Chapter 14

SOLUTIONS OF TRIGONOMETRIC EQUATIONS

EXERCISE 14.1

Trigonometric Equations:

The equations containing at least one trigonometric functions are called trigonometric equations.

e.g., $\sin x = \frac{2}{5}$, $\sec x = \tan x$

Q.1 Find the solutions of the following equation which lie in $[0, 2\pi]$

(i) $\sin x = \frac{-\sqrt{3}}{2}$

(ii) $\operatorname{cosec} \theta = 2$ **(Gujranwala Board 2005, Lahore Board 2006)**

(iii) $\sec x = -2$

(iv) $\cot \theta = \frac{1}{\sqrt{3}}$ **(Lahore Board 2010)**

Solution:

(i) $\sin x = \frac{-\sqrt{3}}{2}$

$$x = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = 60^{o} = \frac{\pi}{3}$$

Since $\sin x$ is –ve in III & IV Quadrants with the reference angle $\pi/3$ thus we have

For III-Quadrant

$x = \pi + \theta$

$x = \pi + \frac{\pi}{3}$,

$x = \frac{4\pi}{3}$,

For IV-Quadrant

$x = 2\pi - \theta$

$x = 2\pi - \frac{\pi}{3}$

$x = \frac{5\pi}{3}$

So thus the required solution is $x = \frac{4\pi}{3}$, $\frac{5\pi}{3}$

(ii) $\mathbf{cosec\,\theta = 2}$

$$\frac{1}{\sin\theta} = 2$$

$$\Rightarrow \quad \sin\theta = \frac{1}{2}$$

$$\boxed{\theta = \sin^{-1}\left(\frac{1}{2}\right) = 30^{o} = \frac{\pi}{6}}$$

Since $\sin\theta$ is +ve in I and II Quadrants with reference angle $\frac{\pi}{6}$ Thus

For I-Quadrant		**For II-Quadrant**
$x = \frac{\pi}{6}$	,	$x = \pi - \theta$
		$x = \pi - \frac{\pi}{6}$
		$x = \frac{5\pi}{6}$

So thus the required solution is $x = \frac{\pi}{6}\,,\ \frac{5\pi}{6}$.

(iii) $\mathbf{sec\,x = -2}$

$$\frac{1}{\cos x} = -2 \quad \Rightarrow \quad \cos x = -\frac{1}{2}$$

$$\boxed{x = \cos^{-1}\left(\frac{1}{2}\right) = 60^{o} = \frac{\pi}{3}}$$

Since $\cos x$ is –ve in II & III Quadrants with reference angle $\frac{\pi}{3}$ thus we have

For II-Quadrant		**For III-Quadrant**
$x = \pi - \theta$	,	$x = \pi + \theta$
$x = \pi - \frac{\pi}{3}$	,	$x = \pi + \frac{\pi}{3}$
$x = \frac{2\pi}{3}$	,	$x = \frac{4\pi}{3}$

Thus the required solution is $x = \frac{2\pi}{3}\,,\ \frac{4\pi}{3}$

(iv) $\mathbf{cot\,\theta = \frac{1}{\sqrt{3}}}$

$$\tan\theta = \sqrt{3}$$

$$\boxed{\theta = \tan^{-1}\left(\sqrt{3}\right) = 60^{o} = \frac{\pi}{3}}$$

For I-Quadrant

$x = \frac{\pi}{3}$

For III-Quadrant

$x = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$

Thus the required solution is $x = \frac{\pi}{3}, \frac{4\pi}{3}$ Ans.

Q.2 Solve the following trigonometric equations:

(i) $\tan^2\theta = \frac{1}{3}$ **(ii)** $\text{cosec}^2\theta = \frac{4}{3}$

(iii) $\sec^2\theta = \frac{4}{3}$ **(iv)** $\cot^2\theta = \frac{1}{3}$ **(Lahore Board 2007)**

Solution:

(i) $\tan^2\theta = \frac{1}{3}$

$\tan\theta = \pm\frac{1}{\sqrt{3}}$

$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$ and $\tan\theta = \frac{-1}{\sqrt{3}}$

$$\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30^o = \frac{\pi}{6}$$

Since $\tan\theta$ is +ve in I & III Quadrants with reference angle $\frac{\pi}{6}$

Therefore

For I-Quad

$\theta = \frac{\pi}{6}$,

$\theta = \frac{\pi}{6} + n\pi$,

For III-Quad

$\theta = \pi + \frac{\pi}{6}$

$\theta = \frac{7\pi}{6}$

$\theta = \frac{7\pi}{6} + n\pi$

Since $\tan\theta$ is –ve in II & IV Quadrants with reference angle $\frac{\pi}{6}$

For II-Quad

$\theta = \pi - \frac{\pi}{6}$,

$\theta = \frac{5\pi}{6}$,

$\theta = \frac{5\pi}{6} + n\pi$,

For IV-Quad

$\theta = 2\pi - \frac{\pi}{6}$

$\theta = \frac{11\pi}{6}$

$\theta = \frac{11\pi}{6} + n\pi \quad \forall n \in z$

$$\left\{\frac{\pi}{6} + n\pi\right\} \cup \left\{\frac{7\pi}{6} + n\pi\right\} \cup \left\{\frac{5\pi}{6} + n\pi\right\} \cup \left\{\frac{11\pi}{6} + n\pi\right\}, \forall n \in z$$

(ii) $\text{cosec}^2\theta = \frac{4}{3}$

$$\theta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = 60° = \frac{\pi}{3}$$

$\text{cosec}\,\theta = \pm\frac{2}{\sqrt{3}}$

$\text{cosec}\,\theta = \frac{2}{\sqrt{3}}$

$\sin\theta = \frac{\sqrt{3}}{2}$

Since $\sin\theta$ is +ve in I & II Quadrants with reference angle $\frac{\pi}{3}$

$\text{cosec}\,\theta = \frac{-2}{\sqrt{3}}$

$\sin\theta = \frac{-\sqrt{3}}{2}$

Since $\sin\theta$ is –ve in III & IV Quadrants with reference angle $\frac{\pi}{3}$

For I-Quad **For II-Quad** | **For III-Quad** **For IV-Quad**

$\theta = \frac{\pi}{3}$, $\theta = \pi - \frac{\pi}{3}$ | $\theta = \pi + \frac{\pi}{3}$, $\theta = 2\pi - \frac{\pi}{3}$

$\theta = \frac{\pi}{3} + 2n\pi$ $\theta = \frac{2\pi}{3} + 2n\pi$ | $\theta = \frac{4\pi}{3} + 2n\pi$, $\theta = \frac{5\pi}{3} + 2n\pi$

$$S.S = \left\{\frac{\pi}{3} + 2n\pi\right\} \cup \left\{\frac{2\pi}{3} + 2n\pi\right\} \cup \left\{\frac{4\pi}{3} + 2n\pi\right\} \cup \left\{\frac{5\pi}{3} + 2n\pi\right\}, \forall n \in z \quad \text{Ans.}$$

(iii) $\sec^2\theta = \frac{4}{3}$ $\boxed{\theta = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = 30° = \frac{\pi}{6}}$

$$\sec\theta = \pm\frac{2}{\sqrt{3}}$$

$\sec\theta = \frac{2}{\sqrt{3}}$ | $\sec\theta = \frac{-2}{\sqrt{3}}$

$\Rightarrow$ $\cos\theta = \frac{\sqrt{3}}{2}$ | $\cos\theta = \frac{-\sqrt{3}}{2}$

Since $\cos\theta$ is +ve in I & IV Quadrants, with reference angle $\frac{\pi}{6}$ therefore we have

For I-Quad **For IV-Quad**

$\theta = \frac{\pi}{6}$, $\theta = 2\pi - \frac{\pi}{6}$

$\theta = \frac{\pi}{6} + 2n\pi$, $\theta = \frac{11\pi}{6} + 2n\pi$

Since $\cos\theta$ is –ve in II & III Quadrants with reference angle $\frac{\pi}{6}$ therefore we have

For II-Quad **For III-Quad**

$\theta = \pi - \frac{\pi}{6}$, $\theta = \pi + \frac{\pi}{6}$

$\theta = \frac{5\pi}{6} + 2n\pi$, $\theta = \frac{7\pi}{6} + 2n\pi$

$$S.S = \left\{\frac{\pi}{6} + 2n\pi\right\} \cup \left\{\frac{4\pi}{6} + 2n\pi\right\} \cup \left\{\frac{5\pi}{6} + 2n\pi\right\} \cup \left\{\frac{7\pi}{6} + 2n\pi\right\}, \forall n \in z \quad \text{Ans.}$$

(iv) $\cot^2\theta = \dfrac{1}{3}$ **(Lahore Board 2007)**

$\Rightarrow \quad \cot\theta = \pm\dfrac{1}{\sqrt{3}}$

$\tan\theta = \pm\sqrt{3}$

$$\boxed{\theta = \tan^{-1}(\sqrt{3}) = 60^\circ = \frac{\pi}{3}}$$

$\tan\theta = \sqrt{3}$

Since $\tan\theta$ is +ve in I & III

Quadrants with reference angle $\dfrac{\pi}{3}$

therefore we have

For I-Quad **For III-Quad**

$\theta = \dfrac{\pi}{3}$, $\theta = \pi + \dfrac{\pi}{3}$

$\theta = \dfrac{\pi}{3} + n\pi$, $\theta = \dfrac{4\pi}{3} + n\pi$

$\tan\theta = -\sqrt{3}$

Since $\tan\theta$ is –ve in II & IV

Quadrants, with reference angle $\dfrac{\pi}{3}$

therefore we have

For II-Quad **For IV-Quad**

$\theta = \pi - \dfrac{\pi}{3}$, $\theta = 2\pi - \dfrac{\pi}{3}$

$\theta = \dfrac{2\pi}{3} + n\pi$, $\theta = \dfrac{5\pi}{3} + n\pi$

$$\text{S.S} = \left\{\frac{\pi}{3} + n\pi\right\} \cup \left\{\frac{4\pi}{3} + n\pi\right\} \cup \left\{\frac{2\pi}{3} + n\pi\right\} \cup \left\{\frac{5\pi}{3} + n\pi\right\}, \forall n \in z \text{ Ans.}$$

Q.3 Find the values of θ satisfying the following equations:

$3\tan^2\theta + 2\sqrt{3}\tan\theta + 1 = 0$ (Gujranwala Board 2006)

Solution:

$3\tan^2\theta + 2\sqrt{3}\tan\theta + 1 = 0$

$a = 3, \quad b = 2\sqrt{3}, \quad c = 1$ by quadratic formula

$$\tan\theta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2\sqrt{3} \pm \sqrt{(2\sqrt{3})^2 - 4(3)(1)}}{2(3)}$$

$$= \frac{-2\sqrt{3} \pm \sqrt{12 - 12}}{6} = \frac{-2\sqrt{3}}{6} = \frac{-1}{\sqrt{3}}$$

$\Rightarrow \quad \tan\theta = -\dfrac{1}{\sqrt{3}}$

$$\boxed{\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30^\circ = \frac{\pi}{6}}$$

Since $\tan\theta$ is –ve in II & IV Quadrants, with reference angle $\dfrac{\pi}{6}$ therefore we have

For II-Quad **For IV-Quad**

$\theta = \pi - \frac{\pi}{6}$, $\theta = 2\pi - \frac{\pi}{6}$

$\theta = \frac{5\pi}{6} + n\pi$, $\theta = \frac{11\pi}{6} + n\pi$

$S.S = \left\{\frac{5\pi}{6} + n\pi\right\} \cup \left\{\frac{4\pi}{6} + n\pi\right\}, \forall n \in z$ Ans.

Q.4 $\tan^2\theta - \sec\theta - 1 = 0$

Solution:

$\tan^2\theta - \sec\theta - 1 = 0$

$\sec^2\theta - 1 - \sec\theta - 1 = 0$

$\sec^2\theta - \sec\theta - 2 = 0$

$\sec^2\theta - 2\sec\theta + \sec\theta - 2 = 0$

$\sec\theta(\sec\theta - 2) + 1(\sec\theta - 2) = 0$

$(\sec\theta - 2)(\sec\theta + 1) = 0$

$\sec\theta = 2$

$\cos\theta = \frac{1}{2}$

Since $\cos\theta$ is +ve in I & IV Quadrants

With reference angle $\frac{\pi}{3}$

For I-Quad. **For IV-Quad.**

$\theta = \frac{\pi}{3}$, $\theta = 2\pi - \frac{\pi}{3}$

$\theta = \frac{\pi}{3} + 2n\pi$, $\theta = \frac{5\pi}{3} + 2n\pi$

$\sec\theta = -1$

$\cos\theta = -1$

$\Rightarrow \theta = \cos^{-1}(-1)$

$\theta = \cos^{-1}(-1)$

$\theta = \pi + 2n\pi$

$S.S = \left\{\frac{\pi}{3} + 2n\pi\right\} \cup \left\{\frac{5\pi}{3} + 2n\pi\right\} \cup \{(2n+1)\pi\}, \forall n \in z$ Ans.

Q.5 $2\sin\theta + \cos^2\theta - 1 = 0$

Solution:

$2\sin\theta + 1 - \sin^2\theta - 1 = 0$

$2\sin\theta - \sin^2\theta = 0$

$\sin\theta(2 - \sin\theta) = 0$

$\Rightarrow \quad \sin\theta = 0 \;,\; 2 - \sin\theta = 0$

$\sin\theta = 0$

$\Rightarrow \quad \theta = n\pi$

$\sin\theta = 2$

$2 - \sin\theta = 0$

$\sin\theta = 0$

Which is not possible because $-1 \le \sin\theta \le 1$

$S.S = \{n\pi, \forall\, n \in z\}$

Q.6 $\quad \mathbf{2\sin^2\theta - \sin\theta = 0}$

Solution:

$\sin\theta(2\sin\theta - 1) = 0$

$\Rightarrow \quad \sin\theta = 0 \text{ and } 2\sin\theta - 1 = 0$

$2\sin\theta = 1$

$\sin\theta = 1/2$

$\sin\theta = 0$

$\Rightarrow \quad \theta = n\pi$

$\sin\theta = 1/2$

Since $\sin\theta$ is positive in I and II quadrants with the reference angle $\pi/6$.

I quad.

$q = \pi/6 + 2n\pi \;, \forall\, n \in z$

II quad.

$\theta = \pi - \pi/6$

$\theta = 5\pi/6$

$\theta = 5\pi/6 + 2n\pi \;, \forall\, n \in z$

$S.S = \{n\pi\} \cup \{\pi/6 + 2n\pi\} \cup \{5\pi/6 + 2n\pi\} \;, \forall\, n \in z$

Q.7 $\quad \mathbf{3\cos^2\theta - 2\sqrt{3}\,\sin\theta\cos\theta - 3\sin^2\theta = 0}$

Solution:

$3\cos^2\theta - 2\sqrt{3}\,\sin\theta\cos\theta - \sqrt{3}\,\sin\theta\cos\theta - 3\sin^2\theta = 0$

$3\cos\theta(\cos\theta - \sqrt{3}\,\sin\theta) + \sqrt{3}\,\sin\theta\,(\cos\theta - \sqrt{3}\,\sin\theta) = 0$

$(\cos\theta - \sqrt{3}\,\sin\theta)\,(3\cos\theta + \sqrt{3}\,\sin\theta) = 0$

$\cos\theta - \sqrt{3}\,\sin\theta = 0$

$\cos\theta = \sqrt{3}\,\sin\theta$

$\tan\theta = \dfrac{1}{\sqrt{3}}$

Since $\tan\theta$ is positive in I to III quadrants with the reference angle $\pi/6$

I quad.

$\theta = \pi/6 + n\pi$

III quad

$\theta = \pi + \pi/6$

$\theta = 7\pi/6$

$\theta = 7\pi/6 + n\pi$

$3\cos\theta + \sqrt{3}\,\sin\theta = 0$

$\sqrt{3}\,\sin\theta = -3\cos\theta$

$\tan\theta = -\sqrt{3}$

Since $\tan\theta$ is negative in III and IV quadrants with the reference angle $\pi/3$.

I quad.

$\theta = \pi - \pi/3$

$\theta = 2\pi/3$

$\theta = 2\pi/3 + n\pi$

III quad

$\theta = 2\pi - \pi/3$

$\theta = 5\pi/3$

$\theta = 5\pi/3 + n\pi$

$S.S = \{\pi/6 + n\pi\} \cup \{7\pi/6 + n\pi\} \cup \{2\pi/3 + n\pi\} \cup \{5\pi/3 + n\pi\} \;, \forall\, n \in z$

Q.8 Find the values of θ $4\sin^2\theta - 8\cos\theta + 1 = 0$

Solution:

$$4\sin^2\theta - 8\cos\theta + 1 = 0$$
$$4(1-\cos^2\theta) - 8\cos\theta + 1 = 0$$
$$4 - 4\cos^2\theta - 8\cos\theta + 1 = 0$$
$$4\cos^2\theta + 8\cos\theta - 5 = 0$$
$$4\cos^2\theta + 10\cos\theta - 2\cos\theta - 5 = 0$$
$$2\cos\theta(2\cos\theta + 5) - 1(2\cos\theta + 5) = 0$$
$$(2\cos\theta + 5)(2\cos\theta - 1) = 0$$

$$2\cos\theta + 5 = 0$$
$$\cos\theta = \frac{-5}{2}$$

i.e. solution is impossible.

$$2\cos\theta - 1 = 0$$
$$\cos\theta = \frac{1}{2} \qquad \boxed{\theta = \cos^{-1}\left(\frac{1}{2}\right) = 60^o = \frac{\pi}{3}}$$

Since $\cos\theta$ is +ve in I & IV Quadrants with reference angle $\frac{\pi}{3}$

For I-Quad. $\theta = \frac{\pi}{3}$, **For IV-Quad.** $\theta = 2\pi - \frac{\pi}{3}$

$\theta = \frac{\pi}{3} + 2n\pi$, $= \frac{5\pi}{3} + 2n\pi$

$$S.S = \left\{\frac{\pi}{3} + 2n\pi\right\} \cup \left\{\frac{5\pi}{3} + 2n\pi\right\}, \forall\ n \in z \text{ Ans.}$$

Q.9 Find the solution set of the following equations.

$\sqrt{3}\tan x - \sec x - 1 = 0$ (Gujranwala Board 2004)

Solution:

$$\sqrt{3}\tan x = 1 + \sec x$$
$$(\sqrt{3}\tan x)^2 = (1+\sec x)^2$$
$$3\tan^2 x = 1 + \sec^2 x + 2\sec x$$
$$3(\sec^2 x - 1) = 1 + \sec^2 x + 2\sec x$$
$$3\sec^2 x - 3 - 1 - \sec^2 x - 2\sec x = 0$$
$$2\sec^2 x - 2\sec x - 4 = 0$$
$$\sec^2 x - \sec x - 2 = 0$$

Q.8 Find the values of θ $4\sin^2\theta - 8\cos\theta + 1 = 0$

Solution:

$4\sin^2\theta - 8\cos\theta + 1 = 0$

$4(1 - \cos^2\theta) - 8\cos\theta + 1 = 0$

$4 - 4\cos^2\theta - 8\cos\theta + 1 = 0$

$4\cos^2\theta + 8\cos\theta - 5 = 0$

$4\cos^2\theta + 10\cos\theta - 2\cos\theta - 5 = 0$

$2\cos\theta(2\cos\theta + 5) - 1(2\cos\theta + 5) = 0$

$(2\cos\theta + 5)(2\cos\theta - 1) = 0$

$2\cos\theta + 5 = 0$

$\cos\theta = \frac{-5}{2}$

i.e. solution is impossible.

$2\cos\theta - 1 = 0$

$\cos\theta = \frac{1}{2}$ $\quad$ $\boxed{\theta = \cos^{-1}\left(\frac{1}{2}\right) = 60^{\circ} = \frac{\pi}{3}}$

Since $\cos\theta$ is +ve in I & IV Quadrants with reference angle $\frac{\pi}{3}$

For I-Quad.

$\theta = \frac{\pi}{3}$,

$\theta = \frac{\pi}{3} + 2n\pi$,

For IV-Quad.

$\theta = 2\pi - \frac{\pi}{3}$

$= \frac{5\pi}{3} + 2n\pi$

$$S.S = \left\{\frac{\pi}{3} + 2n\pi\right\} \cup \left\{\frac{5\pi}{3} + 2n\pi\right\}, \forall n \in z \text{ Ans.}$$

Q.9 Find the solution set of the following equations.

$\sqrt{3}\tan x - \sec x - 1 = 0$ (Gujranwala Board 2004)

Solution:

$\sqrt{3}\tan x = 1 + \sec x$

$(\sqrt{3}\tan x)^2 = (1 + \sec x)^2$

$3\tan^2 x = 1 + \sec^2 x + 2\sec x$

$3(\sec^2 x - 1) = 1 + \sec^2 x + 2\sec x$

$3\sec^2 x - 3 - 1 - \sec^2 x - 2\sec x = 0$

$2\sec^2 x - 2\sec x - 4 = 0$

$\sec^2 x - \sec x - 2 = 0$

By quadratic formula

$$\sin x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\sin x = \frac{-2 \pm \sqrt{4 - 4(4)(-1)}}{2(4)} = \frac{-2 \pm \sqrt{4 + 16}}{8}$$

$$\sin x = \frac{-2 \pm \sqrt{20}}{8}$$

$$x = \sin^{-1}\left(\frac{\sqrt{20} - 2}{8}\right)$$

$$x = 18^{o} = \frac{\pi}{10}$$

Since sin x is +ve in the I and II Quadrants

with reference angle $\frac{\pi}{10}$

For I-Quad. **For II-Quad.**

$$x = \frac{\pi}{10} \quad , \quad x = \pi - \frac{\pi}{10}$$

$$x = \frac{\pi}{10} + 2n\pi \, , \, x = \frac{9\pi}{10} + 2n\pi$$

$$\sin x = \frac{-\sqrt{20} - 2}{8} = -\left(\frac{\sqrt{20} + 2}{8}\right)$$

$$x = \sin^{-1}\left(\frac{\sqrt{20} + 2}{8}\right)$$

$$x = 54^{o} = \frac{3\pi}{10}$$

Since sin x is –ve in III & IV Quadrants

with reference angle $\frac{3\pi}{10}$

For III-Quad. **For IV-Quad.**

$$x = \pi + \frac{3\pi}{10} \quad , \quad x = 2\pi - \frac{3\pi}{10}$$

$$x = \frac{13\pi}{10} + 2n\pi \quad , \quad x = \frac{17\pi}{10} + 2n\pi$$

Also $\sin x = 1$

$$x = \frac{\pi}{2} + 2n\pi \, , \, \forall \, n \in z$$

Hence solution set is

$$\left\{\frac{\pi}{2} + 2n\pi\right\} \cup \left\{\frac{\pi}{10} + 2n\pi\right\} \cup \left\{\frac{9\pi}{10} + 2n\pi\right\} \cup \left\{\frac{13\pi}{10} + 2n\pi\right\} \cup \left\{\frac{17\pi}{10} + 2n\pi\right\} \quad n \in z \quad \text{Ans}$$

Q.11 $\sec 3\theta = \sec \theta$

Solution:

$$\sec 3\theta = \sec\theta$$

$$\frac{1}{\cos 3\theta} = \frac{1}{\cos\theta}$$

$$\cos 3\theta = \cos\theta$$

$$\cos 3\theta - \cos\theta = 0$$

$$-2\sin\left(\frac{3\theta+\theta}{2}\right)\sin\left(\frac{3\theta-\theta}{2}\right) = 0$$

$$-2\sin 2\theta \sin\theta = 0$$

$\Rightarrow \quad \sin 2\theta \sin\theta = 0$

$\Rightarrow \quad \sin 2\theta = 0$ | $\sin\theta = 0$

$\Rightarrow \quad 2\theta = n\pi$ | $\Rightarrow \quad \theta = n\pi$

$\Rightarrow \quad \theta = \dfrac{n\pi}{2}$

Solution set is $\left\{\dfrac{n\pi}{2}\right\} \cup \{n\pi\}$, $n \in Z$ Ans.

Q.12 $\tan 2\theta + \cot\theta = 0$

Solution:

$$\tan 2\theta = -\cot\theta$$

$$\frac{\sin 2\theta}{\cos 2\theta} = -\frac{\cos\theta}{\sin\theta}$$

$$\sin 2\theta \sin\theta = -\cos 2\theta \cos\theta$$

$$\sin 2\theta \sin\theta + \cos 2\theta \cos\theta = 0$$

$\Rightarrow \quad \cos(2\theta - \theta) = 0$

$\Rightarrow \quad \cos\theta = 0$

$\Rightarrow \quad \theta = (2n+1)\dfrac{\pi}{2}$

Solution set is $\left\{(2n+1)\dfrac{\pi}{2}\right\}$, $n \in z$

Q.13 $\sin 2x + \sin x = 0$

Solution:

$$\sin 2x + \sin x = 0$$

$$2\sin x \cos x + \sin x = 0$$

$$\sin x(2\cos x + 1) = 0$$

$\sin x = 0$

$x = n\pi$

$2\cos x + 1 = 0$

$\cos x = \dfrac{-1}{2}$

As $\cos x$ is +ve in I & IV Quadrants with reference angle $\dfrac{\pi}{3}$

For I-Quad. | **For II-Quad.**

$x = \dfrac{\pi}{3}$, | $x = 2\pi - \dfrac{\pi}{3}$

$x = \dfrac{\pi}{3} + 2n\pi$, | $x = \dfrac{5\pi}{3} + 2n\pi$

Therefore solution set is

$$\{n\pi\} \cup \left\{\frac{\pi}{3} + 2\,n\,\pi\right\} \cup \left\{\frac{5\,\pi}{3} + 2\,n\,\pi\right\}, \quad n \in z$$

Q.14 sin 4 x – sin 2 x = cos 3 x

Solution:

$$\sin 4\,x - \sin 2\,x = \cos 3\,x$$

$$2\cos\left(\frac{4\,x + 2\,x}{2}\right)\sin\left(\frac{4x - 2x}{2}\right) = \cos 3\,x$$

$$2\cos 3x \sin x - \cos 3\,x = 0$$

$$\cos 3\,x\,[2\sin x - 1] = 0$$

$\cos 3x = 0$

$\Rightarrow \quad 3x = (2n + 1)\frac{\pi}{2}$

$\Rightarrow \quad x = (2n + 1)\frac{\pi}{6}$

$2\sin x - 1 = 0$

$\Rightarrow \quad 2\sin x = 1$

$\Rightarrow \quad \sin x = \frac{1}{2}$ $\quad \boxed{x = \sin^{-1}\left(\frac{1}{2}\right) = 30^{o} = \frac{\pi}{6}}$

Since sin x is +ve in I and II Quadrants

with reference angle $\frac{\pi}{6}$ so

For I-Quad.	**For II-Quad.**
$x = \frac{\pi}{6}$	$x = \pi - \frac{\pi}{6}$
$x = \frac{\pi}{6} + 2n\pi$,	$x = \frac{5\,\pi}{6} + 2n\pi$, $\forall n \in z$

Hence solution set is

$$\left\{(2n + 1)\frac{\pi}{6}\right\} \cup \left\{\frac{\pi}{6} + 2\,n\,\pi\right\} \cup \left\{\frac{5\,\pi}{6} + 2\,n\,\pi\right\}, n \in Z$$ Ans.

Q.15 sin x + cos 3 x = cos 5 x

Solution:

$$\sin x + \cos 3\,x = \cos 5\,x$$

$$\sin x = \cos 5\,x - \cos 3\,x$$

$$\sin x = -2\sin\left(\frac{5x + 3x}{2}\right)\sin\left(\frac{5x - 3x}{2}\right)$$

$$= -2\sin 4\,x \sin x$$

$\sin x + 2\sin 4x \sin x = 0$

$\sin x(1 + 2\sin 4x) = 0$

$\sin x = 0$

$x = n\pi$

$\sin 4x = -\frac{1}{2} \Rightarrow$ $\boxed{4x = \sin^{-1}\left(\frac{1}{2}\right) = 30^\circ = \frac{\pi}{6}}$

since sin x is –ve in III & IV Quadrants

with reference angle $\frac{\pi}{6}$

For III-Quad. | **For II-Quad.**

$4x = \pi + \frac{\pi}{6}$, $4x = 2\pi - \frac{\pi}{6}$

$4x = \frac{7\pi}{6} + 2n\pi$, $4x = \frac{11\pi}{6} + 2n\pi$

$x = \frac{7\pi}{24} + \frac{n\pi}{2}$, $x = \frac{11\pi}{24} + \frac{n\pi}{2}$

Hence solution set $\{n\pi\} \cup \left\{\frac{7\pi}{24} + \frac{n\pi}{2}\right\} \cup \left\{\frac{11\pi}{24} + \frac{n\pi}{2}\right\}$, $n \in z$ Ans.

Q.16 $\sin 3x + \sin 2x + \sin x = 0$

Solution:

$\sin 3x + \sin 2x + \sin x = 0$

$\sin 3x + \sin x + \sin 2x = 0$

$2\cos\left(\frac{3x - x}{2}\right)\sin\left(\frac{3x + x}{2}\right) + \sin 2x = 0$

$2\sin 2x \cos x + \sin 2x = 0$

$\sin 2x(2\cos x + 1) = 0$

$\sin 2x = 0$

$2x = n\pi$

$x = \frac{n\pi}{2}$

$2\cos x + 1 = 0$ $\boxed{x = \cos^{-1}\left(\frac{1}{2}\right) = 60 = \frac{\pi}{3}}$

$\cos x = \frac{-1}{2}$

Since cos is –ve in II & III Quadrants

with reference angle $\frac{\pi}{3}$

For II-Quad. | **For III-Quad.**

$x = \pi - \frac{\pi}{3}$, $x = \pi + \frac{\pi}{3}$

$x = \frac{2\pi}{3} + 2n\pi$, $x = \frac{4\pi}{3} + 2n\pi$, $\forall n \in z$

Solution set is $\left\{\frac{n\pi}{2}\right\} \cup \left\{\frac{2\pi}{3} + 2n\pi\right\} \cup \left\{\frac{4\pi}{3} + 2n\pi\right\}$, $n \in z$ Ans.

Q.17 sin 7 x – sin x = sin 3 x

Solution:

$$\text{Sin } 7x - \sin x = \sin 3x$$

$$2\cos\left(\frac{7x+x}{2}\right)\sin\left(\frac{7x-x}{2}\right) = \sin 3x$$

$$2\cos 4x \sin 3x = \sin 3x$$

$$2\cos 4x \sin 3x - \sin 3x = 0$$

$$\sin 3x\,(2\cos 4x - 1) = 0$$

$\Rightarrow$ $\sin 3x = 0$

$3x = n\pi$

$x = \frac{n\pi}{3}$

$2\cos 4x - 1 = 0$

$2\cos 4x = 1$

$\cos 4x = \frac{1}{2}$ $\quad$ $\boxed{4x = \cos^{-1}\left(\frac{1}{2}\right) = 60° = \frac{\pi}{3}}$

Since cos x is +ve in I & IV Quadrants.

with reference angle $\frac{\pi}{3}$

For I-Quad. **For IV-Quad.**

$4x = \frac{\pi}{3}$, $4x = 2\pi - \frac{\pi}{3}$

, $4x = \frac{5\pi}{3}$

$4x = \frac{\pi}{3} + 2n\pi$, $4x = \frac{5\pi}{3} + 2n\pi$

$x = \frac{\pi}{12} + \frac{n\pi}{2}$, $x = \frac{5\pi}{12} + \frac{n\pi}{2}$

Therefore solution set is $\left\{\frac{n\pi}{3}\right\} \cup \left\{\frac{\pi}{12} + \frac{n\pi}{2}\right\} \cup \left\{\frac{5\pi}{12} + \frac{n\pi}{2}\right\}$ $\quad n \in z$ Ans.

Q.18 sin x + sin 3 x + sin 5 x = 0

Solution:

$$\sin 5x + \sin x + \sin 3x = 0$$

$$2\sin\left(\frac{5x+x}{2}\right)\cos\left(\frac{5x-x}{2}\right) + \sin 3x = 0$$

$$2 \sin 3x \cos 2x + \sin 3x = 0$$

$$\sin 3x (2 \cos 2x + 1) = 0$$

$$\sin 3x = 0$$

$$3x = n\pi$$

$$x = \frac{n\pi}{3}$$

$$2 \cos 2x + 1 = 0$$

$$\cos 2x = \frac{-1}{2}$$

$$\boxed{2x = \cos^{-1}\left(\frac{1}{2}\right) = 60° = \frac{\pi}{3}}$$

Since $\cos x$ is –ve in II & III Quadrants, with reference angle $\frac{\pi}{3}$ so

For I-Quad.		**For III-Quad.**
$2x = \pi - \frac{\pi}{3}$	,	$2x = \pi + \frac{\pi}{3}$
$2x = \frac{2\pi}{3}$	,	$2x = \frac{4\pi}{3}$
$2x = \frac{2\pi}{3} + 2n\pi$	,	$2x = \frac{4\pi}{3} + 2n\pi$
$x = \frac{\pi}{3} + \frac{2n\pi}{2}$	,	$x = \frac{2\pi}{3} + \frac{2n\pi}{2}$
$x = \frac{\pi}{3} + n\pi$	,	$x = \frac{2\pi}{3} + n\pi$

Hence solution set is

$$\left\{\frac{n\pi}{3}\right\} \cup \left\{\frac{\pi}{3} + \frac{2n\pi}{3}\right\} \cup \left\{\frac{2\pi}{3} + \frac{2n\pi}{3}\right\}, \quad n \in z \quad \text{Ans.}$$

Q.19 $\sin\theta + \sin 3\theta + \sin 5\theta + \sin 7\theta = 0$

Solution:

$$[\sin 7\theta + \sin\theta] + [\sin 5\theta + \sin 3\theta] = 0$$

$$\left[2 \sin\left(\frac{7\theta + \theta}{2}\right) \cos\left(\frac{7\theta - \theta}{2}\right)\right] + \left[2 \sin\left(\frac{5\theta + 3\theta}{2}\right) \cos\left(\frac{5\theta - 3\theta}{2}\right)\right] = 0$$

$$2 \sin 4\theta \cos 3\theta + 2 \sin 4\theta \cos\theta = 0$$

$$2 \sin 4\theta (\cos 3\theta + \cos\theta) = 0$$

$$2 \sin 4\theta \left[2 \cos\left(\frac{3\theta + \theta}{2}\right) \cos\left(\frac{3\theta - \theta}{2}\right)\right] = 0$$

$$2 \times 2 \sin 4\theta (\cos 2\theta \cos\theta) = 0$$

$$\sin 4\theta \cos 2\theta \cos\theta = 0$$

$\Rightarrow \sin 4\theta = 0$ | $\cos 2\theta = 0$ | $\cos\theta = 0$

$4\theta = n\pi$ | $2\theta = (2n+1)\frac{\pi}{2}$ | $\theta = (2n+1)\frac{\pi}{2}$

$\theta = \frac{n\pi}{4}$ | $\theta = (2n+1)\frac{\pi}{4}$

Hence solutions set is

$$\left\{\frac{n\pi}{4}\right\} \cup \left\{(2n+1)\frac{\pi}{4}\right\} \cup \left\{(2n+1)\frac{\pi}{2}\right\}, n \in Z \quad \text{Ans.}$$

OR

$$\left\{\frac{n\pi}{4}\right\} \cup \left\{\frac{\pi}{4} + \frac{n\pi}{2}\right\} \cup \left\{\frac{\pi}{2} + n\pi\right\}, n \in z$$

Q.20 $\cos\theta + \cos 3\theta + \cos 5\theta + \cos 7\theta = 0$

Solution:

$$[\cos 7\theta + \cos\theta] + [\cos 5\theta + \cos 3\theta] = 0$$

$$2\cos\left(\frac{7\theta+\theta}{2}\right)\cos\left(\frac{7\theta-\theta}{2}\right) + 2\cos\left(\frac{5\theta+3\theta}{2}\right)\cos\left(\frac{5\theta-3\theta}{2}\right) = 0$$

$$2\cos 4\theta \cos 3\theta + 2\cos 4\theta \cos\theta = 0$$

$$2\cos 4\theta(\cos 3\theta + \cos\theta) = 0$$

$$2\cos 4\theta\left[2\cos\left(\frac{3\theta+\theta}{2}\right)\cos\left(\frac{3\theta-\theta}{2}\right)\right] = 0$$

$$2 \times 2\cos 4\theta(\cos 2\theta\cos\theta) = 0$$

$\Rightarrow \cos 4\theta = 0$ | $\cos 2\theta = 0$ | $\cos\theta = 0$

$4\theta = (2n+1)\frac{\pi}{2}$ | $2\theta = (2n+1)\frac{\pi}{2}$ | $\theta = (2n+1)\frac{\pi}{2}$

$\theta = (2n+1)\frac{\pi}{8}$ | $\theta = (2n+1)\frac{\pi}{4}$

Hence solution set is $\left\{(2n+1)\frac{\pi}{8}\right\} \cup \left\{(2n+1)\frac{\pi}{4}\right\} \cup \left\{(2n+1)\frac{\pi}{2}\right\}, n \in z$

OR

$$S.S = \left\{\frac{\pi}{8} + \frac{n\pi}{4}\right\} \cup \left\{\frac{\pi}{4} + \frac{n\pi}{2}\right\} \cup \left\{\frac{\pi}{2} + n\pi\right\}, n \in z$$