i) $A \cap U$

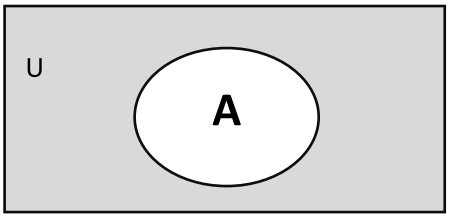
solution: $A \cap U = A$



Shaded area shows $A \cap U$

iii) $A \cup U$

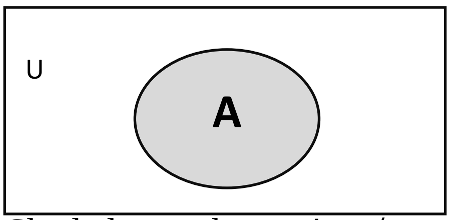
solution: $A \cup U = U$



Shaded area shows $A \cup U$

iv) $A \cup \emptyset$

solution: $A \cup \emptyset = A$



Shaded area shows $A \cup \emptyset$

$v)\ \emptyset \cap \emptyset$

$solution: \emptyset \cap \emptyset = \{\,\}$

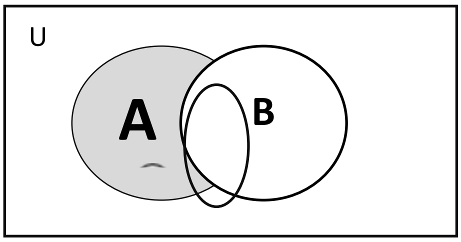
Venn diagram is not necessary.

Q6. Use the Venn diagrams to verify

$i) A - B = A \cap B^c$

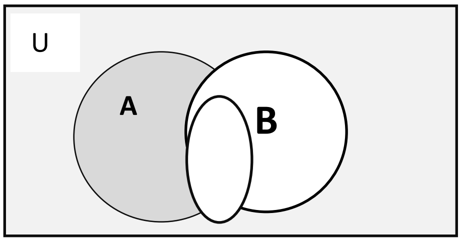
Solution:

$i) A - B = A \cap B^c$ Fig.(i)

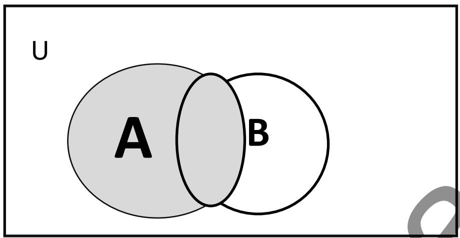


A−B

Fig.(ii)



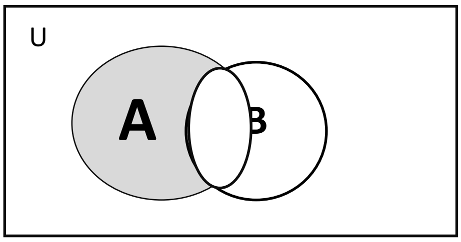
$B^c = U - B$



A

Now from (ii) and fig.(iii)

Fig.(iv)



$A \cap B^c$

Hence from fig (i) and fig. (iv)

We conclude that

$$A - B = A \cap B^c$$

ii) $(\boldsymbol{A} - \boldsymbol{B})^c \cap \boldsymbol{B} = \boldsymbol{B}$

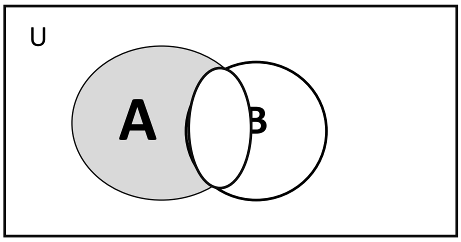


fig.(i)

$A - B$

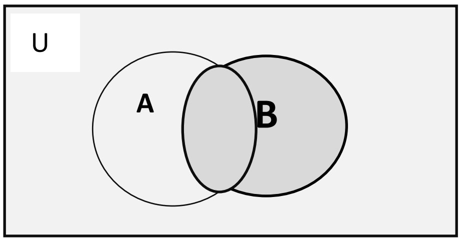


Fig.(ii)

$(A - B)^c$

$= U -$

$(A$

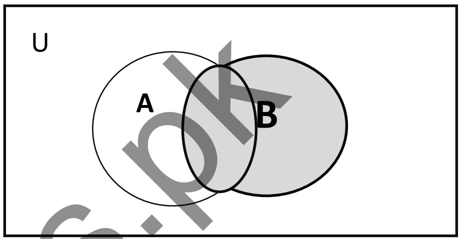
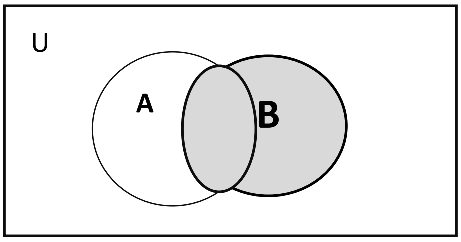


Fig.(iii)

B

From fig(ii) and fig(iii) Fig.(iv)

$$(\boldsymbol{A} - \boldsymbol{B})^c \cap \boldsymbol{B}$$



From fig (iii) and fig(iv)

We conclude that

$$(\boldsymbol{A} - \boldsymbol{B})^c \cap \boldsymbol{B} = \boldsymbol{B}$$

Properties of Union and Intersection:

i) Commutative property of Union
$A \cup B = B \cup A$

proof:

$let\ x \in A \cup B$

⇨ $x \in A\ or\ x \in B$
⇨ $x \in B\ or\ x \in A$
⇨ $x \in B \cup A \quad as\ x \in A \cup B \rightarrow x \in B \cup A$
So $A \cup B \subseteq B \sqcup A$ ... (1)

Conversely,

$let\ y \in B \cup A$

⇨ $y \in B\ or\ y \in A$
⇨ $y \in A\ or\ y \in B$

➪ $y \in A \;\; or\; y \in B \rightarrow y \in A \cup B$

➪ $as\; y \in B \cup A \rightarrow y \in A \cup B$

So $B \cup A \subseteq A \sqcup B \ldots (1)$

From (i) and(ii) we conclude that

$$A \cup B = B \cup A$$

ii) Associative property of Union

$A \cup (B \cup C) = (A \cup B) \cup C$

Proof:

Let $x \in A \cup (B \cup C)$

➪ $x \in A \; or \; x \in (B \cup C)$

➪ $x \in A \; or (x \in B \; or \; x \in C)$

➪ $(x \in A or x \in B) or x \in C$

➪ $x \in (A \cup B) or x \in C$

$x \in (A \cup B) \cup C$

➪ $x \in (A \cup B) or x \in C$

➪ $x \in (A \cup B) \cup C$

➪ $as\;\; x \in (A \cup B) \cup C \rightarrow x \in A \cup (B \cup C)$

➪ $so\; A \cup (B \cup C) \subseteq (A \cup B) \cup C \ldots (i)$

$conversely,$

$let\; y \in (A \cup B) \cup C$

➪ $y \in (A \cup B) or\; y \in C$

➪ $(y \in A\; or y\;\; \in B) or\; y \in C$

➪ $y \in A\; or\; (y \in B\; or\; y \in C)$

➪ $y \in A\; or\; y \in (B \cup C)$

➪ $y \in A \cup (B \cup C)$

➪ $as\; y \in (A \cup B) \cup C \rightarrow y \in A \cup (B \cup C)$

➪ $so\; (A \cup B) \cup C \subseteq A \cup (B \cup C) \ldots (ii)$

$from\; (i) and\; (ii) we\; conclude\; that$

$A \cup (B \cup C) = (A \cup B) \cup C$

iii) Commutative property of intersection

$\boldsymbol{A \cap B = B \cap C}$

Proof :

$$let\; x \in A \cap B$$

➪ $x \in A\; and\; x \in B$

➪ $x \in B\; and\; x \in A$

➪ $x \in B \cap A$

➪ $x \in A \cap B \rightarrow x \in B \cap A$

➪ $so\; A \cap B \subseteq B \cap A \ldots (i)$

Conversely,

➔ $let\; y \in B \cap A$

➔ $y \in B\; and\; y \in A$

➔ $y \in A\; and\; y \in B$

➔ $y \in A \cap B$

➔ $as\; y \in B \cap A \rightarrow y \in A \cap B$

➔ $so\; B \cap A \subseteq A \cap B \ldots (ii)$

From (i) and (ii) we conclude that

$\boldsymbol{A \cap B = B \cap C}$

iv) Associative property of intersection

$A \cap (B \cap C) = (A \cap B) \cap C$

Proof:

Let $x \in A \cap (B \cap C)$

➪ $x \in A\; or\; x \in (B \cap C)$

➪ $x \in A\; or (x \in B\; or\; x \in C)$

➪ $(x \in A or x \in B) or x \in C$

➪ $x \in (A \cap B) or x \in C$

➪ $x \in (A \cap B) \cap C$

➪ $x \in (A \cap B) or x \in C$

➪ $x \in (A \cap B) \cap C$

➪ $as\;\; x \in A \cap (B \cap C) \rightarrow x \in (A \cap B) \cap C$

➪ $so\; A \cap (B \cap C) \subseteq (A \cap B) \cap C \ldots (i)$

$conversely,$

$let\; y \in (A \cap B) \cap C$

➪ $y \in (A \cap B) or\; y \in C$

➪ $(y \in A\; or y\;\; \in B) or\; y \in C$

➪ $y \in A\; or\; (y \in B\; or\; y \in C)$

➪ $y \in A\; or\; y \in (B \cap C)$

➪ $y \in A \cap (B \cap C)$

➪ $as\; y \in (A \cap B) \cap C \rightarrow y \in A \cap (B \cap C)$

➪ $so\; (A \cap B) \cap C \subseteq A \cap (B \cap C) \ldots (ii)$

$from\; (i) and\; (ii) we\; conclude\; that$

$A \cap (B \cap C) = (A \cap B) \cap C$

v) Distributive of union over intersection

$\boldsymbol{A \cup (B \cap C) = (A \cup B) \cap (A \cap C)}$

Proof :

$$let\; x \in A \cup (B \cap C)$$

➪ $x \in A\; or\; x \in (B \cap C)$

➪ $x \in A\; or\; (x \in B\; and\; x \in C)$

➪ $(x \in A\; or\; x \in B) and (x \in A\; or\; x \in C)$

➪ $x \in (A \cup B)\; and\; x \in (A \cup C)$

➪ $x \in (A \cup B) \cap (A \cup C)$

➪ $as\; x \in A \cup (B \cap C) \rightarrow x \in (A \cup B) \cap (A \cap C)$

➪ $so\; A \cup (B \cap\;\; C) \subseteq (A \cup B) \cap (A \cup C) \ldots (ii)$

Conversely,

$$let\; y \in (A \cup B) \cap (A \cap C)$$

➪ $y \in (A \cup B) and\; y \in (A \cap C)$

➪ $(y \in A\; or\; y \in B) and\; (y \in A\; or\; y \in C)$

➪ $y \in A\; or\; (y \in B\; and\; y \in C)$

➪ $y \in A \cup (B \cap C)$

$as\; y \in (A \cup B) \cap (A \cap C)$

$\subseteq A \cup (B \cap C) .. (ii)$

From (i) and (ii) we conclude that

$A \cup (B \cap C) = (A \cup B) \cap (A \cap C)$

vi) **Distributive of intersection over Union**

$\boldsymbol{A \cap (B \cup C) = (A \cap B) \cup (A \cap C)}$

Proof :

$$let\ x \in A \cap (B \cup C)$$

- ⇨ $x \in A\ and\ x \in (B \cup C)$
- ⇨ $x \in A\ and\ (x \in B\ or\ x \in C)$
- ⇨ $(x \in A\ and\ x \in B) or (x \in A\ and\ x \in C)$
- ⇨ $x \in (A \cap B)\ or\ x \in (A \cap C)$
- ⇨ $x \in (A \cap B) \cup (A \cap C)$
- ⇨ $as\ x \in A \cap (B \cup C) \rightarrow x \in (A \cap B) \cup (A \cap C)$
- ⇨ $so\ A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C) \dots (ii)$

Conversely,

$$let\ y \in (A \cap B) \cup (A \cap C)$$

- ⇨ $y \in (A \cap B) and\ y \in (A \cap C)$
- ⇨ $(y \in A\ and\ y \in B) or (y \in A\ and\ y \in C)$
- ⇨ $y \in A\ and\ (y \in B\ or\ y \in C)$
- ⇨ $y \in (A \cup B) \cap C$

$as\ y \in (A \cap B) \cup (B \cap C)$
$\subseteq (A \cup B) \cap C .. (ii)$

From (i) and (ii) we conclude that
$(A \cup B) \cap C = (A \cap B) \cup (A \cap C)$

DE Morgan's Laws

vii) $(\boldsymbol{A} \cup \boldsymbol{B})' = \boldsymbol{A}' \cap \boldsymbol{B}'$
viii) $(\boldsymbol{A} \cap \boldsymbol{B})' = \boldsymbol{A}' \cup \boldsymbol{B}'$

Proof:

vii) $(\boldsymbol{A} \cup \boldsymbol{B})' = \boldsymbol{A}' \cap \boldsymbol{B}'$

$$let\ x \in (\boldsymbol{A} \cup \boldsymbol{B})'$$

- ⇨ $x \notin (A \cup B)$
- ⇨ $x \notin A\ and\ x \notin B$
- ⇨ $x \in A' and\ x \in B'$
- ⇨ $x \in \boldsymbol{A}' \cap \boldsymbol{B}'$
- ⇨ As $x \in (\boldsymbol{A} \cup \boldsymbol{B})' \rightarrow x \in \boldsymbol{A}' \cap \boldsymbol{B}'$
- ⇨ $(\boldsymbol{A} \cup \boldsymbol{B})' \subseteq \boldsymbol{A}' \cap \boldsymbol{B}' \dots (\boldsymbol{1})$

Conversely,

$$let\ y \in A' \cap B'$$

- ⇨ $y \in A' and\ y \in B'$
- ⇨ $y \notin A\ and\ y \notin B$
- ⇨ $y \notin (A \cup B)$
- ⇨ $y \in (A \cup B)'$
- ⇨ As $y \in (A' \cap B') \rightarrow y \in (A \cup B)'$
- ⇨ $so\ A' \cap B \subseteq (A \cup B)' \dots (ii)$

From (i) and (ii) we conclude that

$$(\boldsymbol{A} \cup \boldsymbol{B})' = \boldsymbol{A}' \cap \boldsymbol{B}'$$

Now $(\boldsymbol{A} \cap \boldsymbol{B})' = \boldsymbol{A}' \cup \boldsymbol{B}'$

$$let\ x \in (\boldsymbol{A} \cap \boldsymbol{B})'$$

- ⇨ $x \notin (A \cup B)$
- ⇨ $x \notin A\ and\ x \notin B$
- ⇨ $x \in A' and\ x \in B'$
- ⇨ $x \in (A' \cup B)$
- ⇨ As $x \in (\boldsymbol{A} \cup \boldsymbol{B})' \rightarrow x \in (\boldsymbol{A}' \cap \boldsymbol{B}')$
- ⇨ $(\boldsymbol{A} \cap \boldsymbol{B})' \subseteq \boldsymbol{A}' \cup \boldsymbol{B}' \dots (\boldsymbol{i})$

Conversely,

$$let\ y \in A' \cup B'$$

- ⇨ $y \in A' or\ y \in B'$
- ⇨ $y \notin A\ or\ y \notin B$
- ⇨ $y \notin (A \cap B)$
- ⇨ $y \in (A \cap B)'$
- ⇨ As $y \in (A' \cup B') \rightarrow y \in (A \cap B)'$
- ⇨ $so\ A' \cup B \subseteq (A \cap B)' \dots (ii)$

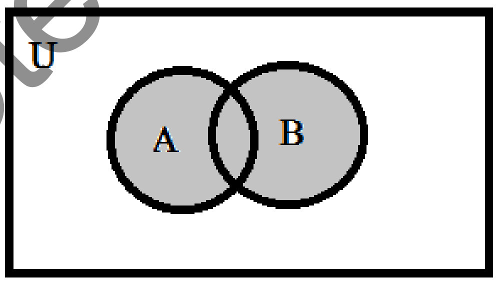
From (i) and (ii) we conclude that

$$(\boldsymbol{A} \cap \boldsymbol{B})' = \boldsymbol{A}' \cup \boldsymbol{B}'$$

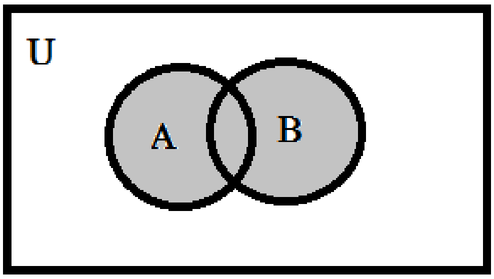
Verification of the properties with the help of Venn Diagram.

(i) $\boldsymbol{A} \cup \boldsymbol{B} = \boldsymbol{B} \cup \boldsymbol{A}$

$A \cup B$

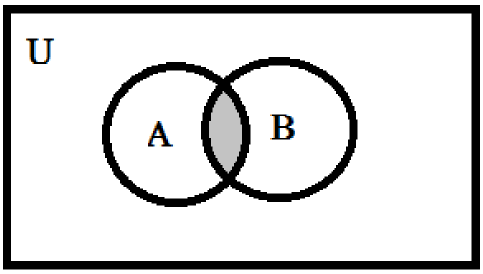


$B \cup A$

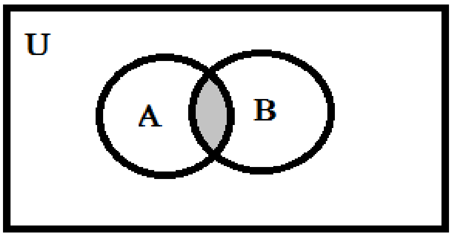


(ii) $A \cap B = B \cap A$

$A \cap B$

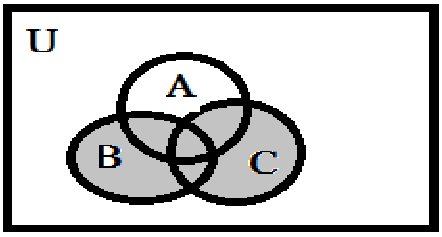


$B \cap A$

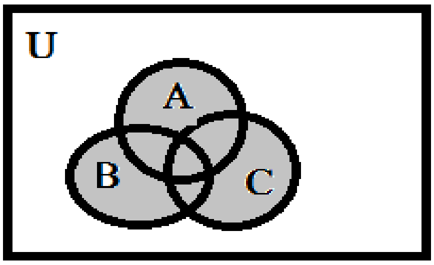


(iii)$A \cup (B \cup C) = (A \cup B) \cup C$

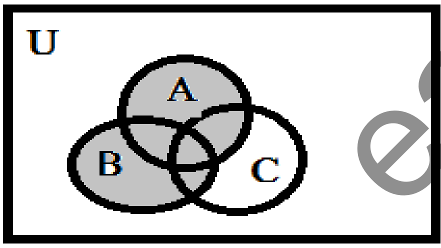
$B \cup C$ fig(i)



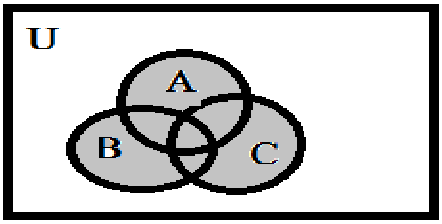
$A \cup (B \cup C)$ fig(ii)



$A \cup B$ fig (iii)



$(A \cup B) \cup C$ fig(iv)



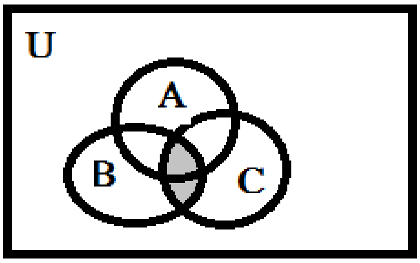
From fig.(ii) and fig(iv)

We conclude that

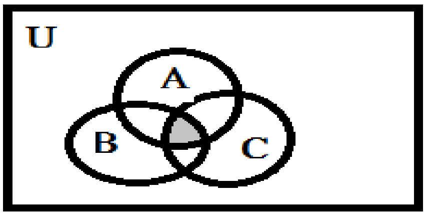
$A \cup (B \cup C) = (A \cup B) \cup C$

(iv) $\boldsymbol{A \cap (B \cap C) = (A \cap B) \cap C}$

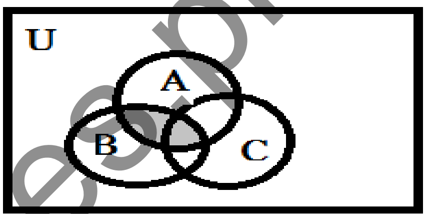
$\boldsymbol{B \cap C}$ **fig.(i)**



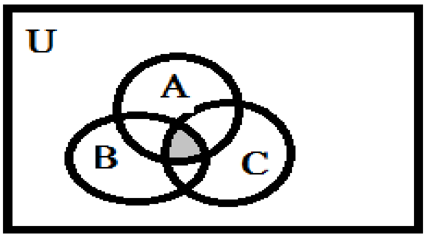
$\boldsymbol{A \cap (B \cap C)}$ **fig(ii)**



$A \cap B$ fig.(iii)



$\boldsymbol{(A \cap B) \cap C}$ **fig.(iv)**

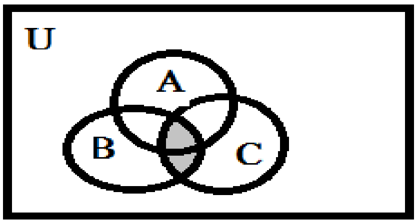


From fig(ii) and fig(iv) it is verified that

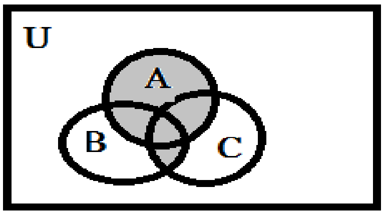
$\boldsymbol{A \cap (B \cap C) = (A \cap B) \cap C}$

(v)$\boldsymbol{A \cup (B \cap C) = (A \cup B) \cap (A \cup C)}$

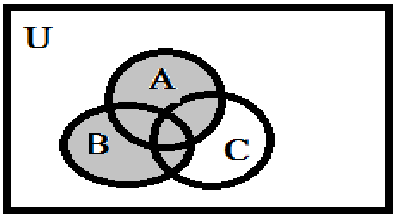
$B \cap C$ fig.(i)



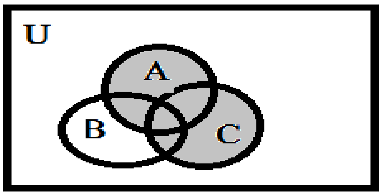
$\boldsymbol{A \cap (B \cap C)}$ **fig.(ii)**



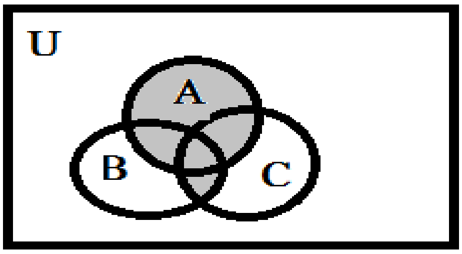
$A \cup B$ fig(iii)



$A \cup C$ fig(iv)



$(\boldsymbol{A} \cup \boldsymbol{B}) \cap (\boldsymbol{A} \cup \boldsymbol{C})$ **fig(v)**

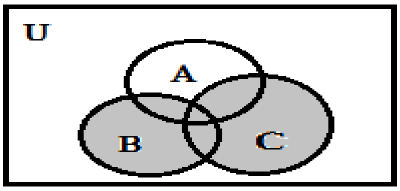


From fig(ii) and fig(v) it is verified it is

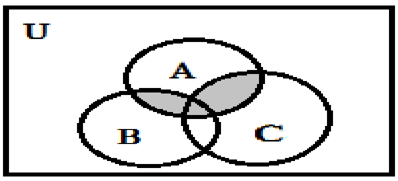
$\boldsymbol{A} \cup (\boldsymbol{B} \cap \boldsymbol{C}) = (\boldsymbol{A} \cup \boldsymbol{B}) \cap (\boldsymbol{A} \cup \boldsymbol{C})$

(vi) $\boldsymbol{A} \cap (\boldsymbol{B} \cup \boldsymbol{C}) = (\boldsymbol{A} \cap \boldsymbol{B}) \cup (\boldsymbol{A} \cap \boldsymbol{C})$

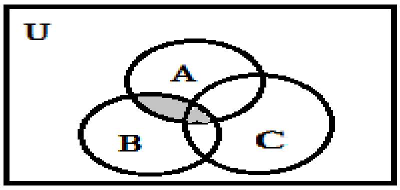
$B \cup C$ fig.(i)



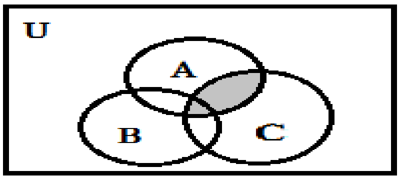
$\boldsymbol{A} \cap (\boldsymbol{B} \cup \boldsymbol{C})$ **fig(ii)**



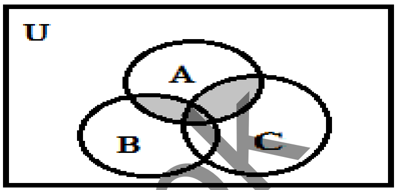
$A \cap B$ fig.(iii)



$A \cap C$ fig(iv)



$(\boldsymbol{A} \cap \boldsymbol{B}) \cup (\boldsymbol{A} \cap \boldsymbol{C})$ **fig(v)**

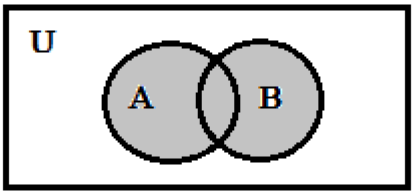


From fig(ii) and fig(v) it is verified that

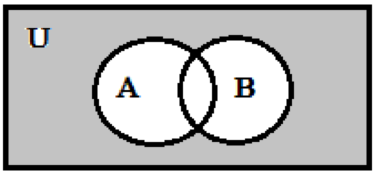
$\boldsymbol{A} \cap (\boldsymbol{B} \cup \boldsymbol{C}) = (\boldsymbol{A} \cap \boldsymbol{B}) \cup (\boldsymbol{A} \cap \boldsymbol{C}$

$(\boldsymbol{vii})(\boldsymbol{A} \cup \boldsymbol{B})' = \boldsymbol{A}' \cap \boldsymbol{B}'$

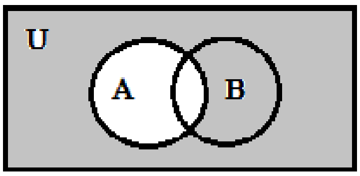
$A \cup B$ fig(i)



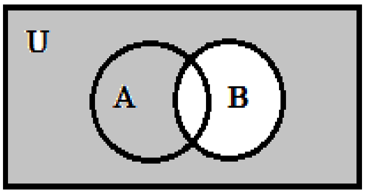
$(A \cup B)'$ fig(ii)



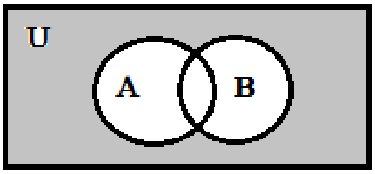
A' fig(iii)



B' fig(iv)



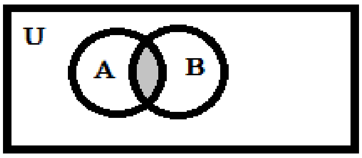
$A' \cap B'$ fig(v)



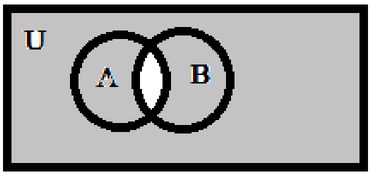
From (ii) and (v) it is verified that $(A \cup B)' = A' \cap B'$

(viii) $(\boldsymbol{A} \cap \boldsymbol{B})' = \boldsymbol{A}' \cup \boldsymbol{B}'$

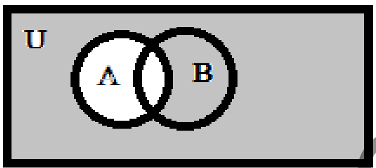
$A \cap B$ fig(i)



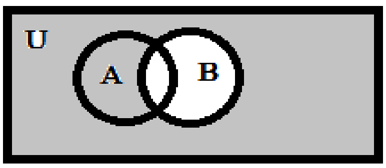
$(\boldsymbol{A} \cap \boldsymbol{B})'$ **fig(ii)**



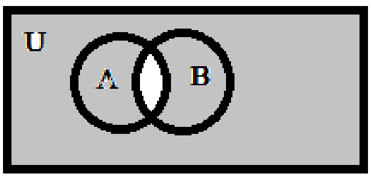
A' fig(iii)



B' fig.(iv)



$\boldsymbol{A}' \cup \boldsymbol{B}'$ **fig(v)**



From fig(ii) and fig(v) it is verified that $(\boldsymbol{A} \cap \boldsymbol{B})' = \boldsymbol{A}' \cup \boldsymbol{B}'$

Exercise 2.3

Q.1:

Verify the poperties of uninon and intersection for the following pairs of sets:

$\boldsymbol{i)\ A = \{1, 2, 3, 4, 5\}\ ,\ \ B = \{4, 6, 8, 10\}}$

SOLUTION:

Commutative Property of Union

$A \cup B = B \cup A$

$L.H.S = A \cup B$

$= \{1,2,3,4,5\} \cup \{4,6,8,10\}$

$= \{1,2,3,4,5,6,8,10\}$

$R.H.S = B \cup A$

$= \{4,6,8,10\} \cup \{1,2,3,4,5\}$

$= \{1,2,3,4,5,6,8,10\}$

Commutative Property of intersection

$A \cap B = B \cap A$

$L.H.S = A \cap B$

$= \{1,2,3,4,5\} \cap \{4,6,8,10\}$

$= \{4\}$

$R.H.S = B \cap A$

$= \{4,6,8,10\} \cap \{1,2,3,4,5\}$

$= \{4\}$

$\boldsymbol{ii)\ A = N\ \ ,\ \ B = Z}$

SOLUTION:

Commutative Property of Union

$A \cup B = B \cup A$

$L.H.S = N \cup Z$

$= Z$

$R.H.S = B \cup A$

$= Z \cup N$

$= Z$

Commutative Property of intersection

$A \cap B = B \cap A$

$L.H.S = N \cap Z$

$= N$

$R.H.S = B \cap A$

$= Z \cap N$

$= N$

$\boldsymbol{iii)\ A = \{x \mid x \in \mathbb{R}\ \wedge x \geq 0\}}$, $\mathbb{R}$

SOLUTION:

$A = Set\ of\ all + ive\ real\ numbers$

$B = \mathbb{R} = Set\ of\ all\ real\ numbers$

Commutative Property of Union

$A \cup B = B \cup A$

$L.H.S = A \cup B$

$= B$

$R.H.S = B \cup A$

$= B$

Commutative Property of intersection

$A \cap B = B \cap A$

$L.H.S = A \cap B$

$= A$

$R.H.S = B \cap A$

$= A$

Q.2: Verify the propeties for the sets A, B, C given below

i) Associativity of union

ii) Associativity of intersection

iii) Distributivity of union over intersection

iv) Distributivity of intersection over union

a) $A=\{1,2,3,4\},\ B=\{3,4,5,6,7,8\},$

$C=\{5,6,7,9,10\}$

i) Associativity of union

$(A\cup B)\cup C=A\cup(B\cup C)$

$L.H.S=(A\cup B)\cup C$

$=(\{1,2,3,4\}\cup\{3,4,5,6,7,8\})\cup\{5,6,7,9,10\}$

$=\{1,2,3,4,5,6,7,8\}\cup\{5,6,7,9,10\}$

$=\{1,2,3,4,5,6,7,8,9,10\}$

$R.H.S=A\cup(B\cup C)=\{1,2,3,4\}\cup(\{3,4,5,6,7,8\}\cup\{5,6,7,9,10\})$

$=\{1,2,3,4\}\cup\{3,4,5,6,7,8,9,10\}=\{1,2,3,4,5,6,7,8,9,10\}$

ii) Associativity of intersection

$(A\cap B)\cap C=A\cap(B\cap C)$

$L.H.S=(A\cap B)\cap C=(\{1,2,3,4\}\cap\{3,4,5,6,7,8\})\cap\{5,6,7,9,10\}$

$=\{3,4\}\cap\{5,6,7,9,10\}=\{\ \}$

$R.H.S=A\cap(B\cap C)$

$=\{1,2,3,4\}\cap(\{3,4,5,6,7,8\}\cap\{5,6,7,9,10\})$

$=\{1,2,3,4\}\cap\{5,6,7\}=\{\ \}$

iii) Distributivity of union over intersection

$A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$

$L.H.S=A\cup(B\cap C)=$

$\{1,2,3,4\}\cup(\{3,4,5,6,7,8\}\cap\{5,6,7,9,10\})$

$=\{1,2,3,4\}\cup\{5,6,7\}=\{1,2,3,4,5,6,7\}$

$R.H.S=(A\cup B)\cap(A\cup C)$

$=(\{1,2,3,4\}\cup\{3,4,5,6,7,8\})\cap(\{1,2,3,4\}\cup\{5,6,7,9,10\})$

$=\{1,2,3,4,5,6,7,8\}\cap\{1,2,3,4,5,6,7,8,9,10\}=\{1,2,3,4,5,6,7\}$

$Hence\quad L.H.S=R.H.S$

iv) Distributivity of intersection over union

$A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$

$L.H.S=A\cap(B\cup C)$

$=\{1,2,3,4\}\cap(\{3,4,5,6,7,8\}\cup\{5,6,7,9,10\})$

$=\{1,2,3,4\}\cap\{3,4,5,6,7,8,9,10\}$

$=\{3,4\}$

$R.H.S=(A\cap B)\cup(A\cap C)$

$=(\{1,2,3,4\}\cap\{3,4,5,6,7,8\})\cup(\{1,2,3,4\}\cap\{5,6,7,9,10\})$

$=\{3,4\}\cap\{\ \}$

$=\{3,4\}$

$Hence\quad L.H.S=R.H.S$

b) $A=\emptyset\ or\ \{\ \},\ B=\{0\},\ C=\{0,1,2\}$

i) Associativity of union

$(A\cup B)\cup C=A\cup(B\cup C)$

$L.H.S=(A\cup B)\cup C$

$=(\{\ \}\cup\{0\})\cup\{0,1,2\}=\{0\}\cup\{0,1,2\}$

$=\{0,1,2\}$

$R.H.S=A\cup(B\cup C)$

$=\{\ \}\cup(\{0\}\cup\{0,1,2\})$

$=\{\ \}\cup\{0,1,2\}=\{0,1,2\}$

$Hence\quad L.H.S=R.H.S$

ii) Associativity of intersection

$(A\cap B)\cap C=A\cap(B\cap C)$

$L.H.S=(A\cap B)\cap C$

$=(\{\ \}\cap\{0\})\cap\{0,1,2\}=\{\ \}\cap\{0,1,2\}=\{\ \}$

$R.H.S=A\cap(B\cap C)$

$=\{\ \}\cap(\{0\}\cap\{0,1,2\})=\{\ \}\cap\{5,6,7\}=\{\ \}$

$Hence\quad L.H.S=R.H.S$

iii) Distributivity of union over intersection

$A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$

$L.H.S=A\cup(B\cap C)=\{\ \}\cup(\{0\}\cap\{0,1,2\})$

$=\{\ \}\cup\{0\}=\{0\}$

$R.H.S=(A\cup B)\cap(A\cup C)$

$=(\{\ \}\cup\{0\})\cap(\{\ \}\cup\{0,1,2\})$

$=\{0\}\cap\{0,1,2\}=\{0\}$

$Hence\quad L.H.S=R.H.S$

iv) Distributivity of intersection over union

$A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$

$L.H.S=A\cap(B\cup C)=$

$\{\ \} \cap (\{0\} \cup \{0,1,2\}) = \{\ \} \cap \{0,1,2\} = \{\ \}$

$R.H.S = (A \cap B) \cup (A \cap C)$

$= (\{\ \} \cap \{0\}) \cup (\{\ \} \cap \{0,1,2\}) = \{\ \} \cap \{\ \} = \{\ \}$

$Hence \quad L.H.S = R.H.S$

) $\boldsymbol{A = \mathbb{N}, \ B = \mathbb{Z}, \ C = \mathbb{Q} \quad OR}$

$\boldsymbol{A = \{1,2,3,...\}, \ B = \{0, \pm1, \pm2, \pm3, ...\},}$

$\boldsymbol{C = \{x | x = \frac{p}{q}, where\ p,q \in \mathbb{Z}\ and\ q \neq 0\}}$

i) Associativity of union

$(A \cup B) \cup C = A \cup (B \cup C)$

$L.H.S = (A \cup B) \cup C = (N \cup Z) \cup Q = Z \cup Q = Q$

$R.H.S = A \cup (B \cup C) = N \cup (Z \cup Q) = N \cup Q = Q$

$Hence \quad L.H.S = R.H.S$

ii) Associativity of intersection

$(A \cap B) \cap C = A \cap (B \cap C)$

$L.H.S = (A \cap B) \cap C = (N \cap Z) \cap Q = N \cap Q = N$

$R.H.S = A \cap (B \cap C) = N \cap (Z \cap Q) = N \cap Z = N$

$Hence \quad L.H.S = R.H.S$

iii) Distributivity of union over intersection

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$L.H.S = A \cup (B \cap C) = N \cup (Z \cap Q) = N \cup Z = Z$

$R.H.S = (A \cup B) \cap (A \cup C)$

$= (N \cup Z) \cap (N \cup Q) = Z \cap Q = Z$

$Hence \quad L.H.S = R.H.S$

iv) Distributivity of intersection over union

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$L.H.S = A \cap (B \cup C) = N \cap (Z \cup Q) = N \cap Q = N$

$R.H.S = (A \cap B) \cup (A \cap C) = (N \cap Z) \cup (N \cap Q)$

$= N \cup N$

$= N$

$Hence \quad L.H.S = R.H.S$

Q.3: Verify De Morgan's Laws for the following sets:

$\boldsymbol{U = \{1,2,3,...,20\}, \ A = \{2,4,6,...,20\},}$
$\boldsymbol{B = \{1,3,5, \ ... \ ,19\}}$

SOLUTION:

$\boldsymbol{De\ Morgan's\ Laws: \ (A \cup B)' = A' \cap B'}$

$L.H.S = (A \cup B)' = U - (AUB)$

$= \{1,2,3, \ ... \ ,20\}$
$-(\{2,4,6, \ ... \ ,20\} \cup \{1,3,5, \ ... \ ,19\})$
$= \{1,2,3, \ ... \ ,20\} - \{1,2,3, \ ... \ ,20\} = \{\ \}$

$R.H.S = A' \cap B' = (U - A) \cap (U - B)$
$= (\{1,2,3, \ ... \ ,20\} - \{2,4,6, \ ... \ ,20\})$
$\cap (\{1,2,3, \ ... \ ,20\} - \{1,3,5, \ ... \ ,19\})$
$= \{1,3,5, \ ... \ ,19\} \cap \{2,4,6, \ ... \ ,20\} = \{\ \}$

$Hence \quad L.H.S = R.H.S$

$\boldsymbol{De\ Morgan's\ Laws: \ (A \cap B)' = A' \cup B'}$

$L.H.S = (A \cap B)' = U - (A \cap B)$
$= \{1,2,3, ... ,20\}$
$-(\{2,4,6, \ ... \ ,20\} \cap \{1,3,5, \ ... \ ,19\})$
$= \{1,2,3, ... ,20\} - \{\ \}$
$= \{1,2,3, \ ... \ ,20\}$

$R.H.S = A' \cup B' = (U - A) \cup (U - B)$
$= (\{1,2,3, \ ... \ ,20\} - \{2,4,6, \ ... \ ,20\})$
$\cup (\{1,2,3, \ ... \ ,20\} - \{1,3,5, \ ... \ ,19\})$
$= \{1,3,5, \ ... \ ,19\} \cup \{2,4,6, \ ... \ ,20\}$
$= \{1,2,3, \ ... \ ,20\}$

$Hence \quad L.H.S = R.H.S$

$\boldsymbol{Q.4: U = The\ set\ of\ English\ alphabets, A = \{x | x\ is\ a\ vowel\}, B = \{y | y\ is\ a\ consonant\}}$

Verify De Morgan's Laws for these sets:

SOLUTION:

$U = \{a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r,s,t,u,v,w,x,y,z\}$

$A = \{a,e,i,o,u\}$

$B = \{b,c,d,f,g,h,j,k,l,m,n,p,q,r,s,t,v,w,x,y,z\}$

$\boldsymbol{De\ Morgan's\ Laws: \ (A \cup B)' = A' \cap B'}$

$L.H.S = (A \cup B)' = U - (AUB) = U - U = \{\ \}$

$R.H.S = A' \cap B' = (U - A) \cap (U - B) = B \cap A = \{\ \}$

$Hence \quad L.H.S = R.H.S$

$\boldsymbol{De\ Morgan's\ Laws: \ (A \cap B)' = A' \cup B'}$

$L.H.S = (A \cap B)' = U - (A \cap B) = U - \{\ \} = U$

$R.H.S = A' \cup B' = (U - A) \cup (U - B) = B \cup A = U$

$Hence \quad L.H.S = R.H.S$

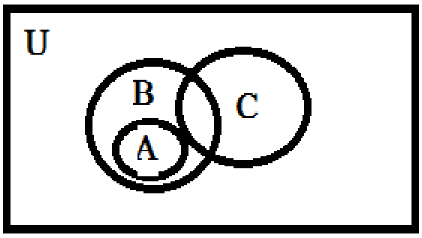
Q5. **With the help of Venn diagrams, verify the two distributive properties in the following cases w.r.t union and intersection**

i) $\boldsymbol{A \subseteq B, A \cap C = \emptyset\ and\ B\ and\ C\ are\ overlappimg}$

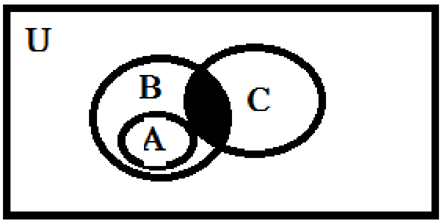
(a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

From given information we have Venn diagram as

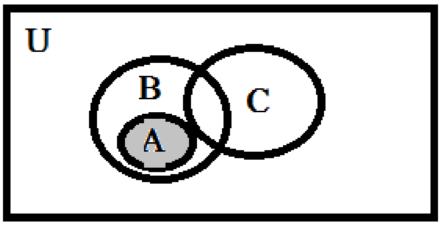
$A \subseteq B, A \cap C = \emptyset$ B and C are overlapping



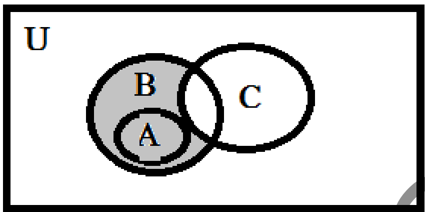
$B \cap C$ fig(i)



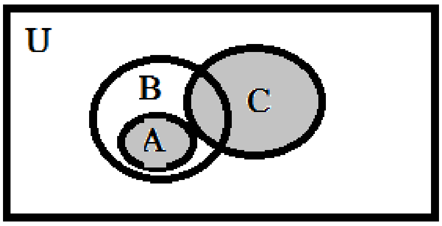
$A \cup (B \cap C)$ fig.(ii)



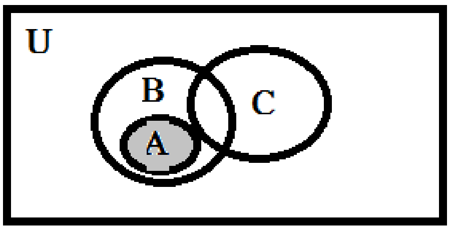
$A \cup B$ Fig.(iii)



$A \cup C$ fig.(iv)



$(A \cup B) \cap (A \cup C)$ fig(v)

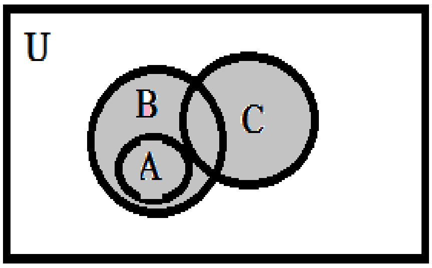


From fig.(iii) and (v) we verified that

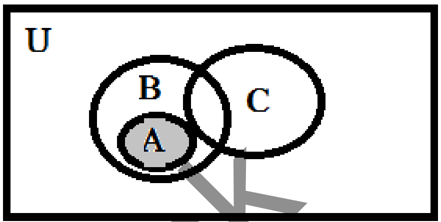
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

(b) $\boldsymbol{A \cap (B \cup C) = (A \cap B) \cup (A \cap C)}$

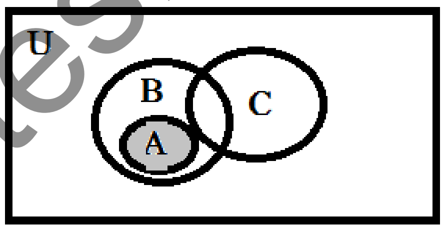
$B \cup C$ fig.(i)



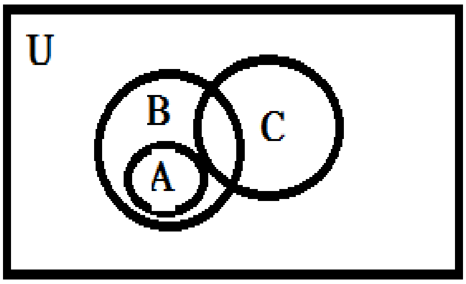
$A \cap (B \cup C)$ fig.(ii)



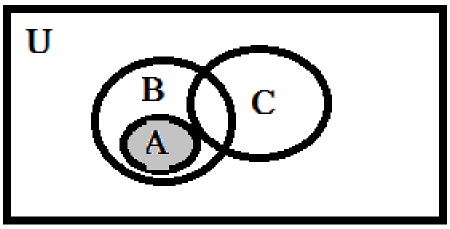
$A \cap B$ fig(iii)



$A \cap C \therefore A \cap C = \emptyset$ fig.(iv)



$\boldsymbol{(A \cap B) \cup (A \cap C)}$ **fig(v)**



From fig(iii) and (v)

It is verified that

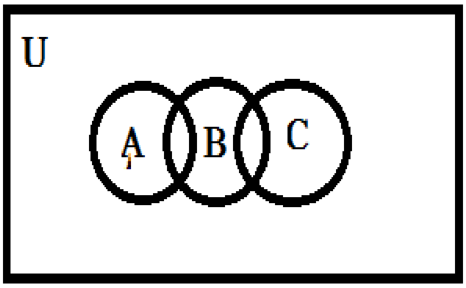
$$\boldsymbol{A \cap (B \cup C) = (A \cap B) \cup (A \cap C)}$$

(iii) A and B are overlapping, B and C are overlapping but A and C are disjoint.

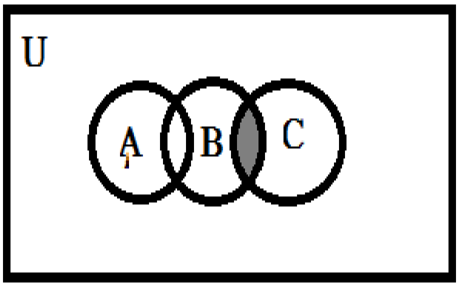
$(\boldsymbol{a}) \boldsymbol{A} \cup (\boldsymbol{B} \cap \boldsymbol{C}) = (\boldsymbol{A} \cup \boldsymbol{B}) \cap (\boldsymbol{A} \cup \boldsymbol{C})$

According to given information we have Venn diagram as

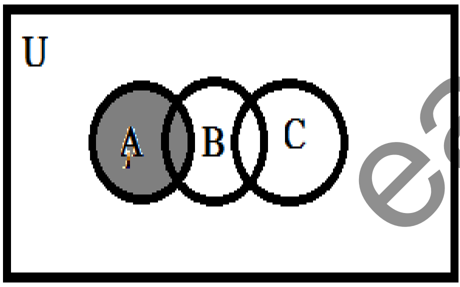
$A \cap B \neq \emptyset, B \cap C = \emptyset, A \cap C = \emptyset$



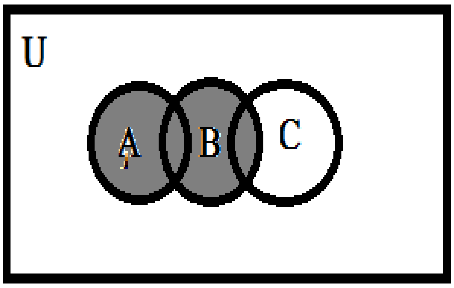
$B \cap C$ fig.(i)



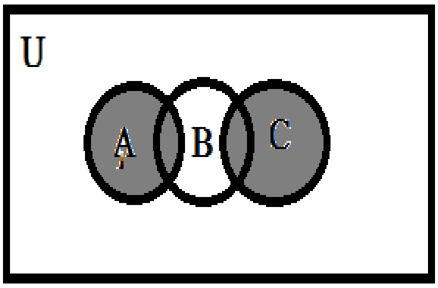
$A \cup (B \cap C)$ fig.(ii)



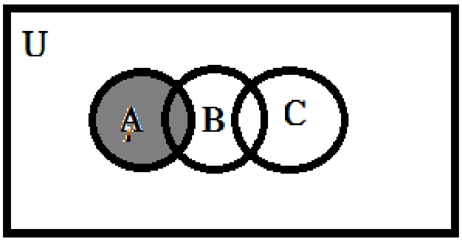
$A \cup B$ Fig(iii)



$A \cup C$ fig(iv)



$(\boldsymbol{A} \cup \boldsymbol{B}) \cap (\boldsymbol{A} \cup \boldsymbol{C})$ **fig.(v)**



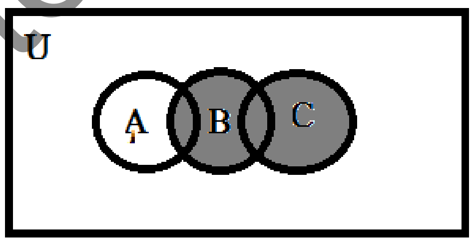
From fig(ii) and fig(v)

It is verified that

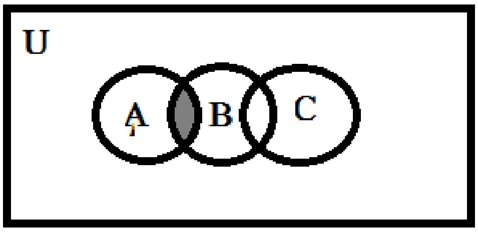
$\boldsymbol{A} \cup (\boldsymbol{B} \cap \boldsymbol{C})(\boldsymbol{A} \cup \boldsymbol{B}) \cap (\boldsymbol{A} \cup \boldsymbol{C})$

(b) $\boldsymbol{A} \cap (\boldsymbol{B} \cup \boldsymbol{C}) = (\boldsymbol{A} \cap \boldsymbol{B}) \cup (\boldsymbol{A} \cap \boldsymbol{C})$

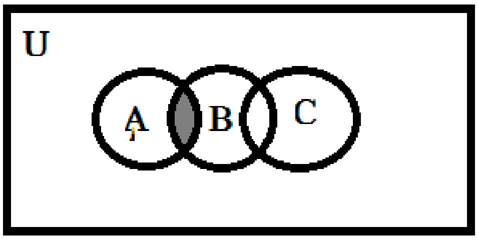
$B \cup C$ fig(i)



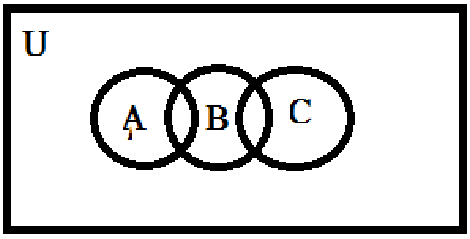
$A \cap (B \cup C)$ fig(ii)



$A \cap B$ fig.(iii)



$A \cap C$ $A \cap C = \emptyset$



From fig(ii) and fig(iv) it is verified that

$A\cap (B\cup C)=(A\cap B)\cup(A\cap C)$

Q.6: *Taking any set, say*

$A=\{1,2,3,4,5\}$ ***verify the following:***

SOLUTION:

i) $A\cup\emptyset=A$

$L.H.S=A\cup\emptyset=\{1,2,3,4,5\}\cup\{\ \}=\{1,2,3,4,5\}=A$

ii) $A\cup A=A$

$L.H.S=A\cup A=\{1,2,3,4,5\}\cup\{1,2,3,4,5\ \}=\{1,2,3,4,5\}=A$

iii) $A\cap A=A$

$L.H.S=A\cap A=\{1,2,3,4,5\}\cap\{1,2,3,4,5\}=A$

Q.7: *If* $U=\{1,2,3,4,\ ...,20\}$ ***and***

$A=\{1,3,5,7,\ ...,19\}$

Verify the following:

SOLUTION:

i) $A\cup A'=U$

$L.H.S=A\cup A'=A\cup(U-A)$

$=\{1,3,5,7,\ ...,19\}\cup(\{1,2,3,4,\ ...,20\}-\{1,3,5,7,\ ...,19\})$

$=\{1,3,5,7,\ ...,19\}\cup\{2,4,6,\ ...,20\}$

$=\{1,2,3,4,\ ...,20\}=U=R.H.S$

Hence $L.H.S=R.H.S$

ii) $A\cap U=A$

$L.H.S=\ A\cap U$

$=\{1,3,5,7,\ ...,19\}\cap\{1,2,3,4,\ ...,20\}$

$=A$

$=R.H.S$

Hence $L.H.S=R.H.S$

iii) $A\cap A'=\emptyset$

$L.H.S=\ A\cap A'$

$=A\cap(U-A)$

$=\{1,3,5,7,\ ...,19\}\cap(\{1,2,3,4,\ ...,20\}-\{1,3,5,7,\ ...,19\})$

$=\emptyset$

$=R.H.S$

Q.8: *From suitable properties of union and intersection deduce the following results:*

SOLUTION:

i) $A\cap(A\cup B)=A\cup(A\cap B)$

$L.H.S=A\cap(A\cup B)$

$=(A\cap A)\cup(A\cap B)$

$=A\cup(A\cap B)$

$=R.H.S$

Hence $L.H.S=R.H.S$

ii) $A\cup(A\cap B)=A\cap(A\cup B)$

$L.H.S=A\cup(A\cap B)$

$=(A\cup A)\cap(A\cup B)$

$=A\cap(A\cup B)$

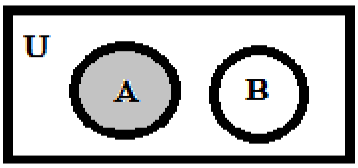
$=R.H.S$

Hence $L.H.S=R.H.S$

Q.9 using Venn diagram, verify the following results.

(i) $A\cap B'=A$ ***iff*** $A\cap B=\emptyset$

Suppose $A\cap B'=A$ fig.(i)

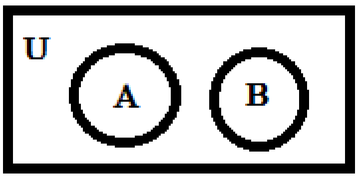


We are to prove that $A\cap B=\emptyset$ *from fig (i)* $A\cap B'=A$

Showing A and B are disjoint. so $A\cap B=\emptyset$

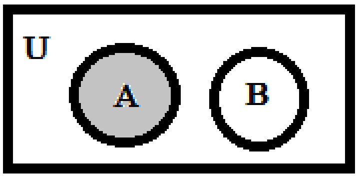
Conversely,

Suppose $A\cap B=\emptyset$



As $A\cap B=\emptyset$

- A and B are disjoint
- A will be subset of B’
- So $A \cap B' = A$ *as shown in fig.(iii)*

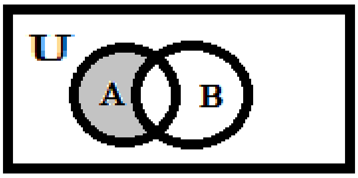


$$A \cap B' = A$$

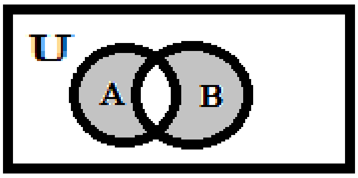
ii) $(\boldsymbol{A}-\boldsymbol{B}) \cup \boldsymbol{B} = \boldsymbol{A} \cup \boldsymbol{B}$

consider A and B are overlapping sets, then

$\boldsymbol{A}-\boldsymbol{B}$ **fig(i)**



$(\boldsymbol{A}-\boldsymbol{B}) \cup \boldsymbol{B}$ **fig(ii)**



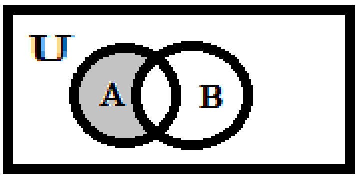
From fig(ii) it is clear that $(\boldsymbol{A}-\boldsymbol{B}) \cup \boldsymbol{B} = \boldsymbol{A} \cup \boldsymbol{B}$

iii)

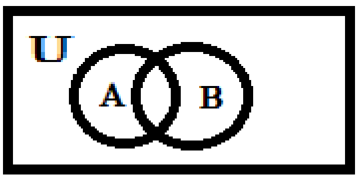
$(A-B) \cap B = \emptyset$

Consider A and B are overlapping sets then

$A-B$ fig.(i)



From fig. $A-B$ and B having nothing common So nothing will be shaded to show $(A-B) \cap B$ *as shown in fig.(ii)*

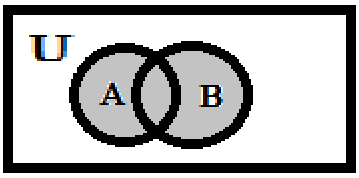


So, $(A-B) \cap B = \emptyset$

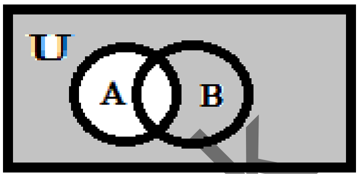
iv) $\boldsymbol{A} \cup \boldsymbol{B} = \boldsymbol{A} \cup (\boldsymbol{A}' \cap \boldsymbol{B})$

consider A and B are overlapping sets, then

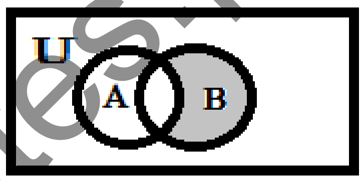
$A \cup B$ fig(i)



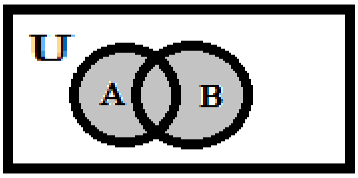
$A' = U - A$ fig(ii)



$A' \cap B$ fig(iii)



$\boldsymbol{A} \cup (\boldsymbol{A}' \cap \boldsymbol{B})$ **fig(iv)**



From fig(i) and fig(iv) it is verified that

$\boldsymbol{A} \cup \boldsymbol{B} = \boldsymbol{A} \cup (\boldsymbol{A}' \cap \boldsymbol{B})$

Induction:

A result on the basis of limited observations is called induction.

Deduction:

A result or (conclusion) on the basis of well-known **facts is called deduction.**

Proposition:

Any statement which is either true or false but not both is called proposition are denoted by $p_1, q_1, r_1 \dots$

Negation:

Negation of a proposition means to rejects that proposition. If p is proposition variable then negation of p is denoted by $\sim p$

Note:

$if\ p\ is\ true\ \sim p\ is\ false$

$if\ p\ is\ false\ , \sim p\ is\ true$

$if\ \sim p\ is\ true\ , p\ is\ false$

$if\ \sim p\ is\ false\ , p\ is\ true$

$the\ truth\ table\ is$

p	$\sim p$
T	F
F	T

Conjunction:

Let P and q be two proposition then their conjunction is denoted by $p \wedge q$ and read as "p and q"

*A conjunction is true only if both p and q are true.

*A conjunction is false if at least one of p and q is false.

The truth table is given as

p	p	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction:

Let P and q be two proposition then their disjunction is denoted by $p \vee q$ and read as "p and q"

*A disjunction is false only if both p and q are false

*A disjunction is true if at least one of p and q is true.

The truth table is given as

p	p	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Implication or conditional:

let p and q be two proposition, then p implication q is denoted by $p \rightarrow q$ $and\ read\ as\ \text{"}p\ implis\ q\text{"}(\ or\ if\ p\ then\ q\}\ where\ p\ is\ called\ hypothesis\ or\ anticedent$ $while\ q\ is\ called\ consequent\ or\ conclusion.$

*A conditional statement is false only when hypothesis is true, otherwise conditional statement is always true.

The truth table is given below.

p	p	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Bi conditional:

The proposition $p \rightarrow q \wedge q \rightarrow p\ is\ shortly\ written\ as$ $p \leftrightarrow q$ read as p if and only if q is called bi conditional or equivalent statement.

*A bi conditional statement is true if both p and q are true.

*a bi conditional statement is true if both p and q are false.

*A bi conditional statement is false when any one of p, q is false.

We draw up its truth table as

P	p	$p \rightarrow q$	$q \rightarrow p$	$p \leftrightarrow q$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Conditionals related with a given conditional

Converse:

Let $p \rightarrow q\ be\ a\ given\ conditional\ then$ $q \rightarrow p\ is\ called\ the\ converse\ of\ p \rightarrow q$

Inverse:

Let $p \rightarrow q\ be\ a\ given\ conditional\ then$ $\sim p \rightarrow \sim q\ is\ called\ the\ inverse\ of\ p \rightarrow q$

Contrapositive :

Let $p \rightarrow q\ be\ a\ given\ conditional\ then$ $\sim q \rightarrow \sim p\ is\ called\ the\ contrapositive\ of\ p \rightarrow q$

The truth table is given as

p	q	$\sim p$	$\sim q$
T	T	F	F
T	F	F	T
F	T	T	F
F	F	T	F

conditional	$converse$	$inverse$	$contrapositive$
$p \longrightarrow q$	$q \longrightarrow p$	~p⟶~q	$\sim q \longrightarrow \sim p$
T	T	T	T
F	T	T	F
T	F	F	T
T	T	T	T

Note:

i) From the table it is clear that convers and inverse are equivalent to each other.

ii) From the table it is clear that any conditional and its contrapositive are equivalent to each other.

Tautologies:

A statement which is necessary true for all the cases is called a tautology.

p	$\sim p$	$p \vee \sim p$

T	F	T
F	T	T

From $p \vee \sim q$ *is true for all cases.*
S0 $p \vee \sim p$ *is tautology*

Contradiction:
A statement which is necessary false for all the cases is called contradiction.

p	$\sim p$	$p \wedge \sim p$
T	F	F
F	T	F

From the table we observe that $p \wedge \sim p$
Is a contradiction.
Contingency:
A Statement which is neither tautology nor contradiction is called contingency.
e.g., $(p \to q) \wedge (p \vee q)$ *is contingency.*

i) Quantifiers: the word or symbol which convey the idea of quantity or number are called quantifiers.
There are two quanrifiers.
Universal quantifiers: the symbol
$\forall$ *is called universal quantifiers.*
ii)Existential Quantifiers: the symbol
$\exists$ *is called existential quantifiers.*

ii) $p \longrightarrow (p \vee q)$

p	q	$p \vee q$	$p \longrightarrow (p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

Since all the entries in the last column are T,
hence it is a taotology.

iii) $\sim(p \longrightarrow q) \longrightarrow p$

p	q	$p \longrightarrow q$	$\sim (p \longrightarrow q)$	$\sim (p \longrightarrow q) \longrightarrow p$
T	T	T	F	T
T	F	F	T	T
F	T	T	F	T
F	F	T	F	T

Since all the entries in the last column are T,
hence it is a taotology.

iv) $\sim q \wedge (p \longrightarrow q) \longrightarrow \sim p$

p	q	$\sim p$	$\sim q$	$p \longrightarrow q$	$\sim q \wedge (p \longrightarrow q)$	$\sim q \wedge (p \longrightarrow q) \longrightarrow \sim p$
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	F	T
F	F	T	T	T	T	T

Since all the entries in the last column are T,
hence it is a taotology.

Q.4: Determine whether : each of the following is a tautology, a contigency, or an absurdity

i) $p \wedge \sim p$

p	$\sim p$	$p \wedge \sim p$
T	F	F
F	T	F

Since all the entries in the last column are F,
hence it is an absurdity.

ii) $p \longrightarrow (q \longrightarrow p)$

p	q	$q \longrightarrow p$	$p \longrightarrow (q \longrightarrow p)$
T	T	T	**T**
T	F	T	**T**
F	T	F	**T**
F	F	T	**T**

Since all the entries in the last column are T,
hence it is a taotology.

iii) $q \vee (\sim q \vee p)$

p	q	$\sim q$	$\sim q \vee p$	$q \vee (\sim q \vee p)$
T	T	F	T	T
T	F	T	T	T
F	T	F	F	T
F	F	T	T	T

Since all the entries in the last column are T,
hence it is a taotology.

Exercise 2.4

Q.1:
Write the converse, inverse and contrapositive of the following conditionals

Parts	conditional	converse	inverse	contrapositive
i)	$\sim p \longrightarrow q$	$q \longrightarrow \sim p$	$p \longrightarrow \sim q$	$\sim q \longrightarrow p$
ii)	$q \longrightarrow p$	$p \longrightarrow q$	$\sim q \longrightarrow \sim p$	$\sim p \longrightarrow \sim q$
iii)	$\sim p \longrightarrow \sim q$	$\sim q \longrightarrow \sim p$	$p \longrightarrow q$	$q \longrightarrow p$
iv)	$\sim q \longrightarrow \sim p$	$\sim p \longrightarrow \sim q$	$q \longrightarrow p$	$p \longrightarrow q$

Q.2: Construct the truth table for the following ststements

$(p \to \sim p) \vee (p \to q)$

p	q	$\sim p$	$p \to \sim p$	$p \to q$	$(p \to \sim p) \vee (p \to q)$
T	T	F	F	T	T
T	F	F	F	F	F
F	T	T	T	T	T
F	F	T	T	T	T

ii) $(p \wedge \sim p) \longrightarrow q$

p	q	$\sim p$	$p \wedge \sim p$	$(p \wedge \sim p) \longrightarrow q$
T	T	F	F	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

iii) $\sim (p \longrightarrow q) \leftrightarrow (p \longrightarrow \sim q)$

p	q	$\sim q$	$p \longrightarrow q$	$\sim (p \longrightarrow q)$	$p \wedge \sim q$	$\sim (p \longrightarrow q) \leftrightarrow (p \longrightarrow \sim q)$
T	T	F	T	F	F	T
T	F	T	F	T	T	T
F	T	F	T	F	F	T
F	F	T	T	F	F	T

Q.3

***Show that each of the following ststement is a tautology*:**

i) $(p \wedge q) \rightarrow q$

p	Q	$p \wedge q$	$(p \wedge q) \rightarrow q$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

Since all the entries in the last column are T, hence it is a taotology..

ii) $p \rightarrow (p \vee q)$

p	q	$p \vee q$	$p \rightarrow (p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

Since all the entries in the last column are T, hence it is a taotology.

iii) $\sim(p \rightarrow q) \rightarrow p$

p	q	$p \rightarrow q$	$\sim(p \rightarrow q)$	$\sim(p \rightarrow q) \rightarrow p$
T	T	T	F	T
T	F	F	T	T
F	T	T	F	T
F	F	T	F	T

Since all the entries in the last column are T, hence it is a taotology.

iv) $\sim q \wedge (p \rightarrow q) \rightarrow \sim p$

p	q	~p	~q	$p \rightarrow q$	$\sim q \wedge (p \rightarrow q)$	$\sim q \wedge (p \rightarrow q) \rightarrow \sim p$
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	F	T
F	F	T	T	T	T	T

Since all the entries in the last column are T, hence it is a taotology.

Q.4: Determine whether each of the following is a tautology, a contigency, or an absurdity

i) $p \wedge \sim p$

p	$\sim p$	$p \wedge \sim p$
T	F	F
F	T	F

Since all the entries in the last column are F, hence it is an absurdity.

ii) $p \rightarrow (q \rightarrow p)$

p	q	$q \rightarrow p$	$p \rightarrow (q \rightarrow p)$
T	T	T	**T**
T	F	T	**T**
F	T	F	**T**
F	F	T	**T**

Since all the entries in the last column are T, hence it is a taotology.

iii) $q \vee (\sim q \vee p)$

p	q	~q	$\sim q \vee p$	$q \vee (\sim q \vee p)$
T	T	F	T	T
T	F	T	T	T
F	T	F	F	T
F	F	T	T	T

Since all the entries in the last column are T, hence it is a taotology.

***Q.5: Prove that*:** $p \vee (\sim p \wedge q) \vee (p \wedge q) = p \vee (\sim p \wedge \sim q)$

p	q	~p	~q	$\sim p \wedge \sim q$	$p \wedge q$	$p \vee (\sim p \wedge \sim q)$	$p \vee (\sim p \wedge q) \vee (p \wedge q)$
T	T	F	F	F	T	T	T
T	F	F	T	F	F	T	T
F	T	T	F	F	F	F	F
F	F	T	T	T	F	T	T

Last two columns show that L.H.S = R.H.S

Exercise 2.5

Convert the following theorems to logical and prove them by constructing truth tables:

$\boldsymbol{Q.1}$: $(\boldsymbol{A} \cap \boldsymbol{B})' = \boldsymbol{A}' \cup \boldsymbol{B}'$

SOLUTION: *Given theorem:* $(A \cap B)' = A' \cup B'$ and *Logical form:* $\sim (p \wedge q) = \sim p \vee \sim q$

P	q	~p	~q	$p \wedge q$	$\sim (p \wedge q)$	$\sim p \vee \sim q$
T	T	F	F	T	T	T
T	F	F	T	F	T	T
F	T	T	F	F	F	F
F	F	T	T	F	T	T

Last two columns show that L.H.S = R.H.S

$\boldsymbol{Q.2}$: $(\boldsymbol{A} \cup \boldsymbol{B}) \cup \boldsymbol{C} = \boldsymbol{A} \cup (\boldsymbol{B} \cup \boldsymbol{C})$

SOLUTION: *Given theorem:* $(A \cup B) \cup C = A \cup (B \cup C)$ and *Logical form:* $(p \vee q) \vee r = p \vee (q \vee r)$

P	q	r	$p \vee q$	$q \vee r$	$(p \vee q) \vee r$	$p \vee (q \vee r)$
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	T	T	T	T
T	F	F	T	F	T	T
F	T	T	T	T	T	T
F	T	F	T	T	T	T
F	F	T	F	T	T	T
F	F	F	F	F	F	F

Last two columns show that L.H.S = R.H.S

$\boldsymbol{Q.3}$: $(\boldsymbol{A} \cap \boldsymbol{B}) \cap \boldsymbol{C} = \boldsymbol{A} \cap (\boldsymbol{B} \cap \boldsymbol{C})$

SOLUTION: *Given theorem:* $(A \cap B) \cap C = A \cap (B \cap C)$ and *Logical form:* $(p \wedge q) \wedge r = p \wedge (q \wedge r)$

p	q	r	$p \wedge q$	$q \wedge r$	$(p \wedge q) \wedge r$	$p \wedge (q \wedge r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	F	F	F
T	F	F	F	F	F	F
F	T	T	F	T	F	F
F	T	F	F	F	F	F
F	F	T	F	F	F	F
F	F	F	F	F	F	F

Last two columns show that L.H.S = R.H.S

$\boldsymbol{Q.4}$: $\boldsymbol{A} \cup (\boldsymbol{B} \cap \boldsymbol{C}) \cap \boldsymbol{C} = (\boldsymbol{A} \cup \boldsymbol{B}) \cap (\boldsymbol{A} \cup \boldsymbol{C})$

SOLUTION:

Given theorem: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Logical form: $p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$

p	q	r	$q \wedge r$	$p \vee q$	$p \vee r$	$p \vee (q \wedge r)$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	T	F	F	F
F	F	T	F	F	T	F	F
F	F	F	F	F	F	F	F

Last two columns show that L.H.S = R.H.S

Relation / Binary relation:

Let A and B be two non-empty sets and $A \times B$ be their Cartesian product then relation from A and B is subset of $A \times B$.

Domain:

The set of the first element of the ordered pairs forming a relation is called its domain.

Range:

The set of the second element of the ordered pairs forming a relation is called its range.

Note:

In general $A \times B \neq B \times A$

Into Function:

If $f: A \rightarrow B$ be function such that Range $of\ f$ is proper subset of B then f is said to be function from A into B.

(1-1) And into (injective) function:

$if\ f: A \rightarrow B\ be\ into\ function\ further$
More there is no repetition in the second element any two ordered pairs(i.e; each element of A have distinct image in B) then f is said to be (1-1) and into function.

Inverse of a function:

$if\ f: A \rightarrow B\ be\ a\ bijective\ function\ then$ its inverse is denoted by f^{-1} and defined as
$f^{-1}: A \rightarrow B$

In this case $Domain\ of\ f^{-1} = Range\ of\ f$

$$Range\ of\ f^{-1} = Domain\ of\ f$$

Example:

$$let\ f = \{(x,y)|y = mx + c\}$$

⇨ $f^{-1} = \{(x,y)|x = my + c\}$

In other words:

$f^{-1}\ can\ be\ obtained\ by\ interchanging$

$components\ of\ ordered\ pairs\ of\ f$

Inverse of a function is not necessary a function.

Function:

Let A and B be two non-empty sets, then f is called function from A to B written as

$f: A \rightarrow B$ and defined as;

i)$Domf = A$

ii) No two ordered pairs of f have first elements equal

Example:

i) Let $A = \{1,2,3\} and\ B = \{a,b,c\} and$
$f = \{(1,a),(2,a),(3,c)\}$
$here\ Domf = \{1,2,3\} = A\ also\ two\ ordered\ pairs$

$of\ f have\ first\ element\ equal. hence\ f\ is\ function.$

Onto(Surjective) function:

$$if\ f: A \rightarrow B\ be\ a\ function\ such\ that$$

Range of $f = B\ then\ f$ is called on to function.

(1-1)and Onto(bijective)Function:
$if\ f: A \rightarrow B\ be\ onto\ function\ further\ more$
There is no repletion in the second element of any two ordered pairs of f then f is said to be an one-one and onto function.

Set Builder Notation for A function:

The function $f = \{(x,y)|y = mx + c\}$is called a linear function.
If we draw a liner function then its graph will be a straight line**.**
The function $f = \{(x,y)|y = ax^2 + bx + c\}$
Is called quadratic function.
If we draw a quadratic function its graph will be a parabola.

Exercise 2.6

Q.1

$\boldsymbol{For\ A = \{1,2,3,4\}, find\ the\ folloing\ relations\ in\ A. State\ the\ domain\ and\ range\ of\ each\ relation. Also\ draw\ the\ graph\ of\ each.}$

$\boldsymbol{i)}\ \{(\boldsymbol{x},\boldsymbol{y}) |\ \boldsymbol{y} = \boldsymbol{x}\}$

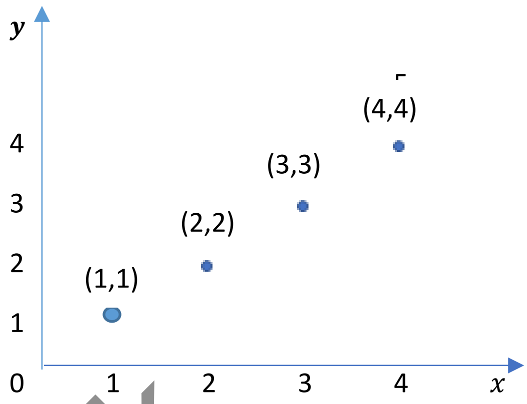
$\boldsymbol{SOLUTION}: \boldsymbol{A} = \{\boldsymbol{1},\boldsymbol{2},\boldsymbol{3},\boldsymbol{4}\}$

$A \times A = \{(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(2,3),(2,4),$
$(3,1),(3,2),(3,3),(3,4),(4,1),(4,2),(4,3),(4,4)\}$

$Let \quad R = \{(1,1),(2,2),(3,3),(4,4)\}$

$Domain\ R = \{1,2,3,4\}$

$Range \quad R = \{1,2,3,4\}$



$\boldsymbol{ii)}\ \{(\boldsymbol{x},\boldsymbol{y}) |\ \boldsymbol{x}+\boldsymbol{y} = \boldsymbol{5}\}$

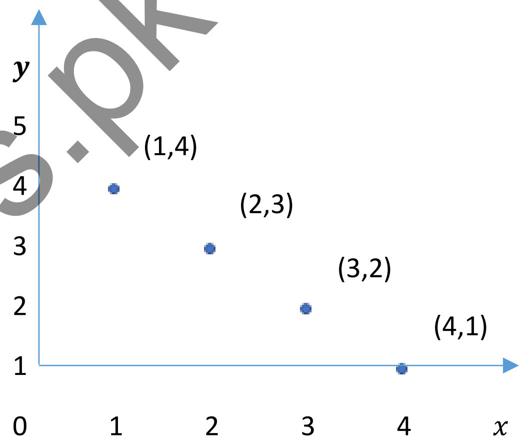
$\boldsymbol{SOLUTION}$:

$A \times A = \{(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(2,3),(2,4),$
$(3,1),(3,2),(3,3),(3,4),(4,1),(4,2),(4,3),(4,4)\}$

$Let \quad R = \{(1,4),(2,3),(3,2),(4,1)\}$

$Domain: \quad R = \{1,2,3,4\}$

$Range: \quad R = \{1,2,3,4\}$



$\boldsymbol{iii)}\ \{(\boldsymbol{x},\boldsymbol{y}) |\ \boldsymbol{x}+\boldsymbol{y} < \boldsymbol{5}\}$

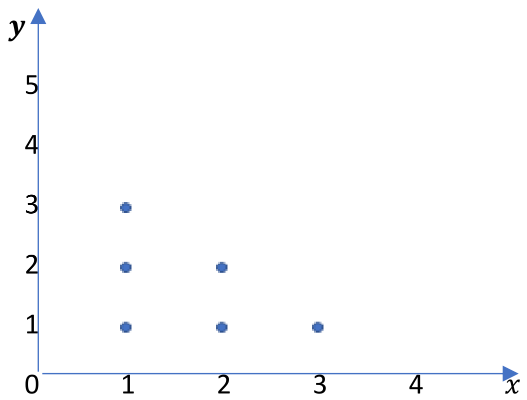
$\boldsymbol{SOLUTION}$:

$A \times A = \{(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(2,3),(2,4),$
$(3,1),(3,2),(3,3),(3,4),(4,1),(4,2),(4,3),(4,4)\}$

$Let \quad R = \{(1,1),(1,2),(1,3),(2,1)(2,2),(3,1)\}$

$Domain: \quad R = \{1,2,3\}$

$Range: \quad R = \{1,2,3\}$



$iv)\ \{(x,y)|\ x+y>5\}$

SOLUTION:

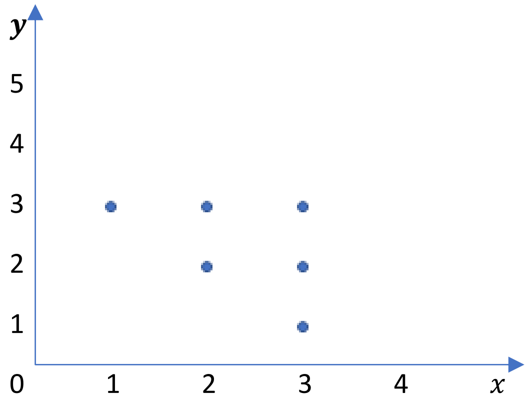
$A\times A=\{(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(2,3),(2,4),$

$(3,1),(3,2),(3,3),(3,4),(4,1),(4,2),(4,3),(4,4)\}$

Let $R=\{(2,4),(3,3),(3,4),(4,2)(4,3),(4,4)\}$

Domain: $R=\{2,3,4\}$

Range: $R=\{2,3,4\}$



$\boldsymbol{Q.2: Repeat\ Q-1\ when\ A=R\ , the\ set\ of\ real\ numbers. Which\ of\ the\ real\ lines\ are\ functions?}$

Solution:

Given $A=\mathbb{R}=set\ of\ real\ no.s$

i) $R=\{(x,y)|y=x\}$
$DomR=\mathbb{R}$
No two orderer pairs of r have first element equal.
So R is a function.

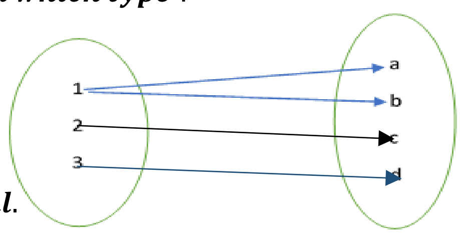
ii) $R=\{(x,y)|x+y=5\}$
$DomR=\mathbb{R}$
No two orderer pairs of R have first element equal.
So R is a function.

iii) $R=\{(x,y)|x+y<5\}$
$DomR=\mathbb{R}$
there are so many orderer pairs of $R(i.e\ (1,2),(1,3),(3,1),(2,2),(3,0),\ldots$
having first element same.
So R is a function.

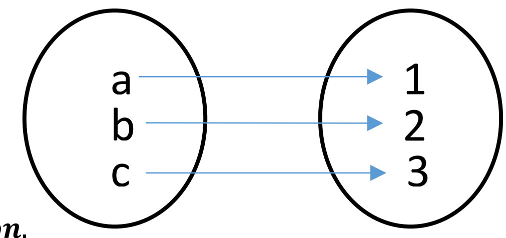
iv) R= $\{(x,y)\ |\ x+y>5\}$
$DomR=R$
there are so many orderer pairs of $R(i.e\ (1,5),(1,6),(3,3),(4,4),\ldots$
having first element equal.
So R is not a function.

$\boldsymbol{Q.3: Which\ of\ the\ following\ diagrams\ represent\ functions\ and\ which\ type?}$

i) **Solution:**
Here $f=\{(1,a),(1,b),(2,c),(3,d)\}$
$Dom\ f=\{1,2,3\}=A$
$Rang\ f=\{a,b,c,d\}=B$
ordered pairs $(1,a)$*and* $(1,b)$*have first element equal.*
so f is not not a function.



ii) **Solution:**
Here $f=\{(a,1),(b,3),(c,5)\}$
$Dom\ f=\{a,b,c\}=A$
$Rang\ f=\{1,3,5\}=B$
f is one − one and also onto function
so f is bijective function.



iii)

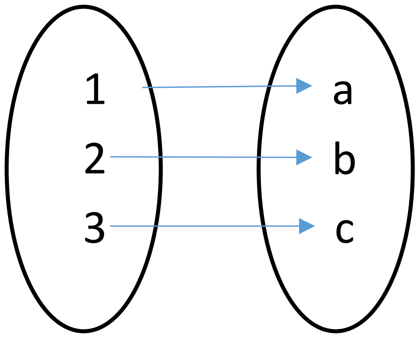
Solution:

$\boldsymbol{here\ f = \{(1,a),(2,b),(3,c)\}}$

$\boldsymbol{Dom\ f = \{1,2,3,\} = A}$

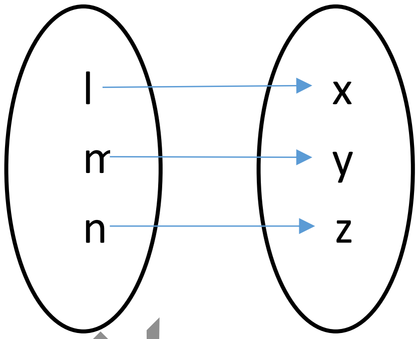
$\boldsymbol{Rang\ f = \{a,b,c\} = B}$

f is one- one as well as onto function. So f is bijective function.



iv) Solution :

$\boldsymbol{here\ f = \{(l,x),(m,y),(n,z)\}}$
$\boldsymbol{Dom\ f = \{l,m,n\} = A}$
$\boldsymbol{Rang\ f = \{x,y,z\} = B}$
So f is into function.



Q4. Find the inverse of the following relation. Tell whether each relation and its inverse is a function or not

$\boldsymbol{i)}$ $\{(\boldsymbol{2},\boldsymbol{1}),(\boldsymbol{3},\boldsymbol{2}),(\boldsymbol{4},\boldsymbol{3}),(\boldsymbol{5},\boldsymbol{4}),(\boldsymbol{6},\boldsymbol{5})\}$

$\boldsymbol{SOLUTION}$:

The inverse is

$\{(1,2),(2,3),(3,4),(4,5),(5,6)\}$

$which\ is\ a\ function.$

$\boldsymbol{ii)}$ $\{(\boldsymbol{1},\boldsymbol{3}),(\boldsymbol{2},\boldsymbol{5}),(\boldsymbol{3},\boldsymbol{7}),(\boldsymbol{4},\boldsymbol{9}),(\boldsymbol{5},\boldsymbol{11})\}$

$\boldsymbol{SOLUTION}$:

The inverse is

$\{(3,1),(5,2),(7,3),(9,4),(11,5)\}$

$which\ is\ a\ function.$

$\boldsymbol{iii)}$ $\{(\boldsymbol{x},\boldsymbol{y}) \mid \boldsymbol{y} = \boldsymbol{2x} + \boldsymbol{3}, \boldsymbol{x} \in \boldsymbol{R}\}$

$\boldsymbol{solution}$:

The inverse is

$\{(x,y) \mid x = 2y + 3, x \in R\}$ $R\}$

$\boldsymbol{iv)}$ $\{(\boldsymbol{x},\boldsymbol{y}) \mid \boldsymbol{y^2} = \boldsymbol{4ax}, \boldsymbol{x} \geq \boldsymbol{0}\}$

$\boldsymbol{SOLUTION}$:

$the\ inverse\ is$

$\{(x,y) \mid x^2 = 4ay,\ y \geq 0\}$

$\therefore x^2 = 4ay \rightarrow x = \pm\sqrt{4ay}$

For each x there is unique element y so it is function.

$\boldsymbol{v)}$ $\{(\boldsymbol{x},\boldsymbol{y}) \mid \boldsymbol{x^2} + \boldsymbol{y^2} = \boldsymbol{9},\ |\boldsymbol{x}| \leq \boldsymbol{3},\quad |\boldsymbol{y}| \leq \boldsymbol{3}\}$

$\boldsymbol{SOLUTION}$:

The inverse is

$\{(x,y) \mid x^2 + y^2 = 9,\ |y| \leq 3,\ |x| \leq 3\}$

$which\ is\ not\ a\ function. \therefore for\ each\ x$,∃ two y.so ordered pairs will have first element equal

Binary Operation:

Let G be a non- empty set then binary operation on G (is a function) denoted by ⁎ (read as star) and is defined as

$*: G \times G \to G$ i.e, for all $a, b \in G \quad a * b \in G$

Remember i) $G,*)$ *will be a non empty set.*

ii)$if \ *$ *is a binary operation onG then G is said to* be closed under binary operation⁎

residue classes Module:

consider "⁎ "be a binary operation on a non-empty set S. let $a, b \in S$ now we find $a * b$

i) If $a * b < n$ then we take $a + b$ as ordinary sum $a * b$ of a and b.

ii) If $a * b \geq n$ *then we take* $a * b = r$ Where r is the remainder obtained after dividing $a + b$ by n. then binary operation ⁎ is called addition modulo "n"

Properties of binary operation:

$$if\ \forall a \in S\ there\ exist\ e \in S\ such\ that\ a * e = e * a$$

$$then\ e\ is\ called\ identity\ w.r.t\ *.$$

Existence of inverse of each element:

$$if\ \forall a \in S\ there\ exist\ a' \in Ssuch\ that$$

$$a * a' = a' * a = e$$

The a' *is called inverse of* a *w.r.t* ⁎

Exercise 2.7

Q.1: Complete the table, indicating by a tick mark those properties which are satisfied by the specified set of numbers.

Set of numbers⟶ Property↓	Natural	Whole	Integers	Rational	Reals
Closure +	yes	Yes	Yes	Yes	Yes
x	yes	Yes	Yes	Yes	Yes
Associative +	yes	Yes	Yes	Yes	Yes
x	yes	Yes	Yes	Yes	Yes
Identity +	No	Yes	Yes	yes	yes
x	yes	Yes	Yes	Yes	Yes
Inverse +	No	No	Yes	Yes	Yes
x	No	No	No	No	No
Commutative +	Yes	Yes	Yes	Yes	Yes
x	yes	yes	yes	Yes	Yes

Q.2: What are the field axioms?

In what respect does the field of real numbers differ from that of complex numbers?

SOLUTION:

A non − empty set F under two binary operations is said to be a field if the following axioms are satisfied

i) F is an abelian group under ' + '

ii) F − {0} is an abelian group under ' × '

iii) Distributive law holds.

Q.3: Show that the adjoining table is that of multiplicatio of the elements of the set of residue classes modulo 5.

⁎	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

SOLUTION:

the xero in C_2 *and* R_2 *are obtained by multiplying of* 1,2,3,4, *with* 0

⇨ It is a x table.

⇨ Every element is less than 5. So the table is a $'x'$ table of the set of element residue classes module 5.

.4: Prepare the table of addition of the elements of the set of residue classes modulo 4.

SOLUTION:

+	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

Clearly {0,1,2,3} is the set of residues classes module 4.

Q.5: Which of the following binary operations shown in tables (a)or (b)is commutative?

(a)

⁎	a	b	c	d
a	a	c	b	d
b	b	c	b	a
c	c	d	b	c
d	a	a	b	b

(b)

⁎	a	b	c	d
a	a	c	b	d
b	c	d	b	a
c	b	b	a	c
d	d	a	c	d

Solu

In take (a)

$a * c = b \to (1)$

And $c * a = b \to (2)$

By (1) and (2) $a * c \neq c * a$

So binary operation is not commutative.
In take (b)
$a * b = c \rightarrow (1)$
And $b * a = c \rightarrow (2)$
By (1) and (2) $a * b = b * a$
So binary operation is commutative.

Q.6: Supply the missing elements of the third row of the given table so that operation $*$ may be associative.

*	a	b	c	d
a	a	b	c	d
b	b	a	c	d
c	-	-	-	-
d	d	c	c	d

We want to find
$c * a = ?$, $c * b = ?$, $c * c = ?$, $c * d = ?$

$$\because c = d * b$$
$$c * a = (d * b) * a$$
$$= d * (b * a) \quad (\because associative)$$
$$c * a = d * b = c \rightarrow c * a = c$$

Also
$$c = d * b$$
➔ $c * c = (d * b) * b$
$$= d * (b * b)$$
$$d * a = d$$
⇨ $c * b = d$
Also $c = d * b$
⇨ $c * d = (d * b) * d$
$$= d * (b * d)$$
$$= d * d = b$$
⇨ $c * d = b$
So third row will be completed as

c	C	d	c	d

Q.7: What operation is represented by the adjoining table?

Name the identity element of the relevent set, if it exists. Is the operation associative?
Find the inverse of 0, 1, 2, 3. If they exist.

*	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

Solution:
i) The operation used the set of residue class mod 4 is '+'
ii) The identity element is zero.
∵o+0=0, 0+1=1, 0+2=2, 0+3=3
iii) The operation is associative e.g;
(1+2)+3=1+(2+3)
3+3=1+1
⇨ 2=2
Similarly, it can be verified for any other choice of elements.

iv) ∵ 1+3=3+1=0 1 and 3 are inverse of each other
$2 + 2 = 0$ *also*
0+0=0

Groups:
Grouped:

A non-empty set which is closed under given binary operation $*$ is closed is called grouped. It is denoted as (S,$*$)
Example:
The $\{E, 0\}$ is closed under addition.
$$\because E + E = E; \quad 0 + e = 0$$
$$E + 0 = 0; \quad 0 + 0 = E$$
$$\therefore \{E, 0\} is\ groupoid$$

Semi-Group:
A non-empty set is called semi group if
i) it is called under binary operation given.
ii) The binary operation is associative.

Example;
The set of natural numbers "N" under binary operation '+' is semi-group.
i) i.e.; B.O '+' is defined in N
ii) for any three elements
$$a, b, c \in N$$
$$(a + b) + c = a + (b + c)$$
ie.; associative law holds.
⇨ Both conditions for semi-group are satisfied.

Monoid:
A non- empty set is called Monoid.
i) It is closed w.r.t given binary operation $*$
ii) Binary operation $*$ is associative
iii) The set has identity element w.r.t Binary operation $*$

Example:
$$if\ Z' = \{0,1,2,3..\}$$
i) $Z' is\ closed\ w.r.t +$
ii) *binary operation+is associative.*
iii) o" *is idenetity element w*.r.t Binary operation"+"
$$\therefore given\ set\ is\ monoid.$$

Group:
A non -empty set G is called a group w.r.t Binary operation$*$ if
i) it is closed under binary operation $*$if
i.e; $\forall a, b \in G ; a * b \in G$
ii) Binary operation is associative
$\forall\ a, b \in G ; (a * b) * c = a * (b * c)$

iii) G has identity element w.r.t Binary operation$*$

i.e.; $\forall a \in G$ $there exist$ $e \in G$ $s.t$

$a * e = e * a$

$= a$ $then$ e is $identity$ $element$

iv) Every element of G has an inverse in G w.r.t Binary operation i.e.;

$where$ $a * a' = a' * a = e$

$where$ $a' \in G$ is $called$ $inverse$ of $a \in G$ $w.r.t$ Binary operation $*$

Abelian Group:

A group G under Binary operation $*$is called abelian group if binary operation is commutative.

$i.e$ $\forall a, b \in G;$

$a * b = b * a$

Finite Group:

A group G having finite number of elements is called finite group.

Infinite Group:

A group G having infinite number of element is called infinite group.

Left cancellation Law:

If a, b, c are elements of group G then

$ab = ac \Rightarrow b = c$

Proof:

$ab = ac$

⇨ $a^{-1}(ab) = a^{-1}(ac)$

⇨ $(a^{-1}a)b = (a^{-1}a)c$ $(\because$ $associative$ $law)$

⇨ $eb = ec$ $(\because a^{-1}a = e)$

⇨ $b = c$ $proved$

Right Cancellation Law

if a, b, c are $element$ of $group$ G $then$

$ba = ca \Rightarrow b = c$

Proof:

$ba = ca$

⇨ $(ba)a^{-1} = (ca)a^{-1}$

⇨ $b(aa^{-1}) = c(aa^{-1})$ $(\because$ $assocaitive$ $law.$

⇨ $be = ce$ $(\because aa^{-1} = e)$

⇨ $b + c$ Proved.

Reversal Law of inverses:

If a, b are elements of a group G, then show that $(ab)^{-1} = b^{-1}a^{-1}$

Proof :

$(ab)(b^{-1}a^{-1}) = a(bb^{-1})a^{-1}$ $(asoc. law)$

$= aea^{-1}$

$= aa^{-1}$

$= e$

$also$ $(b^{-1}a^{-1})(ab) = b^{-1}(a^{-1}a)b$

$= b^{-1}beb$

$= b^{-1}b$

$= e$

⇨ ab and $b^{-1}a^{-1}$ are $inverse$ of $each$ $other.$

⇨ $inverse$ of ab is $b^{-1}a^{-1}$

i.e$(ab)^{-1} = b^{-1}a^{-1}$

Solution of linear Equation:

a, b $being$ $elements$ of a $group$ $G, solve$ the following equations:

Solution:

i) $ax = b$

⇨ $a^{-1}(ax) = a^{-1}b$

⇨ $(a^{-1}a)x = a^{-1}b$ $(assoc. law)$

⇨ $ex = a^{-1}b$ $(\because a^{-1}a = e)$

⇨ $x = a^{-1}b$

ii) $xa = b$

⇨ $(xa)a^{-1} = ba^{-1}$

⇨ $x(aa^{-1}) = ba^{-1}$ $(Assoc. Law)$

⇨ $xe = ba^{-1}$

⇨ $x = b\ a^{-1}$

Exercise 2.8

Q.1:

Operation '+' performed on the two member set $G = \{0, 1\}$ is shown in the adjoinig table.

i) Name the identity element , if it exists ?

ii) What is the inverse of 1 ?

iii) Is the set G, under the given operation a group ?

Abelian or non − abelian ?

SOLUTION:

i) Name the identity element , if it exists ?

⊕	0	1
0	**0**	**1**
1	**1**	**0**

Here 0 is identity in G.

ii) What is the inverse of 1 ?

$\because$ **1 + 1 = 0** (***i.e identity element***))

so inverse of 1 is 1

iii) Is the set G, under the given operation a group ?

Abelian or non − abelian ?

- G is $closed$ $under$ "+"
- G is associative w.r.t "+"
- $o \in G$ is $identity$ $w.r.t$ $+$
- $inverse$ of $each$ $element$ $exist$ in $G.$
  i.e$0 + 0 = 0$ so $o^{-1} = 0$
  $1 + 1 = 0$ $so 1^{-1} = 1$
- Commutative law hold in G
  $i.e$ $1 + 0 = 0 + 1$

⇨ So G is abelian Group under "+"

Q.2:
The opperation ⊕as performed on the set {0, 1, 2, 3}is shown in the adjoining table, show that set is an abelian group.

⊕	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

i) S is closed under "+" (it is clear from table)
ii) It is clear the set of Associative w.r.t"+"
iii) "o" is identity element.
iv) Each element has inverse

$\because 1+3=3+1=0$ *and*

$0+0=0$ *and* $2+2=0$

v) $\forall\ 1{,}0 \in G=\{0{,}1{,}2{,}3\}, 1+0=0+1=1$

G is abelian

Q.3: For each of the following sets, determine whether or not the set forms a group w.r.t indicated operation.

i)

The set of rational numbers under the operation '×'

SOLUTION:

Q = *set of rational no'sis not a group w.r.t '×'*

$\because$ *inverse of O w.r.t xdoes not exist.*

ii) The set of rational no's "+"

Solution:

$(Q,+)$*is a group.*

iii) The set of $+ve$ rational numbers" × "

Solution:

It is group w.r.t " × "

iv) The set of integers s" + "

Solution:

It is group w.r.t "+"

v) The set of integers " × "

Solution:

It is not group

$\because$ *inverse of* '0' does not exist.

Q.4: Shown in the adjoining table represents the sums of the elements of the set{E, O}. What is the identity element of the set? Show that this set is an abelian group.

⊕	E	O
E	E	O
O	O	E

Solution :

i) $E+E=E\ (even)$

$E+O=O(odd)$

$O+O=E(even)$

Here E is identity element.

ii) Table show the set satisfies the closure law w.r.t "+" ∵ all elements of table ∈ $\{E, o\}$

- The set is associative law w.r.t "+"

$(0+E)+0=0+(E+0)$

$0+0=0+0$

$E=E$

- *Eis identity* $\in \{E, 0\}$
- *each element has inverse* $(0+e=E+0$ *and* $E+E=0$*and* $0+0=0)$
- *commuttive law holds* $(0+E=E+0)$

So set $\{E, 0\}$ is abelian group.

Q5.

Show that the set $\{1, w, w^2\}$ ***, when*** $w^3=1$ ***is an abelian group w.r.t ordinary multiplication.***

Solution:

Let $S=\{1, w, w^2\}$ *, where* $w^3=1$

i) Clearly from table S is close " × "

ii) $1, w, w^2 \in S$

$(1.w).w^2=1.(w.w^2)$

$w.w^2=1.w^3$

$w^3=w^3$

⇨ $1=1$

$\therefore$ *Assoc. Law hold under* ×

iii) 1 is identity element under " × "

iv) Inverse of each element exists.

$as\ 1\times 1=1 \Rightarrow 1^{-1}=1$

$w\times w^2=1 \Rightarrow w^{-1}=w^2$

$W^2\times w=1 \Rightarrow (w^2)^{-1}=w$

v) Commutative law holds under"× "

(clear from table)

⇨ S is an abelian group under " × "

Q.6: If G is a group under the operation ∗ and a, b ϵG, find the solution of the equations : a ∗ x = b , x ∗ a = b.

SOLUTION:

Given that G is a group under the operation ∗ .

Since $a \in G$ so $a^{-1} \in G$

i) $a * x = b$

Pre multiplying by a^{-1}

$a^{-1} * (a * x) = a^{-1} * b$

$(a^{-1} * a) * x = a^{-1} * b$

$e * x = a^{-1} * b$

$x = a^{-1} * b$

ii) $x * a = b$

Post multiplying by a^{-1}

$(x * a) * a^{-1} = b * a^{-1}$

$x * (a * a^{-1}) = b * a^{-1}$

$x * e = b * a^{-1}$

$x = b * a^{-1}$

Q.7

Q.7: Show that set consisting of elements of the from $a + \sqrt{3}b$ (a, b being a rational), is an abelian group with respect to addition.

Solution:

$$let\ G = \{a + b\sqrt{3}\ |a, b \in Q\}$$

i) G is closed w.r.t "+"

Let $x = a + b\sqrt{3}$

$a, b, c, d \in Q$

$y = c + d\sqrt{3}$

so $x, y \in G$

now

$$x + y = (a + b\sqrt{3}) + (c + d\sqrt{3})$$
$$= (a + c) + \sqrt{3}(b + d) \in G$$
$$\because (a + C), (b + d) \in Q$$

ii) Associative law w.r.t.'+" hold in G

let $x = a + b\sqrt{3}$ $y = c + d\sqrt{3}$

and $z = e + f\sqrt{3}$

Then $x, y, z \in G$

$$x + (y + z) = (a + b\sqrt{3}) + (c + d\sqrt{3} + e + f\sqrt{3})$$
$$= a + b\sqrt{3} + [(c + e) + \sqrt{3}(d + f)]$$
$$= (a + c + e) + \sqrt{3}(b + d + f)$$
$$= [(a + c) + \sqrt{3}\ (b + d + f)]$$
$$= [(a + c) + \sqrt{3}(b + d)] + (e + f\sqrt{3})$$
$$= (x + y) + z$$

iii) $0 + \sqrt{3}\,0 \in G$ is identity w.r.t. " + "

iv) for $(a + \sqrt{3}\,b) \in G$, thereexist

$$-a - \sqrt{3}b \in G$$
$$\because (a + \sqrt{3}b) + (-a - \sqrt{3}b) = 0 + \sqrt{3}0$$

$\otimes$	1	w	w^2
1	1	w	w^2
w	w	w^2	1
w^3	w^2	1	w

i.e inverse of each element of G exists.

v) Commutative law w.r.t "+" holds inG.

i.e $x + y = y + x\ \forall x, y \in G$

As

$$x + y = (a + b\sqrt{3}) + (c + d\sqrt{3}$$
$$x + y = (a + c) + \sqrt{3}(b + d)$$
$$= (c + a) + \sqrt{3}(d + b)$$
$$= y + x$$

⇨ G is Abelian group.

Q.8: Determine whether $(P(S),*)$, where $*$ stands for intersection is a semi − group, a monoid, or neither. If it is monoid specify its identity.

SOLUTION:

Given that

$$P(S) = power\ set\ of\ set\ S$$
$$here * means\ \cap$$

i) **$P(S)$ = is closed w.r.t $*$**

Let $S_1, S_2 \in P(S)$

then $S_1 * S_2\ P(S)$

i.e $S_1 \cap S_2 \in P(S)$

ii) **$P(S)$ holds associative law w.r.t $*$**

let $S_1, S_2 \in P(S)$

then $S_1 * (S_2 * S_3) = (S_1 * S_2) * S_3$

⇨

$S_1 \cap (S_2 \cap S_3) = (S_1 \cap S_2) \cap S_3$

$P(S)$ has no identity element

So P(S) is semi group under $*$ but P(S) is not monoid under$*$

Q.9: Complete the following table to obtain a semi − group under ' $*$ '

$*$	a	b	c
A	c	a	b
B	a	b	c
C	-	-	a

Solution:

From table

$$a * a = c \rightarrow (i)$$

Now

$$c * a = (a * a) * a \quad by(i)$$
$$= a * (a * a) \quad (Assoc. Law.)$$
$$= a * c$$
$$= b \quad from\ table$$

⇨ $c * a = b$

Also

$$c * b = (a * a) * b \quad by\ (i)$$
$$= a * (a * b)$$
$$= a * a \quad from\ table$$
$$= c \quad by\ (i)$$

⇨ $c * b = c$

⇨ So the third row becomes as.

c	b	c	a

Q.No10

See example at page 46#solved.