1. **Rational number**

A number which can written in the form of $\frac{p}{q}$ where $p, q \in z$ and q≠0 called rational number

Example $\frac{1}{2}, 4, \frac{3}{4}$

2. **Irrational number**

A Number which cannot written in the form of $\frac{p}{q}$ where $p, q \in z$ and q≠0 is called irrational number. Example $\sqrt{2}, \sqrt{3}, \sqrt{5}$

3. ***Decimal Representation of Rational and Irrational Numbers***

1) ***Terminating decimals:***

A decimal that contains finite number of digits in its decimal part is called Terminating decimal

e.g. 2.02.04, 0.000415

** Every terminating decimal can be converted to a common fraction.*

Every terminating decimal represents a rational number.

2) ***Non-Terminating decimals:***

A decimal having infinite number of digits in its decimal part is called non-terminating decimal

e.g., 0.428571.... , 0.33333 …

$\sqrt{2}$=1.414213....

There are two types of non-terminating decimals:

i) Recurring decimals ii) Non-recurring decimals

i) Recurring decimals

A recurring or periodic or cyclic decimal is a decimal in which one or more digits repeat indefinitely.

e.g., $2.\bar{3}$ = 2.3333.... (Rational no.)

$\frac{1}{3}$ = 0.33333.... (Rational no.)

$\frac{3}{7}$ = 0:4285714285714.... (rational)

Every recurring decimal can be converted to a common fraction.

* Every recurring decimal represents la rational number.

ii) Non-recurring decimals:

A decimal which neither terminates nor it is recurring is called non-recurring decimal.

e.g. $\sqrt{2}$ = 1.414213562... , $\sqrt{7}$= 2.645751...

Every non-recurring decimal can not be converted to a common fraction.

* Every non-terminating and non-recurring decimal represents an irrational number.

Example 1

i. 0.25 = $\frac{25}{100}$ rational no.

ii. $\frac{1}{3}$ = 0. 333.... = (recurring decimal (rational no.)

iii. $2.\bar{3}$ = 2.333.... Rational no.

iv. 0.142857142857 ... = $\frac{1}{7}$ (rational no.)

v. 0.01001000100001.... Non-terminating,
Non-periodic so irrational no.

vi. 214.121122111222… Irrational no.

vii. 1.4142135... Is an irrational no.

viii. 7.320 5080... Irrational no.

ix. 1.709975947... Irrational no.

x. 3.141592 654.... Important irrational number calleda π (Pi) and

$$\pi = \frac{circumference\ of\ any\ circle}{length\ of\ its\ diameter}$$

Example 2.

Prove that $\sqrt{2}$ is an irrational number.

Solution:

Suppose $\sqrt{2}$ is a rational number then $\sqrt{2} = \frac{p}{q}$ $where\ p, q \epsilon Z\ \wedge q \neq 0\ if\ HCF(p,q) \neq 1$

$then\ by\ dividing$ $p\ and\ q\ by\ HCF(p.q)$, $\sqrt{2}$ can be reduced as

$\sqrt{2} = \frac{p}{q}$ $where\ HCF\ (a,b) = 1 \rightarrow (1)$

⇨ $\sqrt{2}b = a$

⇨ $2b^2 = a^2$

⇨ $a^2\ is\ divsible\ by\ 2$

⇨ $a\ is\ divisible\ by\ 2 \rightarrow (2)$

⇨ $a = 2c\ where\ ia\ an\ integer$

$\therefore \sqrt{2}b = 2c$

$2b^2 = 4c^2$

⇨ $b^2 = 2c^2$

⇨ $2b^2 = 4c^2$

⇨ $b^2 = 2c^2$

⇨ $b\ is\ divisble\ by\ 2 \rightarrow (3)$

$from\ (2) and\ (3)\ 2\ is\ a\ common\ factor\ of\ a\ and\ b$ $which\ contrdicts\ (1)$

$so\ \sqrt{2} is\ a\ irrational\ number.$

Prove that $\sqrt{3}$ is an rational number.

Solution:

Suppose $\sqrt{3}$ is a rational number then $\sqrt{3} = \frac{p}{q}$ $where\ p, q \epsilon Z\ \wedge q \neq 0\ if\ HCF(p,q) \neq 1$

$then\ by\ dividing$ $p\ and\ q\ by\ HCF(p.q)$, $\sqrt{3}$ can be reduced as

$\sqrt{3} = \frac{p}{q}\ \ where\ HCF\ (a,b) = 1 \rightarrow (1)$

- ⇨ $\sqrt{3}b = a$
- ⇨ $3b^2 = a^2$
- ⇨ $a^2\ is\ divsible\ by\ 3$
- ⇨ $a\ is\ divisible\ by\ 3 \rightarrow (2)$
- ⇨ $a = 3c\ where\ is\ c\ an\ integer$

$$\therefore \sqrt{3}b = 2c$$
$$3b^2 = 9c^2$$

- ⇨ $b^2 = 3c^2$
- ⇨ $b^2 = is\ divisible\ by\ 3$
- ⇨ $b\ is\ divisble\ by\ 3 \rightarrow (3)$

$from\ (2) and\ (3)\ 3\ is\ a\ common\ factor\ of\ a\ and\ b\ which\ contrdicts\ (1)$

$so\ \sqrt{3} is\ a\ irrational\ number.$

Properties of real numbers

Binary operation:-

$A\ binary\ operation\ in\ a\ set\ A\ is\ a\ rule\ usually\ denoted\ by\ * \ that\ assign\ to\ any\ pair\ of\ elements\ of\ A, taken\ in\ a\ definite\ order, another\ element\ of\ A.$

$*\ two\ binary\ operations\ addition\ and\ multiplication\ (.or\ \times) in\ a\ set\ of\ real\ numbers\ (IR) are\ important.$

1. Addition laws

i) Closure law

$\forall\ a,b \in IR, a + b \in IR$

$ii)$ $\boldsymbol{Associative\ law}$

$\forall\ a,b \in IR, a + (b + c) = (a + b) + c$

$\boldsymbol{iii)}$ $\boldsymbol{Additive\ identity}$

$\forall\ a \in IR, \exists\ 0 \in IR\ such\ that$

$a + 0 = 0 + a = a\ \ \ 0\ (zero) is\ called\ identity\ element\ of\ addition.$

$\boldsymbol{iv)}$ $\boldsymbol{Additive\ inverse}$

$\forall\ a \in IR, \exists(-a) \in IR\ such\ that$

$a + (-a) = 0 = (-a) + a$

$\boldsymbol{v)}$ $\boldsymbol{Commutative\ Law}$

$\forall\ a,b \in IR, a + b = b + a$

2. Multiplication Laws

vi) Closure Law

$\forall\ a,b \in IR, a.b \in IR$

$(a,b\ is\ usually\ written\ as\ ab)$

vii) Associative Law

$\forall\ a,b \in IR, a.(b.c) = (a.b).c$

$viii)\ Multiplicative\ Identity$

$\forall\ a \in IR, \exists\ 1 \in IR\ such\ that$

$a.1 = 1.a = a\ \ \ 1\ (one) is\ called\ identity\ element\ of\ multiplicative.$

$ix)\ Multiplicative\ Inverse$

$\forall\ a \in IR, \exists(a^{-1}) \in IR\ such\ that$

$a.a^{-1} = a^{-1}a\ \ = 1\ \ (a^{-1}\ \ is\ also\ written\ as\ \frac{1}{a}$

$x)\ Commutative\ Law$

$\forall\ a,b \in IR, ab + ba$

3. Multiplicative – addition Law

xi)

$\forall\ a,b,c \in IR,$

$a(b + c) = ab + ac\ \ left\ distributive$

$a + b)c = ac + bc\ \ right\ distributive$

4.

$Properties\ of\ equality$

i) Reflexive property

$\forall\ a \in IR,\ \ a = a$

ii) Symmetric Property

$\forall\, a,b \in IR,\;\; a=b \Rightarrow b=a$

iii) *Transitive Property*

$\forall\, a,b,c \in IR,\qquad a=b \;\wedge b=c \Rightarrow a=c$

iv) *Additive property*

$\forall\, a,b,c \in IR,\qquad a=b \;\Rightarrow a+c=b+c$

v) Multiplicative property

$\forall\, a,b,c \in IR,\qquad a=b \;\Rightarrow ac=bc \wedge ca=cb$

vi) cancellation property w.r.t addition

$\forall\, a,b,c \in IR,\qquad a+c=b+c \;\Rightarrow a=b$

vii) Cancellation property w.r.t multiplication

$\forall\, a,b,c \in IR,\qquad ac=bc \;\Rightarrow a=b, c\neq 0$

5. Properties of inequalities
   1) Trichotomy property

$\forall\, a,b \in IR, either\;\; a=b\; or a>b\; or a<b$

2) Transitive property

$\forall\, a,b,c \in IR$

$1.\, a>b \;\wedge b>c \Rightarrow a>c$

$2.\, a<b \;\wedge b<c \Rightarrow a<c$

3) Additive property

$\forall\, a,b,c,d \in IR$

a) $1.\, a>b \;\Rightarrow a+c>b+c$
   $2.\, a<b \;\Rightarrow a+c<b+c$

b) $1.\, a>b \;\wedge c>d \;\Rightarrow a+c>b+d$
   $2.\, a<b \;\wedge c<d \;\Rightarrow a+c<b+d$

4) Multiplicative properties

a) $\forall\, a,b,c \in IR \text{ and } c>0$

$i)\; a>b \Rightarrow ac>bc$

$ii)\; a<b \;\Rightarrow ac<bc$

b) $\forall\, a,b,c \in IR \text{ and } c<0$

$i)\; a>b \Rightarrow ac<bc$

$ii)\; a<b \;\Rightarrow ac>bc$

c) $\forall\, a,b,c,d \in IR$ *and* a,b,c,d *are all positive*

$1.\, a>b \;\wedge c>d \;\Rightarrow ac>bd$

$2.\, a<b \;\wedge c<d \;\Rightarrow ac<bd$

Note:

$*\; a$ *and* $(-a)$ *are additive inverse of each other*

By def. inverse of $(-a)$ *is* $a\quad -(-a)=a$

*a and 1/a are x^{-1}inverse of each other.

inverse of $\frac{1}{a}$ *is* a (*i.e inverse of* a^{-1})$a\neq 0$

$(a^{-1})^{-1}=a \;\; or \frac{1}{\frac{1}{a}}=a$

Exercise No.1.1

1. Which of the following have closure property w.r.t addition and multiplication

i. $\{0\}$

Solution:- As 0+0=0∈$\{0\}$

⇒$\{0\}$ has closure property w.r.t addition

As 0×0 =0∈$\{0\}$

⇒$\{0\}$ has closure property w.r.t multiplication

ii. $\{1\}$

Solution:- As 1+1=2∉$\{1\}$

⇒$\{1\}$ does not have closure property w.r.t addition

As 1×1 =1∈$\{1\}$

⇒$\{1\}$ has closure property w.r.t multiplication

iii. $\{0,-1\}$

Solution:-

0+0=0∈$\{0,-1\}$

0+(-1)=-1∈$\{0,-1\}$

(-1)+0=-1∈$\{0,-1\}$

(-1)+(-1)=-2 ∉ $\{0,-1\}$

⇒$\{0,-1\}$does not have closure property w.r.t addition

As (-1)×(-1) =1∉ $\{0,-1\}$

⇒$\{0,-1\}$ has closure property w.r.t multiplication

iv. $\{1,-1\}$

Solution:-

(-1)+(-1)=-2 ∉ $\{1,-1\}$

$\Rightarrow \{1,-1\}$ does not have closure property w.r.t addition

Also as

1 x1=1 $\in \{1,-1\}$

-1x(-1)=1 $\in \{1,-1\}$

(-1) x 1=-1 $\in \{1,-1\}$

1 x (-1)= -1 $\in \{1,-1\}$

$\Rightarrow \{1,-1\}$ has closure property w.r.t multiplication.

2. Name the property used in the following equations

i. 4+9=9+4

Ans:- commutative property w.r.t addition

ii. $(a+1)+\frac{3}{4} = a+(1+\frac{3}{4})$

Ans:- Associative property w.r.t addition

iii. $(\sqrt{3}+\sqrt{5})+\sqrt{7} = \sqrt{3}+(\sqrt{5}+\sqrt{7})$

Ans:- Associative property w.r.t addition

iv. 100+0=100

Ans:-additive identity

v. 1000x1=1000

Ans:-Multiplicative identity

vi. 4.1+(-4.1)=0

Ans:-Additive inverse

vii. a-a=0

Ans:- Additive inverse

viii. $\sqrt{2}\times\sqrt{5} = \sqrt{5}\times\sqrt{2}$

Ans:- commutative property w.r.t multiplication

ix. a(b-c)=ab-ac

Ans:-Left distribution property

x. (x-y)z=xz-yz

Ans:- Right distribution property

xi. 4x(5x8)=(4x5)x8

Ans:- Associative property w.r.t multiplication

xii. a(b+c-d)=ab+ac-ad

Ans:- Left distribution property

3. Name the property used in the following inequalities

i. $-3 < -2 \Rightarrow 0 < 1$

Ans:- Additive property

ii. $-5 < -4 \Rightarrow 20 > 16$

Ans:- Multiplicative property

iii. $1 > -1 \Rightarrow -3 > -5$

Ans:- Additive property

iv. $a < 0 \Rightarrow -a > 0$

Ans:- Multiplicative property

v. $a > b \Rightarrow \frac{1}{a} < \frac{1}{b}$

Ans:- Multiplicative property

vi. $a > b \Rightarrow -a < -b$

Ans:- Multiplicative property

4. Prove the followings rules 0f addition.

i. $\frac{a}{c}+\frac{b}{c} = \frac{a+b}{c}$

L.H.S$=\frac{a}{c}+\frac{b}{c}$

$=a \times \frac{1}{c} + b \times \frac{1}{c}$

$=(a+b) \times \frac{1}{c}$

$=\frac{a+b}{c}$ =R.H.S

ii. $\frac{a}{b}+\frac{c}{d} = \frac{ad+bc}{bd}$

L.H.S$=\frac{a}{b}+\frac{c}{d}$

$=\frac{a}{b} \times 1 + 1 \times \frac{c}{d}$

$=\frac{a}{b} \times (d \times \frac{1}{d}) + (b \times \frac{1}{b}) \times \frac{c}{d}$

$=\frac{a}{b} \times \frac{d}{d} + \frac{b}{b} \times \frac{c}{d}$

$=\frac{ad}{bd} + \frac{bc}{bd}$

$=ad \times \frac{1}{bd} + bc \times \frac{1}{bd}$

$= (ad+bc) \, x \, 1/bd$

$=\frac{ad+bc}{bd}$ =R.H.S

5. Prove that $-\frac{7}{12}-\frac{5}{18} = \frac{-21-10}{36}$

$L.H.S = -\frac{7}{12}-\frac{5}{18}$

$= -\frac{7}{12}\times 1 - \frac{5}{18}\times 1$

$= -\frac{7}{12} \times (3 \times \frac{1}{3}) - \frac{5}{18} \times (2 \times \frac{1}{2})$

$= -\frac{7}{12} \times \frac{3}{3} - \frac{5}{18} \times \frac{2}{2}$

$= -\frac{21}{36} - \frac{10}{36}$

$= (-21-10) x \frac{1}{36}$

$=\frac{-21-10}{36}$ = R.H.S

6. *Simplify by justify each step.*

i. $\frac{4+16x}{4}$

Solution:- $=\frac{1}{4}\,x\,(4+16x)$

$=\frac{1}{4}\,x\,(4\times 1+4\times 4x)$ *(Multiplicative identity)*

$=\frac{1}{4}\,x\,4(1+4x)$ *(Distributive property)*

$=1\times(1+4x)$ *(Multiplicative inverse)*

$=(1+4x)$ *(Multiplicative identity)*

ii. $\frac{\frac{1}{4}+\frac{1}{5}}{\frac{1}{4}-\frac{1}{5}}$

Solution:- $\frac{\frac{1}{4}\times 1+\frac{1}{5}\times 1}{\frac{1}{4}\times 1-\frac{1}{5}\times 1}$

$=\frac{\frac{1}{4}\times(\frac{1}{5}x\,5)+\frac{1}{5}\times(\frac{1}{4}x\,4)}{\frac{1}{4}\times(\frac{1}{5}x\,5)-\frac{1}{5}\times(\frac{1}{4}x\,4)}$

$=\frac{\frac{1}{4}\times\frac{5}{5}+\frac{1}{5}\times\frac{4}{4}}{\frac{1}{4}\times\frac{5}{5}-\frac{1}{5}\times\frac{4}{4}}$

$=\frac{\frac{5}{20}+\frac{4}{20}}{\frac{5}{20}-\frac{4}{20}}$

$=\frac{5\times\frac{1}{20}+4\times\frac{1}{20}}{5\times\frac{1}{20}-4\times\frac{1}{20}}$

$=\frac{(5+4)\times\frac{1}{20}}{(5-4)\times\frac{1}{20}}$

$=\frac{(5+4)}{(5-4)}$

$=\frac{9}{1}=9$

iii. $\frac{\frac{a}{b}+\frac{c}{d}}{\frac{a}{b}-\frac{c}{d}}$

Solution:- $\frac{\frac{a}{b}\times 1+\frac{c}{d}\times 1}{\frac{a}{b}\times 1-\frac{c}{d}\times 1}$ (multiplicative identity)

$=\frac{\frac{a}{b}\times(d\times\frac{1}{d})+\frac{c}{d}\times(b\times\frac{1}{b})}{\frac{a}{b}\times(d\times\frac{1}{d})-\frac{c}{d}\times(b\times\frac{1}{b})}$(multiplicative inverse)

$=\frac{\frac{ad}{bd}+\frac{bc}{bd}}{\frac{ad}{bd}+\frac{bc}{bd}}$

$=\frac{(ad+bc)\times\frac{1}{bd}}{(ad-bc)\times\frac{1}{bd}}\;\therefore\;\frac{a}{b}=a\times\frac{1}{b}$

$=\frac{ad+bc}{ad-bc}$

iv. $\frac{\frac{1}{a}+\frac{1}{b}}{1-\frac{1}{a}\cdot\frac{1}{b}}$

$=\frac{\frac{1}{a}\times 1+\frac{1}{b}\times 1}{1-\frac{1}{a}\cdot\frac{1}{b}}$(multiplicative identity)

$=\frac{\frac{1}{a}\times(b\times\frac{1}{b})+\frac{1}{b}\times(a\times\frac{1}{a})}{1-\frac{1}{a}\cdot\frac{1}{b}}$(multiplicative inverse)

$=\frac{\frac{1}{a}\times(\frac{b}{b})+\frac{1}{b}\times(\frac{a}{a})}{1-\frac{1}{a}\cdot\frac{1}{b}}$

$=\frac{\frac{b}{ab}+\frac{a}{ab}}{1-\frac{1}{ab}}$

$=\frac{b\times\frac{1}{ab}+a\times\frac{1}{ab}}{ab\times\frac{1}{ab}-1.\frac{1}{ab}}$(multiplicative inverse and multiplicative identity)

$=\frac{(b+a)\frac{1}{ab}}{(ab-1)\frac{1}{ab}}$(dist. property)

$=\frac{b+a}{ab-1}$(cancellation law)

Complex Numbers

The numbers of the form $x+iy$ where $x,y\in R$ *and* $i=\sqrt{-1}$ *are called complex numbers. here x is called real part and y is called imaginary part of the complex numbers e.g* $3+4i,\,2-\frac{5}{7}i$

- Every real number is a complex number with O as its imaginary part. Consider the equation

$$x^2+1=0$$

⇨ $x^2=-1$

⇨ $x=\pm\sqrt{-1}$

$\sqrt{-1}\notin R$ *for convenience call it imaginary number and denote it by i (read i as iota)*

Power of *i*

$i^2=-1$ *by def.*

$i^3=i^2.i=(-1)i=-i$

$i^4=i^2.i^2=(-1)(-1)=1$

$i^{13}=i^{12}.i=(i^2)^6.i=(-1)^6.i$

$(1)i=i$

$i^6=(i^2)^3=(-1)^3=-1$

thus any power of i must be equal to $i,-i,1,$ *and* -1

Operation on complex numbers

1) $a+bi=c+di$

⇨ $a=c\wedge b=d$

2) Addition

$(a+bi)+(c+di)=(a+c)+(b+d)i$

3) $k(a+bi)=ka+kbi$

4) $(a+bi)-(c+di)$

$=(a-c)+(b-d)i$

5) $(a+bi).(c+di)$

$=ac+adi+bci+bdi^2$

$=(ac-bd)+(ad+bc)i$

Conjugate complex Numbers:

for $z=a+ib$ *then its conjugate is denoted by* $\overline{Z}$ and is defined as

$\overline{Z}=\overline{a+ib}=a-ib$

- A real number is self –conjugate.

Complex Numbers of Ordered pairs of Real Numbers.

$$i)\ (a,b)=(c,d) \Leftrightarrow a=c \wedge b=d$$
$$ii)\ (a,b)+(c,d)=(a+c,b+d)$$
$$iii)\ if\ k\ is\ any\ real\ number\ , then$$
$$k(a,b)=(ka,kb)$$
$$iv)\ (a,b)(c,d)=(ac-bd,ad+bc)$$
$$v)\ (a,b)-(c,d)=(a-c,b-d)$$

Properties of the fundamental operation on complex Numbers.

$i)$ *the additive identity in C is* $(0,0)$

$ii)$ *every complex number* (a,b) *has the additive inverse* $(-a,-b)$

i.e $(a,b)+(-a,-b)=(0,0)$

$iii)$ *the multiplicative identity is* $(1,0)$

i.e $(a,b)(1,0)=(a.1-b.0,b.1+a.0)$

$iv)$ *every non zero complex number i.e number not equal to* $(0,0)$ *has a multiplicative inverse.*

Q.

Prove that the multiplicative inverse of(a,b)

Is $\left(\frac{a}{a^2+b^2},-\frac{b}{a^2+b^2}\right)$

Proof:

let $Z=(a,b)$ *or* $Z=a+ib$

$$Z^{-1}=\frac{1}{a+ib}=\frac{1}{a+ib}\times\frac{a-bi}{a+ib}$$
$$Z^{-1}=\frac{a-bi}{(a+ib)(a-ib)}=\frac{a-ib}{a^2-i^2b^2}$$
$$Z^{-1}=\frac{a-ib}{a^2-(-1)b^2}=\frac{a-ib}{a^2+b^2}$$
$$Z^{-1}=\frac{a}{a^2+b^2}-\frac{b}{a^2+b^2}i$$

Hence

$$Z^{-1}=\left(\frac{a}{a^2+b^2},\frac{-b}{a^2+b^2}\right)$$

v) $(a,b)[(c,d)\pm(e,f)]$
$$=(a,b)(c,d)\pm(a,b)(e,f)$$

A special subset of C *For all* $(a,0),(b,0)\epsilon C$

$$(a,0)+(b,0)=(a+b+0)$$
$$(a,0).(b,0)=(ac,0)$$
$$K(a,0)=(ka,0)$$

Multiplicative inverse of $(a,0)$ *is*

$(\frac{1}{a},0)$ *provide* $a\neq 0$

Exercise 1.2

1. **Verify the addition properties of complex Number**

i. Closure property

For $(a,b),(c,d)\in C$

$(a,b)+(c,d)=(a+c,b+d)\in C$

ii. Associative property

For $(a,b),(c,d),(e,f)\in C$

$=[(a,b)+(c,d)]+(e,f)$

$=(a+c,b+d)+(e+f)$

$=[(a+c)+e,(b+d)+f]$

$=[a+(c+e),b+(d+f)]$

$\therefore$ ' + ' is associative in R

$=(a,b)+(c+e\ ,d+f)$

$=(a,b)+[(c,d)+(e,f)]$

iii. Additive identity

$\forall\ (a,b)\in C$ there is $(0,0)\in C$

Such that $(a,b)+(0,0)$

$=(a+0\ ,b+0)=(a,b)$

iv. Additive inverse

$\forall\ (a,b)\in C$ there is $(-a,-b)\in C$

Such that $(a,b)+(-a,-b)$

$=(a-a\ ,b-b)=(0,0)$

v. Commutative property

$\forall\ (a,b),(c,d)\in C$

$=(a,b)+(c,d)$

$=(a+c)+(b+d)$

$=(c+a)+(d+b)$

$=(c,d)+(a,b)$

Q2. Verify the multiplication properties of the complex numbers.

Solution:

1.Close w.r.t "x"

$$(a+ib),(c+id)\epsilon C\ then$$
$$(a+ib)(c+id)=ac+iad+ibc+i^2bd$$
$$=ac+i(ad+bc)-bd$$
$$=(ac-bd)+i(ad+bc)\epsilon C$$

2.Associative w.r.t "x"

$$(a+ib),(c+id),(e+if)\epsilon C$$
$$[(a+ib)(c+id)](e+if)$$
$$=[(ac-bd)+i(bc+ad)](e+if)$$
$$=[e(ac-bd)-f(bc+ad)]$$
$$+i[f(ac-bd)+e(bc+ad]$$
$$=[eac-ebd-fbc-fad]$$
$$+i[fac-fbd+ebc+ead]$$
$$=[a(ec-df)-b(df+de)]$$
$$+i[a(cf+de)+b(ec-df)]$$
$$=(a+ib)[(ec-df)+i(cf+de)]$$
$$=(a+ib)[(c+id)(e+f)]$$

iii)Identity

$$(a+ib),(1+i0)\in C \ then$$
$$(a+ib)(a+i0)=a+0+ib+0$$
$$a+ib\in C$$

iv)Inverse

$$(a+ib),\left(\frac{a}{a^2+b^2},\frac{ib}{a^2+b^2}\right)\in C$$

Then

$$=(a+ib)\left(\frac{a}{a^2+b^2}-\frac{ib}{a^2+b^2}\right)$$
$$=(a+ib)\left(\frac{a-ib}{a^2+b^2}\right)$$
$$=\frac{a^2-(ib)^2}{a^2+b^2}=\frac{a^2+b^2}{a^2+b^2}=1=1+i0$$

v) Commutative

$$(a+ib),(c+id)\in C$$
$$(a+ib)(c+id)$$
$$=(ac-bd)+i(ad+bc)$$
$$=ca-db)+i(ad+cb)$$
$$=(c+id)(a+ib)$$

Q.3 verify the distributive law of complex numbers

$$(a,b)[(c,d)+(e,f)]$$
$$=(a,b)(c,d)+(a,b)(e,f)$$

Solution :

L.H.S

$$(a,b)[(c,d)+(e,f)]$$
$$=(a,b)(c+e,d+f)$$
$$=(a(c+e)-b(d+f),a(d+f)+b(c+e)$$
$$=(ac+ae-bd-bf,ad+af+bc+ba)$$

R.H.S

$$=(a,b)(c,d)+(a,b)(e,f)$$
$$=(ac-bd,ad+bc)+(ae-bf,af+bc$$
$$(ac+ae-bd-bf,ad+af+bc+ba$$

Hence proved.

2. Simplify the following.

i. i^9

Solution:- $(i^2)^4.i$

$=(-1)^4.i$

$=1.i$

$=i$

ii. i^{14}

Solution:- $(i^2)^7$

$=(-1)^7$

$=-1$

iii. $(-i)^{19}$

Solution:-$-(i^2)^9.i$

$=-(-1)^9.i$

=-(-1). i

$=1.i$

$=i$

iv. $(-1)^{-\frac{21}{2}}$

Solution:

$$\frac{1}{(-1)^{\frac{21}{2}}}=\frac{1}{\left[(-1)^{\frac{1}{2}}\right]^{21}}$$
$$=\frac{1}{i^{21}}=\frac{1}{(i^2)^{10}.i}$$
$$=\frac{1}{(-1)^{10}.i}=\frac{1}{1.i}$$
$$=\frac{1}{i}\times\frac{i}{i}$$
$$=\frac{i}{i^2}=\frac{i}{(-1)}$$
$$=-i$$

Q5.

Write into the term of i

i. $\sqrt{-1b}$

Solution:- ib

ii. $\sqrt{-5}$

solution:- $\sqrt{-1\times 5}$

$$\sqrt{-1}\ \sqrt{5}=\sqrt{5}\,i$$

iii. $\sqrt{-\frac{16}{25}}$

Solution:- $\sqrt{-1\times\frac{16}{25}}$

$=\sqrt{-1}\times\sqrt{\frac{16}{25}}$

$=\frac{4}{5}i$

iv. $\sqrt{\frac{1}{-4}}$

Solution:- $\sqrt{-1\times\frac{1}{4}}$

$=\sqrt{-1}\times\sqrt{\frac{1}{4}}=i\times\frac{1}{2}$

$$= \frac{i}{2} \quad Ans.$$

Q6.

Solve (7,9)+(3,-5)

Solution:-

$$(7,9) + (3,-5)$$
$$= (7+3, 9-5)$$
$$= (10,4$$
$$= \frac{-15 + 40i + 12i - 32i^2}{9 - 64i^2}$$
$$= \frac{-15 + 52i + 32}{9 + 64} \qquad \therefore i^2 = -1$$
$$\frac{17 + 52i}{73} = \frac{17}{73} + \frac{52}{73}i = \left(\frac{17}{73}, \frac{52}{73}\right)$$

Q7.

Solve (8,-5)-(-7,4)

Solution:- (8+7,-5-4)

=(15,-9)

Q8. *Solve* $(2,6).(3,7)$

Solution:

$$(2+6i).(3+7i)$$
$$= (2.3 + 2.7i + 6i.3 + 6i.7i)$$
$$= (6 + 14i + 18i + 42i^2)$$
$$= (6 + 42(-1) + 32i)$$
$$= (6 - 42 + 32i)$$
$$= (-36 + 32i) = (-36,32)$$

Q9.

$$(\mathbf{5}, -\mathbf{4})(-\mathbf{3}, -\mathbf{2})$$

S0lution:

$$(5,-4)(-3,-2)$$
$$= \big(5(-3) - (-4)(-2), 5(-2) + (-4)(-3)\big)$$
$$= (-15 - 8, -10 + 12) = (-23,2)$$

Q10.

(0,3)(0,5)

Solution:

(0,3)(0,5)

$$= (0.3 - 3.5, 0.5 + 3.0) = (0.15, 0 + 0)$$
$$= (-15,0)$$

Q.11

Solve (2,6) ÷ (3,7)

Solution:-

$$\frac{(2,6)}{(3,7)} = \frac{2+6i}{3+7j}$$
$$= \frac{2+6i}{3+7i} \times \frac{3-7i}{3-7i}$$
$$= \frac{(2+6i)(3-7i)}{(3)^2 - (7i)^2}$$
$$= \frac{(2.3 - 2.7i + 6i.3 - 6i.7i)}{9 - 49(i)^2}$$
$$= \frac{6 - 14i + 18i - 42i^2}{9 - 49(-1)}$$
$$= \frac{6 + 4i - 42(-1)}{9 + 49} = \frac{6 + 42 + 32i}{58}$$
$$= \frac{48 + 4i}{58} = \left(\frac{24}{29}, \frac{2}{32}\right)$$

Q.12

Solution

$$(5,-4) \div (-3,-8)$$
$$= \frac{\mathbf{5}, -\mathbf{4}}{(-\mathbf{3}, -\mathbf{8})} = \frac{\mathbf{5} - \mathbf{4i}}{-\mathbf{3} - \mathbf{8i}}$$
$$= \frac{\mathbf{5} - \mathbf{4i}}{-\mathbf{3} - \mathbf{8i}} \times \frac{(-\mathbf{3} + \mathbf{8i})}{(-3 + 8\mathrm{i})}$$
$$= \frac{5(-3) + (5)(8\mathrm{i}) + (-4\mathrm{i})(-3) + (-4\mathrm{i})(8\mathrm{i})}{(-3)^2 - (8\mathrm{i})^2}$$

Q.13

Prove that sum as well as product of two conjugate complex number is real.

Solution:-

let two conjugate complex number be

$$Z = a + ib \text{ and } \bar{z} = a - ib \text{ where } a,b \in \mathbb{R}$$
$$Sum = z + \bar{z} = a + ib + a - ib$$
$$= 2a \in \mathbb{R} \quad \therefore a \in \mathbb{R}$$
$$Product = z.\bar{z}$$
$$= (a+ib).(a-ib)$$
$$= (a^2 - i^2b^2)$$
$$= a^2 - (-1)b^2$$
$$= a^2 + b^2 \in \mathbb{R} \therefore x,b \in \mathbb{R}$$

Q14.

Find the multiplicative inverse of the following

i. (-4,7)

Solution:- let $z = (-4,7)$

Multiplicative inverse of $z = \frac{1}{z}$

$$= \frac{1}{(-4,7)} = \frac{1}{-4+7i}$$
$$= \frac{1}{-4+7i} = \frac{1}{-4+7i} \times \frac{-4-7i}{-4-7i}$$
$$= \frac{-4-7i}{(-4)^2 - (7i)^2} = \frac{-4-7i}{16 - 49(i)^2}$$
$$= \frac{-4-7i}{16+49} = \frac{-4-7i}{16-49(i)^2}$$
$$= \frac{-4-7i}{65} = \frac{-4}{65} - \frac{7}{65}i$$
$$= \left(-\frac{4}{65}, -\frac{7}{65}\right)$$

ii. $(\sqrt{2}, \sqrt{5})$

Solution:- let z=$(\sqrt{2}, -\sqrt{5})$

Multiplicative inverse of z=$\frac{1}{z}$

$$= \frac{1}{(\sqrt{2}, -\sqrt{5})} = \frac{1}{\sqrt{2} - \sqrt{5}i}$$
$$= \frac{1}{\sqrt{2} - \sqrt{5}i} \times \frac{\sqrt{2} + \sqrt{5}i}{\sqrt{2} + \sqrt{5}i}$$
$$= \frac{\sqrt{2} + \sqrt{5}i}{(\sqrt{2})^2 - (\sqrt{5}i)^2}$$

$= \frac{\sqrt{2}+\sqrt{5}i}{2-5i^2}$

$= \frac{\sqrt{2}+\sqrt{5}i}{2-5(-1)} = \frac{\sqrt{2}+\sqrt{5}i}{2+5}$

$= \frac{\sqrt{2}+\sqrt{5}i}{7} = \left(\frac{\sqrt{2}}{7}, \frac{\sqrt{5}}{7}\right)$

iii. (1,0)

Solution:

$let\ z = (1,0)$

Multiplicative inverse of z=$\frac{1}{z}$

$\frac{1}{(1,0)} = \frac{1}{1+0i}$

$= \frac{1}{1+0} = \frac{1}{1} = 1$

$= (1,0)$

Q15.

Factorize the following.

i. $\boldsymbol{a^2 + 4b^2}$

Solution:- $\boldsymbol{a^2 - (-1)4b^2}$

$= a^2 - (i)^2 2^2 b^2$

$= (a)^2 - (i2b)^2$

$= (a + i2b)(a - i2b)$

$= (a + 2bi)(a - 2bi)$

ii. $\boldsymbol{9a^2 + 16b^2}$

Solution:- $\boldsymbol{9a^2 - (-1)16b^2}$

$= (3a)^2 - (i)^2 4^2 b^2$

$= (3a)^2 - (i4b)^2$

= (3a+i4b)(3a-i4b)

= (3a+4bi)(3a-4bi)

iii. $\boldsymbol{3x^2 + 3y^2}$

Solution:- $\boldsymbol{3x^2 - (-1)3y^2}$

$= 3(x)^2 - (i)^2 3y^2$

$= 3[(x)^2 - (iy)^2]$

$= 3[(x + iy)(x - iy)]$

$= 3(x + iy)(x - iy)$

Q17. Separate real and imaginary part.

i.

$$\frac{2-7i}{4+5i}$$

Solution:- $\frac{2-7i}{4+5i} \times \frac{4-5i}{4-5i}$

$= \frac{8-10i-28i+35i^2}{(4)^2-(5i)^2}$

$= \frac{8-38i+35(-1)}{16-25i^2}$

$= \frac{8-38i-35}{16-25(-1)}$

$= \frac{-27-38i}{16+25} = \frac{-27-38i}{41}$

$= \frac{-27}{41} - \frac{38}{41}i$

$$\frac{(-2+3i)^2}{1+i}$$

solution:- $\frac{(-2)^2+(3i)^2+2(-2)(3i)}{1+i}$

$= \frac{4+9i^2-12i}{1+i} = \frac{4+9(-1)-12i}{1+i}$

$= \frac{4-9-12i}{1+i} = \frac{-5-12i}{1+i}$

$= \frac{-5-12i}{1+i} \times \frac{1-i}{1-i}$

$= \frac{-5+5i-12i+12i^2}{(1)^2-(i)^2}$

$= \frac{-5+5i-12i+12(-1)}{1-(-1)}$

$= \frac{-17-7i}{2} = \frac{-17}{2} - \frac{7}{2}i$

iii. $$\frac{i}{1+i}$$

Solution:- $\frac{i}{1+i} \times \frac{1-i}{1-i}$

$= \frac{i-i^2}{1^2-i^2} = \frac{i-(-1)}{1-(-1)}$

$= \frac{i+1}{1+1} = \frac{i+1}{2}$

$= \frac{1}{2} + \frac{i}{2}$

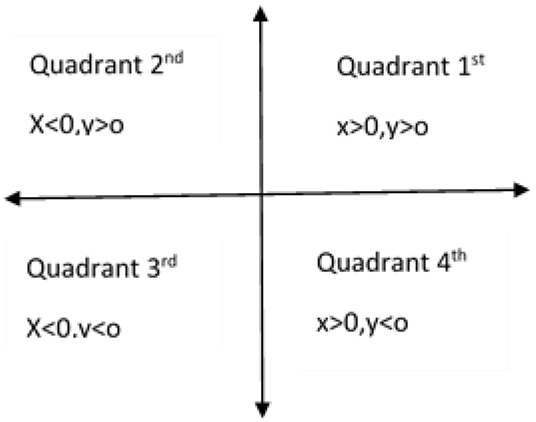
The Real Line

The set or real numbers are represented by a straight line XOX' *as shown.*

The Real Plane /coordinate plane

The plane made by two mutually perpendicular lines is called coordinate plane. Let us draw two mutually ⊥ *lines* XX' *and* YY' *such as O be their* Point of interaction. The lines XX' *and* YY' *are togather* coordinates axes. The common point O is called origin or initial point. XOX' is called X-axis, which is horizontal line and YOY' is called Y-axis. Which is vertical line. Thus the plane made by both x-axis and y-axis is called xy-plane or real plane.

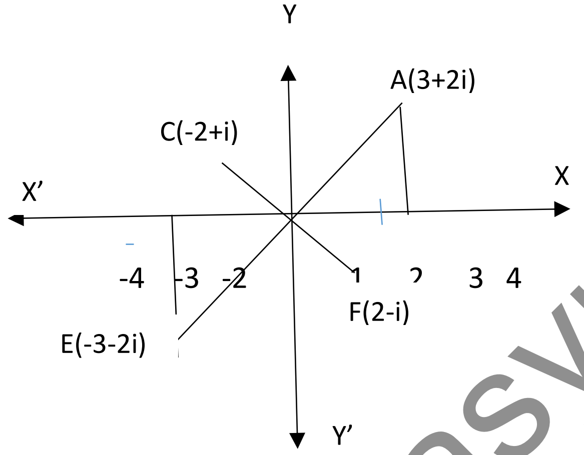
If (a,b) are called coordinates of a point p then a is called x-coordinate or abscissa of point p and b is called y-coordinate are ordinate of point p. the coordinate plane into four equal parts, called quadrants.



Geometrical representation of complex Numbers

Geometrical representation of complex Number $a + ib$ is represented by a point P(a,b) on the coordinate plane. When are represent a complex number on coordinate plane, then the coordinate plane is called complex plane or Z plane.

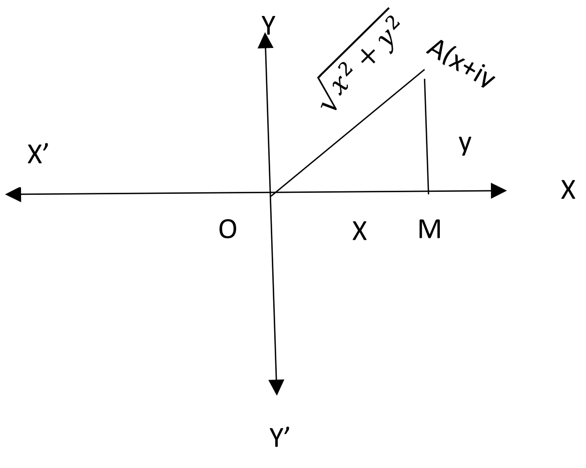
- In the representation the x-axis is called real axis and y-axis is called imaginary axis.



Argand diagram:

Figure representing one or more complex numbers on complex plane is called Argand diagram.

Modulus of the Complex Number $a + ib$



In Cartesian plane distance of A(a,b) from origin O(0,0)

$$|OP| = \sqrt{(a-0)^2 + (b-0)^2}$$

$$\sqrt{a^2 + b^2}$$

Draw $\perp$ $AM\ from\ A\ on\ x-axis\ \ then\ |OM| = a$

$$|AM| = b\ \ and\ |OA| = z\ \ by\ pathagoras\ therom\ \ on\ \Delta\ AOM$$

$$|OA|^2 = |OM|^2 + |AM|^2$$

$$Z^2 = a^2 + b^2$$

$$|Z| = \sqrt{a^2 + b^2}$$

Thus the modules of a complex number from the origin.

Exercise 1.3

Question Find multiplicative inverse of each of the following numbers.

i. ***-3i***

Solution:-let z= -3i then its multiplicative inverse is $\frac{1}{z}$

$\frac{1}{z} = \frac{1}{-3i} \times \frac{i}{i}$

$= \frac{i}{-3(i)^2} = \frac{i}{-3(-1)}$

$= \frac{i}{3}$

ii. **1-2i**

Solution:-let z= -3i then its multiplicative inverse is $\frac{1}{z}$

$\frac{1}{z} = \frac{1}{1-2i}$

$= \frac{1}{1-2i} \times \frac{1+2i}{1+2i}$

$= \frac{1+2i}{(1)^2-(2i)^2}$

$= \frac{1+2i}{1-4(i)^2} = \frac{1+2i}{1-4(-1)}$

$= \frac{1+2i}{1+4} = \frac{1+2i}{5}$

$= \frac{1}{5} + \frac{2}{5}i$

iii. $\boldsymbol{-3-5i}$

Solution:

$$let\ Z = -3 - 5i = (-3, -5)$$

$$\therefore\ for\ Z = (a, b)$$

⇨ $z^{-1} = \left(\frac{a}{a^2+b^2}, -\frac{b}{a^2+b^2}\right)$

$so\ for\ Z = (-3, -5)$

⇨ $z^{-1} = \left(-\frac{3}{(-3)^2+(-5)^2}, -\frac{-5}{(-3)^2+(-5)^2}\right)$

$$= \left(\frac{-3}{9+25}, \frac{5}{9+25}\right) = \left(\frac{-3}{34}, \frac{5}{34}\right)$$
$$= \frac{-3}{34} + \frac{5}{34}i$$

iv. (1,2)

Solution:

$$let\ Z = (1,2)$$
$$\therefore for\ Z = (a,b)$$
$$z^{-1} = \left(\frac{a}{a^2+b^2}, -\frac{b}{a^2+b^2}\right)$$

For Z=(1,2)

$$\Rightarrow z^{-1} = \left(\frac{1}{(1)^2+(2)^2}, \frac{-2}{(1)^2+(2)^2}\right)$$
$$= \left(\frac{1}{1+4}, \frac{-2}{1+4}\right) = \left(\frac{1}{5}, \frac{-2}{5}\right) = \frac{1}{5} + \frac{-2}{5}i$$

Question No.3

Simplify

i. i^{101}

solution:- $i^{100}.i$

$=(i^2)^{50}.i$

$=(-1)^{50}.i$

$=1.i=i$

ii. $(-ai)^4$

$solution$:- $(-a)^4 i^4$

$=a^4.(i^2)^2$

$=a^4.(-1)^2$

$=a^4.1=a^4$

iii. i^{-3}

$solution$:-$\frac{1}{i^3}$

$=\frac{1}{i^2.i}$

$=\frac{1}{(-1).i}$

$=\frac{1}{(-1).i}=\frac{-1}{i}$

$=\frac{-1}{i} \times \frac{i}{i}$

$=\frac{-i}{i^2} = \frac{-i}{(-1)}$

$=i$

iv. i^{-10}

$solution$:- $\frac{1}{i^{10}}$

$=\frac{1}{(i^2)^5}$

$=\frac{1}{(-1)^5}$

$=\frac{1}{-1}$ = -1

Question No.4 Prove that $\bar{z} = z$ iff z is real.

Proof :- let z=a-bi and $\bar{z} = a - bi$

Let $\bar{z} = z$

Then a-bi= $a + bi$

⇒-bi=bi

⇒ $bi + bi$ =0

⇒2bi=0

⇒b=0 so z=a (real)

Conversely let z be a real number that is z=a, a ϵ ℝ

Then $\bar{z} = \bar{a} = a$ because 'a' is a real number.

Hence $\bar{z} = z$

Question No.5

Simplify by expressing in the form of $a + bi$

i. 5+2$\sqrt{-4}$

Solution:- 5+2$\sqrt{4}\sqrt{-1}$

= 5+2(2)i=5+4i

ii. (2+$\sqrt{-3}$)(3+$\sqrt{-3}$)

Solution:- (2+$\sqrt{-3}$)(3+$\sqrt{-3}$)

=(6+2$\sqrt{-3}$ + 3$\sqrt{-3}$ + $(\sqrt{-3})^2$)

=(6+5$\sqrt{-3}$ + (−3))

=(6+5$\sqrt{3}i$ − 3)

=(3+5$\sqrt{3}i$)

iii. $\frac{2}{\sqrt{5}+\sqrt{-8}}$

Solution:- $\frac{2}{\sqrt{5}+\sqrt{-8}}$

$=\frac{2}{\sqrt{5}-\sqrt{8}i} \times \frac{\sqrt{5}+\sqrt{8}i}{\sqrt{5}+\sqrt{8}i}$

$=\frac{2(\sqrt{5}-\sqrt{8}i)}{(\sqrt{5})^2-(\sqrt{8}i)^2}$

$=\frac{2(\sqrt{5}-\sqrt{8}i)}{5-8(i)^2} = \frac{2(\sqrt{5}-\sqrt{8}i)}{5-8(-1)}$

$=\frac{2(\sqrt{5}-\sqrt{8}i)}{5+8} = \frac{2(\sqrt{5}-\sqrt{8}i)}{13}$

$=\frac{2\sqrt{5}}{13} - \frac{2\sqrt{8}}{13}$

iv. $\frac{3}{\sqrt{6}-\sqrt{-12}}$

Solution:- $\frac{3}{\sqrt{6}-\sqrt{-12}}$

$=\frac{3}{\sqrt{6}-\sqrt{-12}} \times \frac{\sqrt{6}+\sqrt{-12}}{\sqrt{6}+\sqrt{-12}}$

$=\frac{3(\sqrt{6}+\sqrt{-12})}{(\sqrt{6})^2-(\sqrt{12}i)^2}$

$=\frac{3(\sqrt{6}+\sqrt{-12})}{6-12(i)^2} = \frac{2(\sqrt{5}+\sqrt{8}i)}{6-12(-1)}$

$=\frac{3\sqrt{6}+\sqrt{12}i)}{6+12} = \frac{3(\sqrt{6}+\sqrt{12}i)}{18}$

$=\frac{3\sqrt{6}}{18}\frac{3.2\sqrt{3}i}{18}$

$=\frac{\sqrt{6}}{6}+\frac{2\sqrt{3}i}{6}$

$=\frac{1}{\sqrt{6}}+\frac{\sqrt{3}i}{3}$

$=\frac{1}{\sqrt{6}}+\frac{1}{\sqrt{3}}i$

Question No.6 Show that

i. $\bar{z}^2+z^2$ is a real number

Solution:- let z=a+ib then

$\bar{z}$=a-ib

$\bar{z}^2+z^2=(a+ib)^2+(a-ib)^2$

$=a^2+b^2i^2+2(a)(ib)+a^2+b^2i^2-2(a)(ib)$

$=2a^2+2b^2(-1)$

$=2a^2-2b^2$ which is real.

ii. $\bar{z}^2-z^2$ is a imaginary number

Solution:- let z=a+ib then

$\bar{z}$=a-ib

$\bar{z}^2-z^2=(a+ib)^2-(a-ib)^2$

$=a^2+b^2i^2+2(a)(ib)-a^2-b^2i^2+2(a)(ib)$

$=4abi$

$=2a^2-2b^2$ Which is imaginary number.

Question No.7 Simplify

i. $(-\frac{1}{2}+\frac{\sqrt{3}}{2}i)^3$

solution:- $(\frac{-1+\sqrt{3}i}{2})^3$ ∴ ω

$=\frac{-1+\sqrt{3}i}{2}$

$=(\omega)^3$ ∴ $\omega^3=1$

=1

ii. $(-\frac{1}{2}-\frac{\sqrt{3}}{2}i)^3$

solution:- $(\frac{-1-\sqrt{3}i}{2})^3$ ∴ ω^2

$=\frac{-1-\sqrt{3}i}{2}$

$=(\omega^2)^3$

$=(\omega^3)^2$

$=(1)^2$ ∴ $\omega^3=1$

=1

iii. $\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)^{-2}(-\frac{1}{2}-\frac{\sqrt{3}}{2}i)$

Solution:- $\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)^{-2}(-\frac{1}{2}-\frac{\sqrt{3}}{2}i)$

$=\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)^{-2+1}$

$=a^3+b^3(-1).i+3a^2bi+3ab^2(-1)$

$=\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)^{-1}$

$=\frac{1}{-\frac{1}{2}-\frac{\sqrt{3}}{2}i}\times\frac{-\frac{1}{2}+\frac{\sqrt{3}}{2}i}{-\frac{1}{2}+\frac{\sqrt{3}}{2}i}$

$=\frac{-\frac{1}{2}+\frac{\sqrt{3}}{2}i}{(\frac{1}{2})^2-(\frac{\sqrt{3}}{2}i)^2}$

$=\frac{-\frac{1}{2}+\frac{\sqrt{3}}{2}i}{\frac{1}{4}-\frac{3}{4}(i)^2}=\frac{-\frac{1}{2}+\frac{\sqrt{3}}{2}i}{\frac{1}{4}-\frac{3}{4}(-1)}$

$=\frac{-\frac{1}{2}+\frac{\sqrt{3}}{2}i}{\frac{1}{4}+\frac{3}{4}}=\frac{-\frac{1}{2}+\frac{\sqrt{3}}{2}i}{\frac{4}{4}}$

$=\frac{-1+\sqrt{3}i}{2}$

iv. $(a+bi)^2$

solution:-$(a)^2+(bi)^2+2abi$

$=a^2+b^2(i)^2+2abi$

$=a^2+b^2(-1)+2abi$

$=a^2-b^2+2abi$

v. $(a+bi)^{-2}$

solution:- $(a+bi)^{-2}$

$=\frac{1}{(a+bi)^2}$

$=\frac{1}{(a)^2+(bi)^2+2abi}$

$=\frac{1}{a^2+b^2(i)^2+2abi}$

$=\frac{1}{a^2+b^2(-1)+2abi}$

$=\frac{1}{(a^2-b^2)+2abi}\times\frac{(a^2-b^2)-2abi}{(a^2-b^2)-2abi}$

$=\frac{(a^2-b^2)-2abi}{(a^2-b^2)^2-4a^2b^2i^2}$

$=\frac{(a^2-b^2)-2abi}{a^4+b^4-2a^2b^2-4a^2b^2(-1)}$

$=\frac{(a^2-b^2)-2abi}{a^4+b^4-2a^2b^2+4a^2b^2}$

$=\frac{(a^2-b^2)-2abi}{(a^2+b^2)^2}$

$=\frac{(a^2-b^2)}{(a^2+b^2)^2}-\frac{2ab}{(a^2+b^2)^2}i$

vi. $(a+bi)^3$

Solution:- $(a+bi)^3$

$=(a)^3+(bi)^3+3(a)(bi)(a+bi)$

$=a^3+b^3(i)^3+3abi(a+bi)$

$=a^3+b^3(i)^2.i+3a^2bi+3ab^2(i)^2$

$=a^3 - b^3 i + 3a^2 bi - 3ab^2$

vii. $(a - bi)^3$

Solution:- $(a - bi)^3$

$=(a)^3 - (bi)^3 - 3(a)(bi)(a - bi)$

$=a^3 - b^3(i)^3 - 3abi(a - bi)$

$=a^3 - b^3(i)^2.i - 3a^2bi + 3ab^2(i)^2$

$=a^3 - b^3(-1).i - 3a^2bi + 3ab^2(-1)$

$=a^3 - b^3 i - 3a^2bi - 3ab^2$

viii. $(3 - \sqrt{-4})^{-3}$

$Solution: -\ (a - \sqrt{4}i)^{-3}$

$$= \frac{1}{(a - 2i)^3}$$

$$= \frac{1}{(3)^3 - (2i)^3 - 3(3)(2i)(3 - 2i)}$$

$$= \frac{1}{27 - 2^3(i)^3 - 18i(3 - 2i)}$$

$$= \frac{1}{27 - 8(i)^2.i - 54i + 36i^2}$$

$$= \frac{1}{27 - 8(-1).i - 54i + 36(-1)}$$

$$= \frac{1}{27 + 8i - 54i - 36}$$

$$= \frac{1}{-9 - 46i} \times \frac{-9 + 46i}{-9 + 46i}$$

$$= \frac{-9 + 46i}{(-9)^2 - (46i)^2}$$

$$= \frac{-9 + 46i}{81 - 2116(i)^2}$$

$$= \frac{-9 + 46i}{81 - 2116(-1)}$$

$$= \frac{-9 + 46i}{81 + 2116}$$

$$= \frac{-9 + 46i}{2197}$$

$$= \frac{-9}{2197} + \frac{46}{2197}$$