10th CLASS MATH

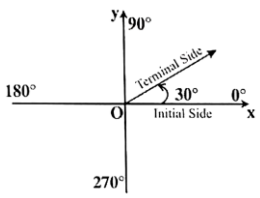
CHAPTER 7

SOLUTION NOTES

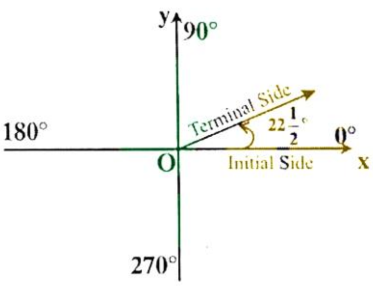
Exercise 7.1

Q.1: Locate the following angles:

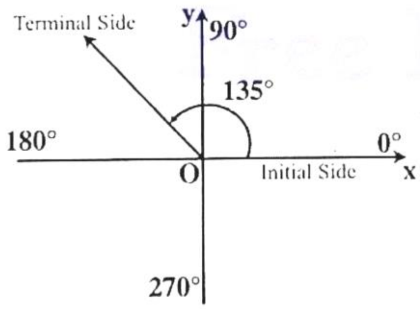
i. 30^0



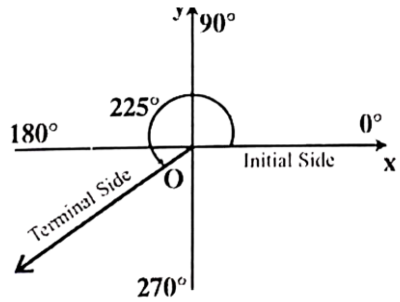
ii. $22\frac{1}{2}^0$



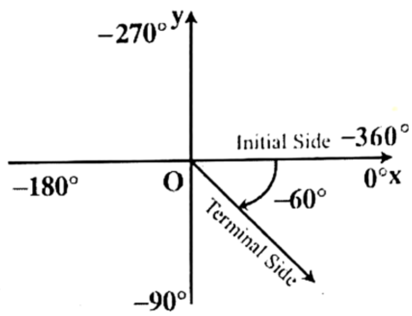
iii. 135^0



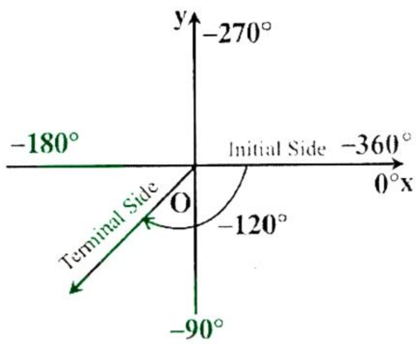
iv. -225^0



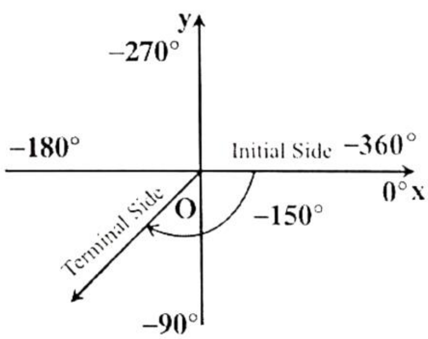
v. -60^0



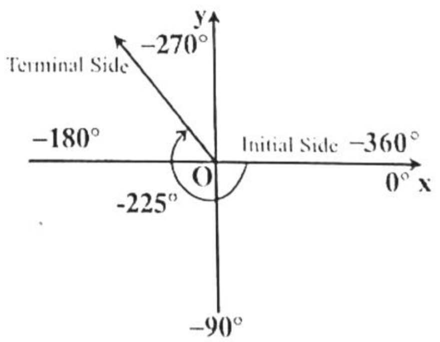
vi. -120^0



vii. -150^0



viii. -225^0



Q.2:
Express the following sexagesiml measures of angles in decimal form.

i. $45^0\ 30'$
Solution:
$$= 45^o + \frac{30^0}{60^0}$$
$$= 45^o + 0.5^0$$
$$= 45.5^o$$

ii. $60^0 30' 30''$
Solution:
$$= 60^0 + \frac{30^o}{60^0} + \frac{30^o}{60^o \times 60^o}$$
$$= 60^o + 0.5^o + 0.008^0$$
$$= 60.508^0$$

iii. $125^o 22' 50''$
Solution:
$$= 125^o + \frac{22^o}{60^o} + \frac{50^o}{60^o \times 60^o}$$
$$= 125^0 + 0.367^0 + 0.0139^o$$
$$= 125.3808^0$$

Q.3: Express the following in $D^o M' S''$:

i. 47.36^0
Solution:
$$= 47^0 + 0.36^0$$
$$= 47^0 + (0.36 \times 60)'$$
$$= 47^0 + 21' + (0.6 \times 60)''$$
$$= 47^0 + 21' + 36''$$
$$= 47^0 21' 36''$$

ii. 125.45^o
Solution:
$$= 125^0 + 0.45^0$$
$$= 125^0 + (0.45 \times 60)'$$
$$= 225^0 + 27'$$
$$225^0 27' 0''$$

iii. 225.75^o
Solution:
$$= 225^o + o.75^o$$
$$= 125^o + (0.75 \times 60)'$$
$$= 125^0 + 45'$$
$$= 225^o 45' 0''$$

iv. -22.5^0
Solution:
$$= -[22^0 + 0.5^0]$$
$$= -[22^0 + (0.5 \times 60)']$$
$$= -[22^o + 30']$$
$$= -22^0 30'$$

v. -67.58^0
Solution:
$$-(67^o + 0.58^0)$$
$$= -[67^0 + (0.58 \times 60)']$$
$$= -[67^0 + 34' + 0.8']$$
$$= [67^0 + 34' + (0.8 \times 60)'']$$
$$= -[67^0 + 34' + 48'']$$
$$= -67^o 34' 48''$$

vi. 315.18^0
$$= 315^o + 0.18^o$$
$$= 315^0 + (0.18 \times 60)'$$
$$= 315 + 10.8'$$
$$= 315^o + 10' + (0.8 \times 60)''$$
$$= 315^0 + 10' + 48''$$
$$= 315^o 10' 48''$$

Q.4: Express the following angles into radians.

i. 30^0
$$= 30 \frac{\pi}{180} radians$$
$$= 30 \frac{\pi}{30 \times 6} radians$$
$$= \frac{\pi}{6}\ radians$$

ii. 60^0
$$= 60\ \times \frac{\pi}{180}\ radian$$
$$= 60 \frac{\pi}{60 \times 3} radian$$
$$= \frac{\pi}{3} radians$$

iii. 135^o
Solution:
$$225^0$$
$$= 225\ \frac{\pi}{180} radians$$
$$= 45 \times 3 \frac{\pi}{45 \times 4} radians$$
$$= \frac{3\pi}{4} radians$$

iv. 225^0
Solution:225^o
$$= 225\ \frac{\pi}{180} radians$$
$$= 45 \times 5 \frac{\pi}{45 \times 4} radians$$
$$= \frac{5\pi}{4} radians$$

v. -150^0
Solution:
$$-150^0$$

$$= -150\,\frac{\pi}{180} radians$$
$$= -5 \times 30\,\frac{\pi}{30 \times 6} radians$$
$$= \frac{-5\pi}{6} radians$$

vi. -225^0

Solution:

$$= -225\,\frac{\pi}{180} radians$$
$$= -5 \times 45\,\frac{\pi}{45 \times 4} radians$$
$$= \frac{-5\pi}{4} radians$$

vii. 300^0

Solution:

$$= 300\,\frac{\pi}{180} radians$$
$$= 60 \times 5\,\frac{\pi}{60 \times 3} radians$$
$$= \frac{5\pi}{3} radians$$

viii. 315^0

Solution:

$$= 315\,\frac{\pi}{180} radians$$
$$= 45 \times 7\,\frac{\pi}{45 \times 4} radians$$
$$= \frac{7\pi}{4} radians$$

Q.5: Convert each of the following to degrees.

i. $\frac{3\pi}{4}$

Solution:

$$\frac{3\pi}{4} radians$$
$$= \frac{3\pi}{4}\,\frac{180}{\pi}\; degree$$
$$= \frac{3\pi}{4}\,\frac{180}{\pi}\quad degree$$
$$= 3 \times 45\; degrees$$
$$= 135^o$$

ii. $\frac{5\pi}{6}$

Solution:

$$\frac{5\pi}{6} radians$$
$$= \frac{5\pi}{6}\,\frac{180}{\pi}\; degree$$
$$= \frac{5\pi}{6}\,\frac{180}{\pi}\quad degree$$
$$= 5 \times 30\; degrees$$
$$= 150^o$$

iii. $\frac{7\pi}{8}$

Solution:

$$\frac{7\pi}{8} radians$$
$$= \frac{7\pi}{8}\,\frac{180}{\pi}\; degree$$
$$= \frac{7 \times 180}{8}\quad degree$$
$$= \frac{1260}{8}\; degrees$$
$$= 157.5^o$$

iv. $\frac{13\pi}{16}$

Solution:

$$\frac{13\pi}{16} radians$$
$$= \frac{13\pi}{16}\,\frac{180}{\pi}\; degree$$
$$= \frac{13 \times 180}{16}\quad degree$$
$$= \frac{2340}{16}\; degrees$$
$$= 146.25^o$$

v. $3\, radians$

Solution:

$$3\, radians$$
$$= 3\,\frac{180}{\pi}\; degree$$
$$= \frac{540}{\pi}\; degrees$$
$$= 171.887^o$$

vi. 4.5

Solution:

$$4.5\, radians$$
$$= 4.5\,\frac{180}{\pi}\; degree$$
$$= \frac{810}{\pi}\; degrees$$
$$= 257.831^0$$

vii. $-\frac{7\pi}{8}$

Solution:

$$-\frac{7\pi}{8}\quad radians$$
$$= -\frac{7\pi}{8}\,\frac{180}{\pi}\; degree$$
$$= \frac{-1260}{8}\; degrees$$
$$= 157.5^o$$

viii. $-\frac{13}{16}\pi$

Solution:

$$-\frac{13\pi}{16}\quad radians$$
$$= -\frac{13\pi}{16}\,\frac{180}{\pi}\; degree$$
$$= \frac{-2340}{16}\; degrees$$
$$= 146.25^o$$

Exercise 7.2

Question No.1 Find θ *when* :

i. $\boldsymbol{I = 2cm, \ r = 3.5cm}$

Solution: using rule

$$I = r\theta$$
$$2 = 3.5\theta$$
$$\frac{2}{3.5} = \theta$$
$$\theta = 0.57\ radian$$

ii. $\boldsymbol{I = 4.5m, \ r = 2.5\ m}$

Solution: $\boldsymbol{using\ rule}$

$$I = r\theta$$
$$\frac{I}{r} = \theta$$
$$\frac{4.5}{2.5} = \theta$$
$$\theta = 1.8radian$$

Question No.2 find $\boldsymbol{I\ \ when}$

i. $\boldsymbol{\theta = 180^0, r = 4.9cm}$

Solution:

$As\ \theta\ should\ be\ in\ radius\ so$

$$\theta = 180^0$$
$$= 180\ \frac{\pi}{180}\ radian$$
$$= \pi\ radian$$

Using rule $I = r\theta$

$$= 4.9cm \times \pi$$
$$= 15.4cm$$

ii. $\boldsymbol{\theta = 60^o\ \ 30', \ \ r = 15mm}$

Solution:

$As\ \theta\ should\ be\ in\ radian, so$

$$\theta = 60^o\ 30'$$
$$= 60^0 + \frac{30^0}{60^o}$$
$$= 60.5^o$$
$$= 60.5\ \frac{\pi}{180} radian$$
$$\theta = 1.056\ radian$$
$$\theta = 1.056\ radian$$
$$using\ rule\ I = r\theta$$
$$= 15mm \times 1.056$$
$$= 15.84mm$$

Questions No.3 find $r, when$

i. $I = 4cm, \ \ \theta = \frac{1}{4}\ \ radian$

Solution:

$$Using\ Rule\ I = r\ \theta$$
$$4cm = r\frac{1}{4}$$
$$4cm \times 4 = r$$
$$r = 16cm$$

ii. $I = 52cm\ , \theta = 45^o$

Solution:$As\ \theta\ \ should\ be\ in\ radians.$

$$\theta = 45^o$$
$$= 45\frac{\pi}{180}\ radian$$

$$= \frac{\pi}{4} radian$$

Now using rule $I = r\theta$

$$52cm = r\frac{\pi}{4}$$

$$\frac{52cm \times 4}{\pi} = r$$

$$r = 66.21\ cm$$

Question No.4 In a circle of radius 12cm, find the length of an arc which subtends a Central angle $\boldsymbol{\theta = 1.5\ radian}$

Solution:

$$Radius = r = 12cm$$

$$Arc\ length = ?$$

$$Central\ angle = \theta = 1.5\ radian$$

Using rule $I = r\theta$

$$I = 12m \times 1.5$$

$$I = 18m$$

Question No.5 In a circle of radius 10m, find the distance travelled by a point moving on this circle if the point makes 3.5 revolution.

Solution:

$Radius = r = 10m$

$Number\ of\ revolutions = 3.5$

$Angle\ of\ one\ revolution = 2\pi$

$Angle\ of\ 3.5\ revolution = \theta$

$$= 3.5 \times 2\ \pi radian$$

$$\theta = 7\pi\ radian$$

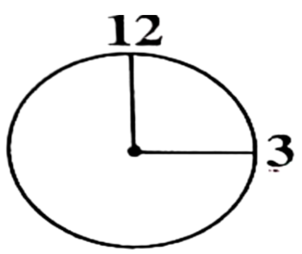
Distance travelled= $I = ?$

$$Using\ rule\ I = r\theta$$

$$I = 10m \times 7\pi$$

$$I = 220m$$

Question No.6 What is the circular measure of the angle between the hands of the watch at 3 O' clock?



Solution:

At 3 O' clock the minute hand will be at 12 and hour hand will be at 3 i.e the angle between the hands of watch will be one quarter of the central angle of full circle.

$$i.e = \frac{1}{4}\ of\ 360^o$$

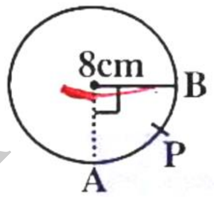
$$\frac{1}{4} \times 360^o$$

$$= 90^o$$

$$= 90\frac{\pi}{180} radian$$

$$= \frac{\pi}{2} radian$$

Question No.7 What is the length of arc APB?



Solution:

From the figure we see that

$$Radius = r = 8cm$$

$$Central\ angle = \theta$$

$$= 90^0$$

$$= \frac{\pi}{2} radian$$

Arc length $I = ?$

By rule $I = r\theta$

$$I = 8cm \times \frac{\pi}{2}$$

$$I = 4cm \times \pi$$

$$I = 12.57\ cm$$

So, length of arc APB is 12.57 cm

Question No.8 In a circle 12cm, how long an arc subtended a central angle of $\boldsymbol{84^0}$?

Solution:

Radius $= r = 12cm$

$Arc\ length = I = ?$

$$Central\ angle = \theta = 84^0$$

$$= 84\frac{\pi}{180}\ radian$$

$$= 1.466\ radian$$

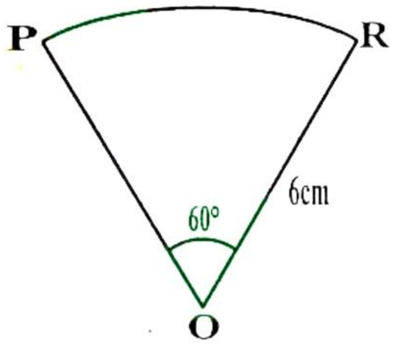
Now by rule $l = r\theta$

$$12cm \times 1.466$$

$$= \mathbf{17.6\ cm}$$

Question No.9 Find the area of sector OPR

(a)



Radius$= r = 6cm$

Central angle $= \theta = 60^o$

$$= 60\frac{\pi}{180}\ radian$$

$$= \frac{\pi}{3} radian$$

Area of sector =?

As area of sector $= \frac{1}{2}r^2\theta$

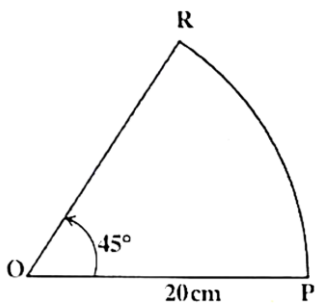
$$= \frac{1}{2} \times (6cm)^2 \times \frac{\pi}{3}$$

$$= \frac{1}{6} \times 36cm^2 \times \pi$$

$$= 6\pi\ cm^2$$

$$= 18.85cm^2$$

(b)



Radius $= r = 20cm$

$$Central\ angle = \theta = 45^0$$

$$= 45\frac{\pi}{180}\ radian$$

$$= \frac{\pi}{4} radian$$

Area of sector =?

Area of Sector $= \frac{1}{2}r^2\theta$

$$= \frac{1}{2}(20cm)^2 \times \frac{\pi}{4}$$

$$= \frac{400cm^2}{8} \times \pi$$

$$= 50\pi cm^2$$

$$= 157.1\ cm^2$$

Question No.10 Find area of sector inside a central angle of 20^0 *in a circle of radius* $7m$.
Solution:
Area of sector =?

$$Radius = r = 7m$$

Central angle$= \theta = 20^o$

$$= 20\frac{\pi}{180}\ radian$$

$$= \frac{\pi}{9} radian$$

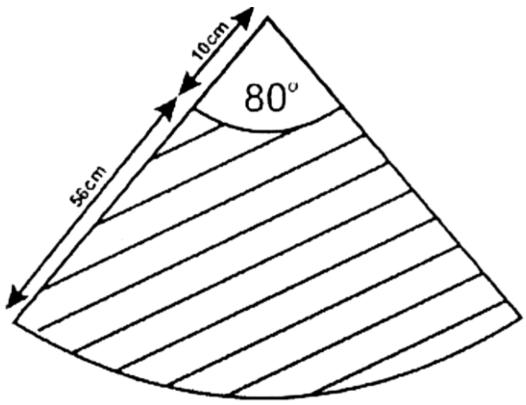
$$Area\ of\ sector = \frac{1}{2}r^2\theta$$

$$= \frac{1}{2} \times (7m)^2 \times \frac{\pi}{9}$$

$$= \frac{49\pi}{18} m^2$$

$$= 8.55m^2$$

Question No.11 Sehar is making skirt. Each panel of this skirt is of the shape shown shaded in the diagram. How much material (cloth) is required for each panel?



Solution:
Central angle $= \theta = 80^0$

$$= 80\ \frac{\pi}{180} radian$$

$$= \frac{4\pi}{9}\ radian$$

Radius of bigger sector $= R = (65 + 10)cm$

$$R = 66cm$$

Radius of smaller sector$= r = 10cm$

$$Shaded\ area = ?$$

$$Area\ of\ bigger\ sector = \frac{1}{2}R^2\theta$$

$$= \frac{1}{2} \times (66cm)^2 \times \frac{4\pi}{9}$$

$$= 4356cm^2 \times \frac{2\pi}{9}$$
$$968\pi cm^2$$

Area of smaller $sector$ $= \frac{1}{2} r^2\theta$

$$= \frac{1}{2} r^2\theta$$
$$= \frac{1}{2}(10cm)^2 \times \frac{4\pi}{9}$$
$$= \frac{200}{9}\pi cm^2$$

Shaded area $968\pi - \frac{200}{9}\pi$

$$= \frac{8712\pi - 200\pi}{9}$$
$$= \frac{8512}{9}\pi cm^2$$
$$= 2971.25cm^2$$

Question No.12 Find the area of a sector with central angle of $\frac{\pi}{5}$ radian in a circle of radius 10cm.

Solution:

Area of sector =?

$$Central\ angle = \theta = \frac{\pi}{5}\ radian$$
$$Radius = r = 10cm$$

Area of sector $= \frac{1}{2}r^2\theta$

$$= \frac{1}{2}(10cm)^2 \times \frac{\pi}{5}$$
$$= \frac{1}{10} \times 100cm^2 \times \pi$$
$$= \frac{1}{10} \times 100cm^2 \times \pi$$
$$= 10\pi cm^2$$
$$= 31.43\ cm^2$$

Question No.13 The area of sector with central angle $\boldsymbol{\theta}$ in circle of radius 2m is 10 square meter. Find $\boldsymbol{\theta\ in\ radius}$

Solution:

Area of sector $= 10m^2$

Radius$= r = 2m$

Central angle$= \theta =?$

As $area\ of\ sector = \frac{1}{2}r^2\theta$

$$10m^2 = \frac{1}{2}(2m)^2\theta$$
$$10m^2 = \frac{1}{2}(4m^2)\theta$$
$$10m^2 = 20m^2$$
$$\theta = \frac{10m^2}{2m^2}$$
$$\theta = 5\ radian$$

Exercise 7.3

Question No.1 Locate each of the following angles in standard position using a protector or fair free hand guess, also find a positive and a negative angle conterminal with each given angle:

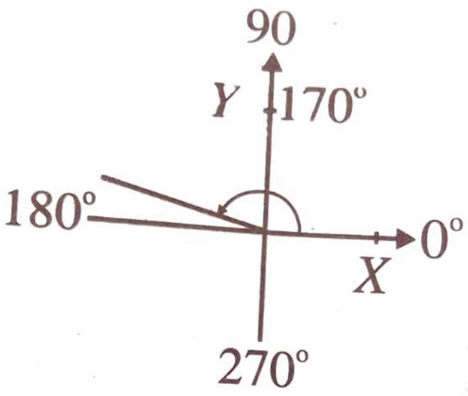
Solution:

i. 170^o

Positive coterminal angle $= 360^o + 170^o$

$= 530^o$

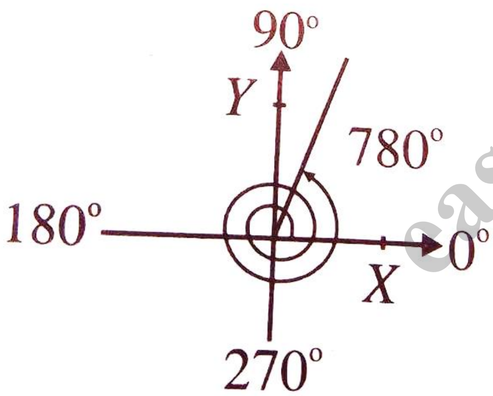
Negative coterminal angle $= -190^0$



ii. 780^0

Positive coterminal angle $780^0 + 2[360^o] = 60^0$

Negative coterminal angle $= -300^0$

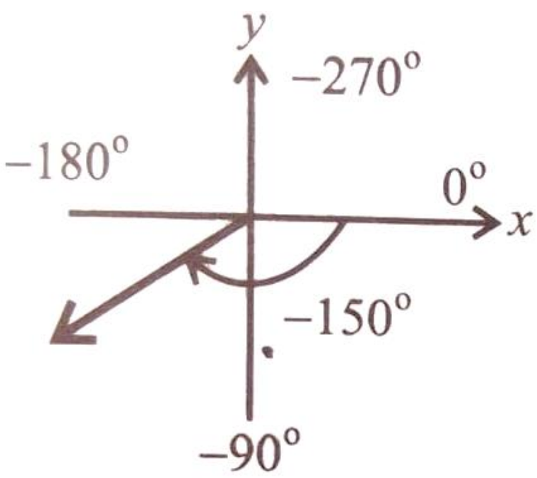


iii. -100^0

Positive coterminal angle 260^0

Negative coterminal angle $= -360^o - 100^o$

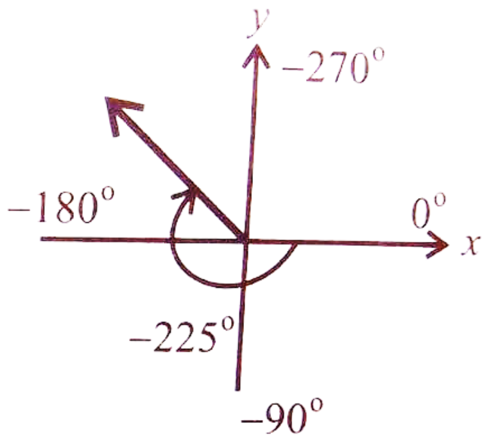
$= -460^0$



iv. -500^o

Positive coterminal angle $= 220^0$

Negative coterminal angle$= -140^o$



Question No.2 Identity closest quadrantile angles between which the following angles lie.

i. 156^0

Answer: 90^0 and 180^0

ii. 318^0

Answer: 270^0 and 360^0

iii. 572^0

Answer: 540^0 and 630^0

iv. -330^0

Answer: 0^o and 90^o

Question No.3 Write the closest quadrantal angles between which the angles lie. Write your answer in radian measure.

i. $\frac{\pi}{3}$

Answer : o and $\frac{\pi}{2}$

ii. $\frac{3\pi}{4}$

Answer: $\frac{\pi}{2}$ and π

iii. $-\frac{\pi}{2}$

Answer: 0 and $-\frac{\pi}{2}$

iv. $-\frac{3\pi}{4}$

Answer : $-\frac{\pi}{2}$ and $-\pi$

Question No.4 in which quadrant θ lies, when

i. $sin\theta > 0, tan < 0$

Answer II $quadrant$

ii. $cos\theta < 0, sin\theta < 0$

Answer: III $quadrant$

iii. $sec\theta > 0, sin\theta < 0$

Answer : IV $quadrant$

iv. $co\theta < 0$, $tan\theta < 0$

Answer:II $quadrant$

v. $cosec\theta > 0, cos\theta > 0$

Answer:I $quadrant$

vi. $sin\theta < 0, sec\theta < 0$

Answer: III $quadrant$

Question No.5 Fill in the blanks:

i. $\cos(-150^0) =$ _______ $cos150^o$
ii. $\sin(-310^0) =$ _______ $sin310^o$
iii. $\tan(-210^0) =$ _______ $tan210^o$
iv. $\cot(-45^0) =$ _______ $cot45^0$
v. $\sec(-60^0) =$ _______ $sec60^o$
vi. $\text{cosec}(-137^0) =$ _______ $cosec137^o$

Answers:

i. $+ve$ ii. $-ve$ iii. $-ve$
iv. $-ve$ v. $+ve$ vi. $-ve$

Question No.6 The given point p lies on the terminal side of θ, Find quadrant of θ and all six trigonometric ratios.

i. $(-2,3)$

$we\ have\ x = -2$ and $y = 3, so\ \theta$ lies in quadrant II.

$$r = \sqrt{x^2 + y^2}$$
$$= \sqrt{(-2)^2 + (3)^2}$$
$$= \sqrt{4 + 9}$$
$$= \sqrt{13}$$

Thus,

$$sin\theta = \frac{y}{r} = \frac{3}{\sqrt{13}} \qquad cosec\theta = \frac{\sqrt{13}}{3}$$
$$cos\theta = \frac{x}{r} = -\frac{2}{\sqrt{13}} \qquad sec\theta = -\frac{\sqrt{13}}{2}$$
$$tan\theta = \frac{y}{x} = -\frac{3}{2} \qquad cot\theta = -\frac{2}{3}$$

ii. $(-3,4)$

$we\ have\ x = -3$ and $y = 4, so\ \theta$ lies in quadrant III.

$$r = \sqrt{x^2 + y^2}$$
$$= \sqrt{(-3)^2 + (4)^2}$$
$$= \sqrt{9 + 16}$$
$$= \sqrt{25}$$
$$= 5$$

Thus,

$$sin\theta = \frac{y}{r} = \frac{-4}{5} \qquad cosec\theta = \frac{-5}{4}$$
$$cos\theta = \frac{x}{r} = \frac{-3}{5} \qquad sec\theta = -\frac{5}{3}$$
$$tan\theta = \frac{y}{x} = \frac{4}{3} \qquad cot\theta = \frac{3}{4}$$

iii. $(\sqrt{2}, 1)$

We have $x = \sqrt{2}$ and $y = 1$ so θ lies in quadrant II.

$$r = \sqrt{x^2 + y^2}$$
$$r = \sqrt{\left(\sqrt{2}\right)^2 + (1)^2}$$
$$= \sqrt{2 + 1}$$
$$= \sqrt{3}$$

Thus,

$$sin\theta = \frac{y}{r} = \frac{1}{\sqrt{3}} \qquad cosec\theta = \sqrt{3}$$
$$cos\theta = \frac{x}{r} = \frac{\sqrt{2}}{\sqrt{3}} \qquad sec\theta = \frac{\sqrt{3}}{\sqrt{2}}$$
$$tan\theta = \frac{y}{x} = \frac{1}{\sqrt{2}} \qquad cot\theta = \sqrt{2}$$

Question No.7 if $\cos\theta = -\frac{2}{3}$ and terminal arm of the angle θ is in quadrant II, find the valves of remaining trigonometric functions.

In any right triangles XYZ

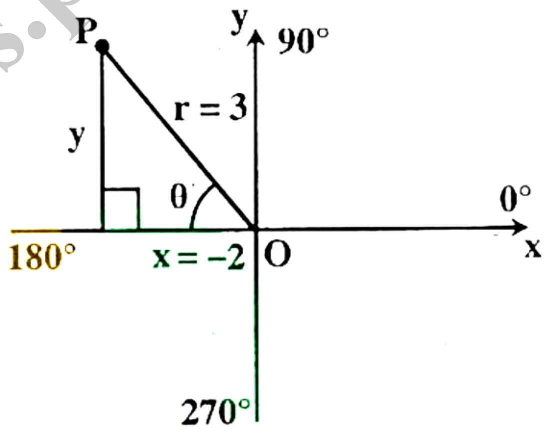
$cos\theta = -\frac{2}{3} = \frac{x}{r}\ \ then\ x = -2\ \ and\ r = 3$

Also,

$$sec\theta = \frac{1}{cos\theta} = -\frac{3}{2}$$

As we know

$$r^2 = x^2 + y^2$$
$$(3)^2 = (-2)^2 + y^2$$
$$9 = 4 + y^2$$
$$y^2 = 5$$
$$y = \pm\sqrt{5}\ \ so\ \ y = \sqrt{5}$$



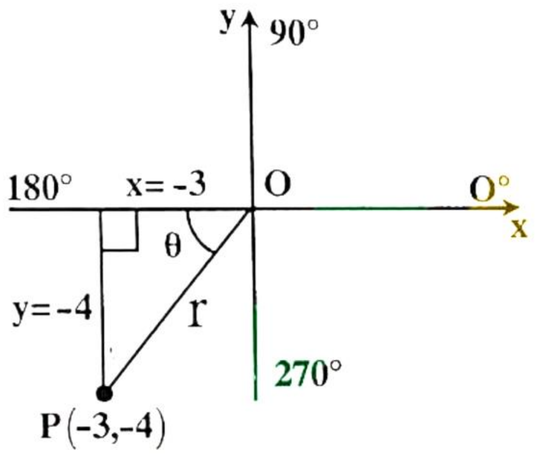
Now

$$sin\theta = \frac{y}{r} = \frac{\sqrt{5}}{3} \qquad cosec\theta = \frac{r}{y} = \frac{3}{\sqrt{5}}$$
$$cos\theta = \frac{x}{r} = \frac{-2}{3} \qquad sec\theta = \frac{r}{x} = \frac{-3}{2}$$
$$tan\theta = \frac{y}{x} = \frac{-\sqrt{5}}{2} \qquad cot\theta = \frac{-2}{\sqrt{5}}$$

Question No.8 $if\ tan\theta = \frac{4}{3}\ and\ sin\theta < 0, find\ the$ valves of other trigonometric functions at θ

Solution:

As $\tan\theta = \frac{3}{4}\ and\ sin\theta\ is - ve, which$ is possible in quadrant III only. We complete the figure.



From the figure $x = -3\ and\ y = -4$
By Pythagorean theorem

$$r^2 = x^2 + y^2$$
$$r = \sqrt{x^2 + y^2}$$
$$r = \sqrt{(-3)^2 + (-4)^2}$$
$$r = \sqrt{9+6}$$
$$r = \sqrt{25}$$
$$r = 5$$

Now,

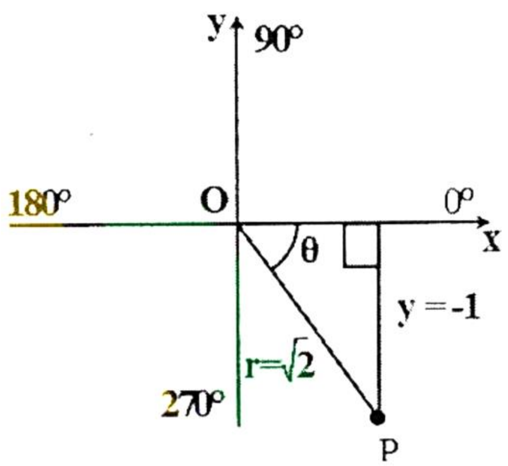
$$sin\theta = \frac{y}{r} = -\frac{4}{5}$$
$$cos\theta = \frac{x}{r} = \frac{-3}{5}$$
$$tan\theta = \frac{y}{x} = \frac{4}{3}$$

$$cosec\theta = \frac{r}{y} = \frac{-5}{4}$$
$$sec\theta = \frac{r}{x} = \frac{-5}{3}$$
$$cot\theta = \frac{3}{4}$$

Question No. 9 $\boldsymbol{if\ sin\theta = -\frac{1}{\sqrt{2}}}$**, and terminal side of the angle is not in quadrant** $\boldsymbol{III}$**, find the valves of** $\boldsymbol{tan\theta}$**,** $\boldsymbol{sec\theta}$ **and** $\boldsymbol{cosec\theta}$**.**

Solution:

As $\sin = -\frac{1}{\sqrt{2}}$ *and terminal* side of angle is not in III quadrant, so it lies in quadrant IV.



From the figure $y = -1\ and\ r = \sqrt{2}$
By Pythagorean theorem

$$r^2 = x^2 + y^2$$
$$x^2 = r^2 - y^2$$
$$x = \sqrt{r^2 - y^2}$$
$$r = \sqrt{\left(\sqrt{2}\right)^2 - (-1)^2}$$
$$r = \sqrt{2-1}$$
$$r = \sqrt{1}$$
$$r = 1$$

Now,

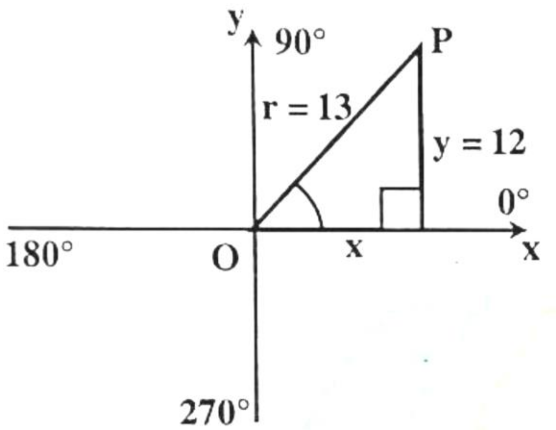
$$Tan\theta = \frac{y}{x} = -\frac{1}{1} = -1$$
$$sec\theta = \frac{r}{x} = \frac{\sqrt{2}}{1} = \sqrt{2}$$
$$cosec\theta = \frac{r}{y} = \frac{\sqrt{2}}{1} = -\sqrt{2}$$

Question No.10 If $\boldsymbol{cosec\theta = \frac{13}{12}}$ $\boldsymbol{and\ sec\theta > 0\ find}$

The remaining trigonometric functions.

Solution:

As, $cosec\theta = \frac{13}{12}$ $and\ also\ sec\theta\ is\ +ve$, which is only possible in quadrant I



From the figure $y = 12\ and\ r = 13$
By Pythagorean theorem

$$r^2 = x^2 + y^2$$
$$x^2 = r^2 - y^2$$
$$x = \sqrt{r^2 - y^2}$$
$$r = \sqrt{(13)^2 - (12)^2}$$
$$r = \sqrt{169 - 144}$$
$$r = \sqrt{25}$$
$$r = 5$$

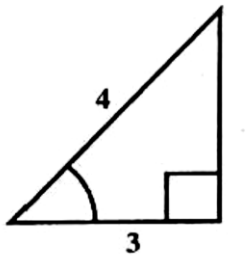
Now,

$$sin\theta = \frac{y}{r} = \frac{12}{13}$$
$$cos\theta = \frac{x}{r} = \frac{5}{13}$$
$$tan\theta = \frac{y}{x} = \frac{12}{5}$$

$$cosec\theta = \frac{r}{y} = \frac{13}{12}$$
$$sec\theta = \frac{r}{x} = \frac{13}{5}$$
$$cot\theta = \frac{5}{12}$$

Question No.11 Find the valves of trigonometric functions at the indicated angles θ in the right triangles.

i.



From the figure Hypotenuse $= 4$ *and Base* $= 3$

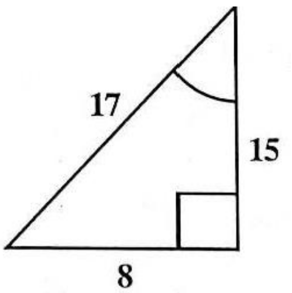
By Pythagorean theorem we can find perpendicular.

$$(Perp)^2 + (Base)^2 = (Hyp.)^2$$
$$(perp.)^2 + (3)^2 = (4)^2$$
$$(perp)^2 = 16 - 9$$
$$(perp)^2 = 7$$
$$perpendicual = \sqrt{7}$$

Now

$$sin\theta = \frac{Per.}{Hyp.} = \frac{\sqrt{7}}{4} \qquad cosec\theta = \frac{Hyp.}{Per.} = \frac{4}{\sqrt{7}}$$
$$cos\theta = \frac{Base}{Hyp.} = \frac{3}{4} \qquad sec\theta = \frac{Hyp.}{Base} = \frac{4}{3}$$
$$tan\theta = \frac{Per.}{Base} = \frac{\sqrt{7}}{3} \qquad cot\theta = \frac{Base}{Per.} = \frac{3}{\sqrt{7}}$$

ii. From the figure



$Hypertenous = 17$

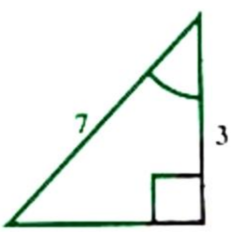
$Perperdicular = 8$

$Base = 15$

Now

$$sin\theta = \frac{Per.}{Hyp.} = \frac{8}{17} \qquad cosec\theta = \frac{Hyp.}{Per.} = \frac{17}{8}$$
$$cos\theta = \frac{Base}{Hyp.} = \frac{15}{17} \qquad sec\theta = \frac{Hyp.}{Base} = \frac{15}{17}$$
$$tan\theta = \frac{Per.}{Base} = \frac{8}{15} \qquad cot\theta = \frac{Base}{Per.} = \frac{15}{8}$$

iii. From the figure



$$hypotenous = 7 \;\; Base = 3$$

we can find perpendicular by Pythagorean theorem.

$$(Base)^2 + (PerP)^2 = (Hyp.)^2$$
$$(Perp.)^2 + (3)^2 = (7)^2$$
$$(perp.)^2 = 40 - 9$$
$$(perp.)^2 = 40$$
$$Perp. = \sqrt{40}$$
$$Perp. = \sqrt{4 \times 10}$$

Now.

$$sin\theta = \frac{Per.}{Hyp.} = \frac{2\sqrt{10}}{7} \qquad cosec\theta = \frac{Hyp.}{Per.} = \frac{2\sqrt{10}}{\sqrt{7}}$$
$$cos\theta = \frac{Base}{Hyp.} = \frac{3}{7} \qquad sec\theta = \frac{Hyp.}{Base} = \frac{7}{3}$$
$$tan\theta = \frac{Per.}{Base} = \frac{2\sqrt{10}}{3} \qquad cot\theta = \frac{Base}{Per.} = \frac{3}{2\sqrt{10}}$$

Question No.12 Find the value of the trigonometric functions. Do not use trigonometric table or calculator.

Solution:

we know that $2k\pi + \theta = \theta, where\ k \in Z$

i. $tan30^o$

$$30^o = 30\ \frac{\pi}{180}\ radian = \frac{\pi}{6} radian$$
$$tan30^o = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$$

ii. $tan330^o$

$$tan330^o = \tan(360^o - 30^o)$$
$$= tan2\pi - \frac{\pi}{6}$$
$$= \tan\left(-\frac{\pi}{6}\right)$$
$$= -\tan\frac{\pi}{6}$$
$$= -\frac{1}{\sqrt{3}}$$

iii. $sec\ 330^0$

$$sec330^o = \sec(360^0 - 30^0)$$
$$= \sec 2\pi - \frac{\pi}{6}$$
$$= sec - \frac{\pi}{6}$$
$$= \sec\frac{\pi}{6}$$
$$= \frac{2}{\sqrt{3}}$$

iv. $\cot\frac{\pi}{4}$

$$= \frac{1}{\tan\frac{\pi}{4}}$$
$$= \frac{1}{tan45^o} = \frac{1}{1} = 1$$

v. $\cos\frac{2\pi}{3}$

$$\cos(120^0) = -\frac{1}{2}$$

vi. $cosec\frac{2\pi}{3}$

$$cosec\frac{2\pi}{3} = cosec120^0 = \frac{1}{\sin(120^0)} = \frac{1}{\frac{\sqrt{3}}{2}}$$
$$= \frac{2}{\sqrt{3}}$$

vii. $\cos(-450^0)$

$$\cos(-450^0) = \cos(-360^0 - 90^0)$$
$$cos - 2\pi - \frac{\pi}{2}$$
$$= cos2(-1)\pi - \frac{\pi}{2}$$
$$cos\frac{\pi}{2} = 0$$

viii. $\tan(-9\pi)$

$$\tan(-9\pi) = \tan(-8\pi - \pi)$$
$$= \tan[2(-4)\pi - \pi]$$
$$= \tan(-8\pi + (-\pi))$$
$$= \tan(-\pi)$$
$$= 0$$

ix. $cos\left(-\frac{5\pi}{6}\right)$

$$= \cos\left(-\frac{5\pi}{6}\right)$$
$$= -\cos\frac{\pi}{6} = -\frac{\sqrt{3}}{2}$$

x. $\sin\frac{7\pi}{6}$

$$\sin\frac{7\pi}{6} = \sin\left(2\pi - \frac{5\pi}{6}\right)$$
$$= \sin\left[2\pi + \left(-\frac{5\pi}{6}\right)\right]$$
$$= \sin\left(-\frac{5\pi}{6}\right) = \sin(-150^0) = -\frac{1}{2}$$

xi. $\cot\frac{7\pi}{6}$

$$\cot\frac{7\pi}{6} = \cot\left[2\pi + \left(-\frac{5\pi}{6}\right)\right]$$
$$= \cot\left(-\frac{5\pi}{6}\right)$$
$$= \frac{1}{\tan\left(-\frac{5\pi}{6}\right)} = \frac{1}{\tan(-150^0)} = \frac{1}{\frac{1}{\sqrt{3}}} = \sqrt{3}$$

xii. $\cos 225^0$

$$\cos(225^0) = \cos(180^0 + 45^0)$$
$$= \cos\pi + \frac{\pi}{4}$$
$$= -\cos\frac{\pi}{4} = -\frac{1}{\sqrt{2}}$$

Exercise 7.4

Things to know:

$$\cos^2\theta + \sin^2\theta = 1$$
$$1 + \tan^0\theta = \sec^2\theta$$
$$1 + \cot^2\theta = cosec^2\theta$$

In problem $1 - 6$ simplify each expression to a single trigonometric functions.

1. $\frac{\sin^2 x}{\cos^2 x}$

Solution:

$$\because \frac{\sin^2 x}{\cos^2 x} = \tan^2 x$$

2. $tanxsinxsecx$

Solution:

$$tanxsinxsecx = tanxsinx\left(\frac{1}{cosx}\right)$$
$$= \frac{sinx}{cosx}\sin x\frac{1}{cosx}$$
$$= \frac{\sin^2 x}{\cos^2 x}$$
$$= \tan^2 x$$

3. $\frac{tanx}{secx}$

Solution:

$$\frac{tanx}{secx} = \frac{\frac{sinx}{cosx}}{\frac{1}{\cos x}} = \frac{sinx}{cosx} \times \frac{cosx}{1} = sinx$$

4. $1 - \cos^2 x$

Solution:

$$1 - \cos^2 x = \cos^2 x + \sin^2 x - \cos^2 x = \sin^2 x$$

5. $\sec^2 x - 1$

Solution:

$$\sec^2 x - 1 = \sec^2 x - (\sec^2 - \tan^2 x)$$
$$= \sec^2 x - \sec^2 x + \tan^2 x$$
$$= \tan^2 x$$

6. $\sin^2 x . \cot^2 x$

Solution:

$$\sin^2 x . \cot^2 x = \sin^2 x . \frac{\cos^2 x}{\sin^2 x}$$
$$= \cos^2 x$$

in problem 7-24 verify the identities.

7. $(1 - sin\theta)(1 + sin\theta) = \theta$

Solution:

$$\boldsymbol{L.H.S = (1 - sin\theta)(1 + sin\theta)}$$
$$= 1 - \sin^2\theta$$
$$= \cos^2\theta$$
$$= R.H.S$$

8. $\frac{sin\theta + cos\theta}{cos\theta} = 1 + tan\theta$

Solution:

$$L.H.S = \frac{sin\theta + cos\theta}{cos\theta}$$
$$= \frac{sin\theta}{cos\theta} + \frac{cos\theta}{cos\theta}$$
$$= tan\theta + 1$$

$= R.H.S$

9. $(tan\theta + cot\theta)tan\theta = \sec^2\theta$

Solution:

$$L.H.S = (tan\theta + cot\theta)tan\theta$$
$$= \left(\frac{sin\theta}{cos\theta} + \frac{cos\theta}{sin\theta}\right)\frac{sin\theta}{cos\theta}$$
$$= \left(\frac{\sin^2\theta + \cos^2\theta}{sin\theta cos\theta}\right)\frac{sin\theta}{cos\theta}$$
$$= \left(\frac{1}{sin\theta cos\theta}\right)\frac{sin\theta}{cos\theta}$$
$$= \frac{1}{\cos^2\theta}$$
$$= \sec^2\theta$$

10. $(cot\theta + cosec\theta)(tan\theta - sin\theta) = sec\theta - cos\theta$

Solution:

$$L.H.S = (cot\theta + cosec\theta)(tan\theta - sin\theta)$$
$$= \left(\frac{cos\theta}{sin\theta} + \frac{1}{sin\theta}\right)\left(\frac{sin\theta}{cos\theta} - sin\theta\right)$$
$$= \left(\frac{cos\theta + 1}{sin\theta}\right)\left(\frac{sin\theta - sin\theta cos\theta}{cos\theta}\right)$$
$$= \left(\frac{1 + cos\theta}{sin\theta}\right)\left(\frac{sin\theta(1 - cos\theta)}{cos\theta}\right)$$
$$= (1 + cos\theta)\frac{(1 - \cos\theta)}{cos\theta}$$
$$= \frac{1 - \cos^2\theta}{cos\theta}$$
$$= \frac{1}{cos\theta} - \frac{\cos^2\theta}{cos\theta}$$
$$= sec\theta - cos\theta$$
$$= R.H.S$$

11. $\frac{sin\theta + cos\theta}{\tan^2\theta - 1} = \frac{\cos^2\theta}{sin\theta - cos\theta}$

Solution:

$$L.H.S = \frac{sin\theta + cos\theta}{\tan^2\theta - 1}$$
$$= \frac{sin\theta + cos\theta}{\frac{\sin^2\theta}{\cos^2\theta} - 1}$$
$$= \frac{sin\theta + cos\theta}{\frac{\sin^2\theta - \cos^2\theta}{\cos^2\theta}}$$
$$= \frac{sin\theta + cos\theta}{\sin^2\theta - \cos^2\theta} \times \cos^2\theta$$
$$= \frac{sin\theta + cos\theta}{(sin\theta + cos\theta)(sin\theta - cos\theta)} \times \cos^2\theta$$
$$= \frac{1}{sin\theta - cos\theta} \times \cos^2\theta$$
$$= \frac{\cos^2\theta}{sin\theta - \cos\theta}$$
$$= R.H.S$$

12. $\frac{\cos^2\theta}{sin\theta} + sin\theta = cosec\theta$

Solution:

$$L.H.S = \frac{\cos^2\theta}{sin\theta} + sin\theta$$
$$= \frac{\cos^2\theta + \sin^2\theta}{sin\theta}$$
$$= \frac{1}{sin\theta} = cosec\theta$$

13. $sec\theta - cos\theta = tan\theta sin\theta$

Solution:

$$L.H.S = sec\theta - cos\theta$$
$$= \frac{1}{cos\theta} - cos\theta$$
$$= \frac{1 - \cos^2\theta}{cos\theta}$$
$$= \frac{\sin^2\theta}{cos\theta}$$
$$= \frac{sin\theta}{cos\theta} \times sin\theta$$
$$= tan\theta sin\theta$$

14. $\frac{\sin^2\theta}{cos\theta} + cos\theta = sec\theta$

Solution:

$$L.H.S = \frac{\sin^2\theta}{cos\theta} + cos\theta$$
$$= \frac{\sin^2\theta + \cos^2\theta}{cos\theta}$$
$$= \frac{1}{cos\theta}$$
$$= sec\theta$$

15. $tan\theta + cot\theta = sec\theta cosec\theta$

Solution:

$$L.H.S = tan\theta + cot\theta$$
$$= \frac{sin\theta}{cos\theta} + \frac{cos\theta}{sin\theta}$$
$$= \frac{\sin^2\theta + \cos^2\theta}{cos\theta sin\theta}$$
$$= \frac{1}{sin\theta cos\theta}$$
$$= \frac{1}{sin\theta} \times \frac{1}{\cos\theta}$$
$$= sec\theta cosec\theta$$

16. $(tan\theta + cot\theta)(cos\theta + sin\theta) = sec\theta + cosec\theta$

Solution:

$$L.H.S = (tan\theta + cot\theta)(cos\theta + sin\theta)$$
$$= \left(\frac{sin\theta}{cos\theta} + \frac{cos\theta}{sin\theta}\right)(cos\theta + sin\theta)$$
$$= \left(\frac{\sin^2\theta + \cos^2\theta}{cos\theta sin\theta}\right)(cos\theta + sin\theta)$$
$$= \left(\frac{1}{cos\theta sin\theta}\right)(cos\theta + sin\theta)$$
$$= \frac{cos\theta}{cos\theta sin\theta} + \frac{sin\theta}{cos\theta sin\theta}$$
$$= \frac{1}{sin\theta} + \frac{1}{cos\theta}$$
$$= cosec\theta + sec\theta$$
$$R.H.S$$

17. $sin\theta(tan\theta + cot\theta) = sec\theta$

Solution:

$$L.H.S = sin\theta(tan\theta + cot\theta)$$
$$= \sin\theta\left(\frac{sin\theta}{cos\theta} + \frac{cos\theta}{sin\theta}\right)$$
$$= \left(\frac{\sin^2\theta + \cos^2\theta}{cos\theta sin\theta}\right)sin\theta$$
$$= \left(\frac{1}{cos\theta sin\theta}\right)sin\theta$$
$$= \frac{1}{cos\theta}$$
$$= sec\theta$$

18. $\frac{1+cos\theta}{sin\theta} + \frac{sin\theta}{1+cos\theta} = 2cosec\theta$

Solution:

$$L.H.S = \frac{1+cos\theta}{sin\theta} + \frac{sin\theta}{1+cos\theta}$$
$$= \frac{(1+cos\theta)^2 + \sin^2\theta}{sin\theta(1+cos\theta)}$$
$$= \frac{(1+2cos\theta+\cos^2\theta+\sin^2\theta)}{sin\theta(1+cos\theta)}$$
$$= \frac{1+2cos\theta+1}{sin\theta}$$
$$= \frac{2+2cos\theta}{sin\theta(1+cos\theta)}$$
$$= \frac{2(1+cos\theta)}{sin\theta(1+cos\theta)}$$
$$= \frac{2}{sin\theta}$$
$$= 2cosec\theta$$
$$= R.H.S$$

19. $\frac{1}{1-cos\theta} + \frac{1}{1+cos\theta} = 2cosec^2\theta$

Solution:

$$L.H.S = \frac{1}{1-cos\theta} + \frac{1}{1+cos\theta}$$
$$= \frac{1+cos\theta+1-cos\theta}{(1-cos\theta)(1+cos\theta)}$$
$$= \frac{2}{1-\cos^2\theta}$$
$$= \frac{2}{\sin^2\theta}$$
$$= 2cosec^2\theta$$
$$= R.H.S$$

20. $\frac{1+sin\theta}{1-sin\theta} - \frac{1-sin\theta}{1+sin\theta} = 4tan\theta sec\theta$

Solution:

$$L.H.S = \frac{1+sin\theta}{1-sin\theta} - \frac{1-sin\theta}{1+sin\theta}$$
$$= \frac{(1+\sin\theta)^2 - (1-sin\theta)^2}{(1-sin\theta)(1+sin\theta)}$$
$$= \frac{1+2sin\theta+\sin^2\theta-1+2sin\theta-\sin^2\theta}{1-\sin^2\theta}$$
$$= \frac{4sin\theta}{\cos^2\theta}$$
$$= \frac{4sin\theta}{\cos\theta} \times \frac{1}{cos\theta}$$
$$= 4tan\theta sec\theta$$

21. $\sin^3\theta = sin\theta - sin\theta\cos^2\theta$

Solution:

$$R.H.S = sin\theta - sin\theta\cos^2\theta$$
$$= sin\theta(1-\cos^2\theta)$$
$$= sin\theta(\sin^2\theta)$$
$$= \sin^3\theta$$
$$= L.H.S$$

22. $\cos^4\theta - \sin^4\theta = \cos^2\theta - \sin^2\theta$

Solution:

$$L.H.S = \cos^4\theta - \sin^4\theta$$
$$= (\cos^2\theta)^2 - (\sin^2\theta)^2$$
$$= (\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta)$$
$$= (\cos^2\theta - \sin^2\theta)(1)$$
$$= R.H.S$$

23. $\sqrt{\frac{1+cos\theta}{1-sin\theta}} = \frac{sin\theta}{1-cos\theta}$

$$L.H.S = \sqrt{\frac{1+cos\theta}{1-sin\theta}}$$
$$= \sqrt{\frac{(1+cos\theta)(1-cos\theta)}{(1-cos\theta)(1-cos\theta)}}$$
$$= \sqrt{\frac{1-\cos^2\theta}{(1-cos\theta)^2}}$$
$$= \sqrt{\frac{\sin^2\theta}{(1-cos\theta)^2}}$$
$$= \frac{sin\theta}{1-cos}$$
$$= R.H.S$$

24. $\sqrt{\frac{sec\theta+1}{sec\theta-1}} = \frac{sec\theta+1}{tan\theta}$

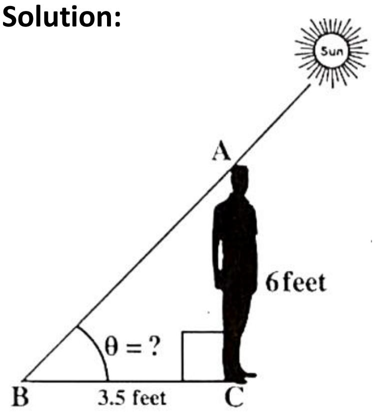
Solution:

$$= \sqrt{\frac{sec\theta+1}{sec\theta-1}}$$
$$= \sqrt{\frac{(sec\theta+1)(sec\theta+1)}{(sec\theta-1)(sec\theta+1)}}$$
$$= \sqrt{\frac{(sec\theta+1)^2}{\sec^2\theta-1}}$$
$$= \sqrt{\frac{(sec\theta+1)^2}{\tan^2\theta}}$$
$$R.H.S$$

Exercise 7.5

Question No.1 Find the angle of elevation of the sun if a 6feet man casts a 3.5 feet shadow.

Solution:

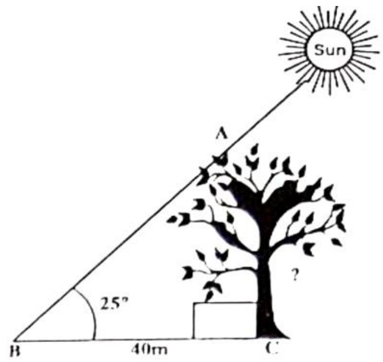


From figure we have

$$tan\theta = \frac{AB}{BC}$$
$$tan\theta = \frac{6}{3.5}$$
$$tan\theta = 1.714$$
$$\theta = \tan^{-1}(1.7143)$$
$$\theta = 59.7437^0$$
$$\theta = 59^0 44' 37''$$

Question No.2 A true casts a 40 meter shadow when the angle of elevation of the sun is 25^0. Find the height of the tree.

Solution:



From the figure

Height of tree $= m\overline{AC} = ?$

Length of shadow$= m\overline{BC} = 40m$

Angle of Elevation $= \theta = 25^0$

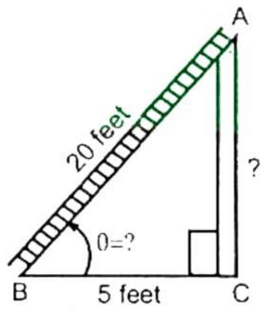
Angle of fact that

$$tan\theta = \frac{m\overline{AC}}{m\overline{BC}}$$
$$tan\theta = \frac{m\overline{AC}}{40}$$
$$m\overline{AC} = 40 \times tan25^0$$
$$m\overline{AC} = 18.652m$$

So, height of tree is $18.652\ m$

Question No.3. A feet long ladder is learning against a wall. The bottom of the wall. Find the acute angle (angle of elevation) the ladder makes with the ground.

Solution:



from the figure

Length of ladder $= m\ \overline{AB} = 20feet$

Distance of ladder from the wall$= m\ \overline{BC} = 5\ feet$

$$Angle\ of\ elevation = \theta = ?$$

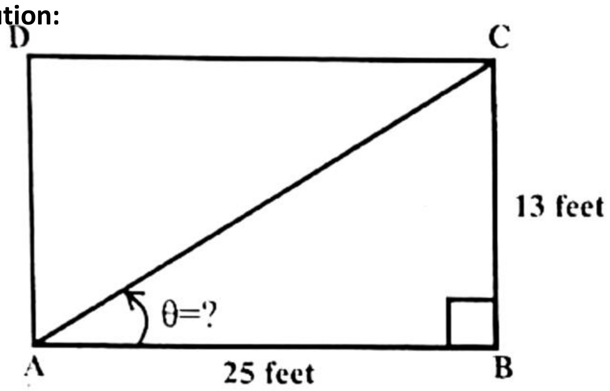
Using the fact that

$$cos\theta = \frac{m\ \overline{BC}}{m\ \overline{AB}}$$
$$cos\theta = \frac{5ft.}{20ft.}$$
$$cos\theta = 0.25$$
$$\theta = \cos^{-1} 0.25$$
$$\theta = 75.5225$$
$$\theta = 75.5^0$$
$$or \quad \theta = 75^0 30'$$

So, angle of elevation is $75^0 30'$

Question No.4 The base of rectangular is 25 feet and the height of rectangular is 13 feet. Find the angle that diagram of the rectangular makes with the base.

Solution:



From the figure

Base of rectangular $= m\overline{BC} = 25feet$

Height of rectangular $= m\overline{BC} = 13feet$

Diagonal $\overline{AC}$ *is taken*

Angle between diagonal and base= θ

Using the fact that

$$tan\theta = \frac{m\overline{BC}}{m\overline{AB}}$$

$$tan\theta = \frac{13}{25}$$

$$\theta = \tan^{-1}\frac{13}{25}$$

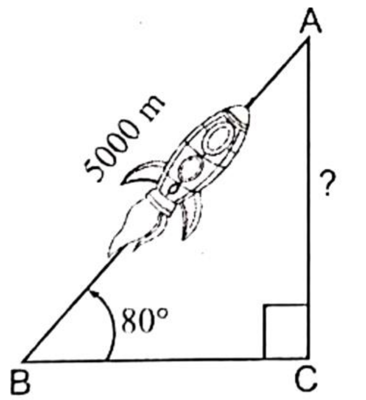
$$\theta = 27.4744$$

$$\theta = 27.47^0$$

So, angle between diagonal and base is 27.47^0

Question No.5 A rocket is launched and climbs at a constant angle of 80^0. Find the altitude of the rocket after it travels 5000 meter.

Solution:



From the figure

Distance travelled by rocket $= m\overline{AB} = 5000m$

$$Altitude\ of\ rocket = m\overline{AC} = ?$$

Angle of elevation $= \theta = 80^0$

Using

$$sin\theta = \frac{m\overline{AC}}{m\overline{AB}}$$

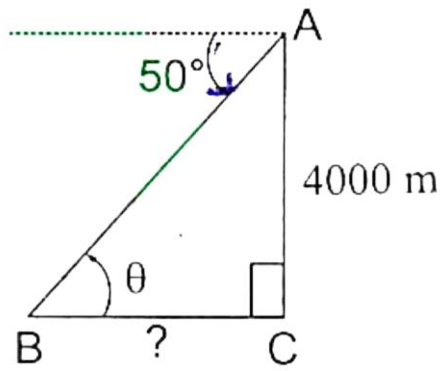
$$sin80^0 = \frac{m\overline{AC}}{5000}$$

$$m\overline{AC} = 5000 \times sin80^0$$

$$m\overline{AC} = 4924.04m$$

Question No.6 An aero plane pilot flying at an altitude of 4000m wishes to make an approaches to an airport at an angle of 50^0 *with the* plane be when the pilot begins to descend?

Solution:



From the figure

Altitude of aero plane= $m\overline{AC} = 4000m$

Distance of plane from airport= $m\overline{BC} = ?$

Angle of depression= 50^0

As the altimeter angles of parallel lines are equal, so angle

$$\theta = 50^0$$

Using the fact that

$$tan\theta = \frac{m\overline{AC}}{m\overline{BC}}$$

$$tan50^0 = \frac{4000m}{m\overline{BC}}$$

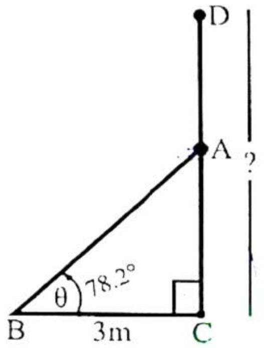
$$m\overline{BC} = \frac{4000m}{tan50^0}$$

$m\overline{BC} = 33356.4$m

So, the distance of aero plane from airport is 3356.4m

Question No.7 A guy wire (supporting wire) runs from the middle of a utility pole to ground. The wire makes an angle of 78.2^0 *with the ground* and touch the ground 3 meters from the base of the pole. Find the height of the pole.

Solution:



From the figure

Height of pole= $m\overline{CD} = ?$

Distance of wire from the base of the pole

$$= m\overline{BC} = 3m$$

Angle of elevation= $\theta = 78.2^0$

As the wire is attached with the pole at its middle point A so first we find $m\overline{AC}$

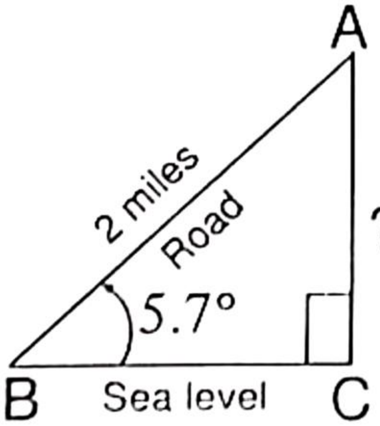
Using the fact that

$$tan\theta = \frac{m\overline{AC}}{m\overline{Bc}}$$
$$tan78.2 = \frac{m\overline{AC}}{3}$$
$$m\overline{AC} = 3m \times tan78.2^0$$
$$m\overline{AC} = 14.36m$$

So, Height of pole is $= m\overline{DC} = 2(m\overline{AC})$
$$= 2 \times 14.36m$$
$$= 28.72m$$

Question No.8 A road is inclined at an angle 5.7^0 suppose that we drive 2 miles up this road starting from sea level. How high above sea level are we?

Solution:



From the figure

Distance covered on road $= m\overline{AB} = 2miles$

Angle of inclination $= \theta = 5.7^0$

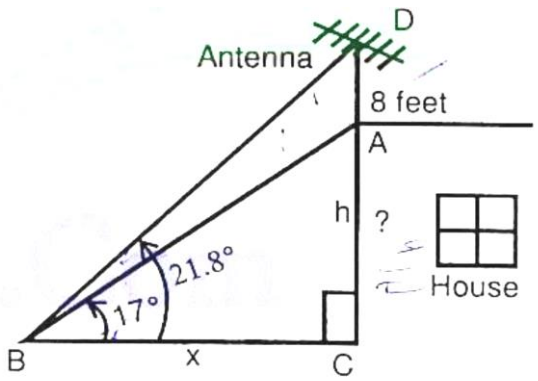
Height from sea level $= m\overline{AC} = ?$

Using the fact that,
$$sin\theta = \frac{m\overline{AC}}{m\overline{AB}}$$
$$sin5.7^0 = \frac{m\overline{AC}}{2}$$
$$m\overline{AC} = 2 \times sin5.7^0$$
$$m\overline{AC} = 0.199mile$$

Hence , we are at height of 0.199 mile from the sea level.

Question No.9 A television antenna of 8 feet height is point on the top of a house. From a point on the ground the angle of elevations to the top of the house is 17^0
And the angle of elevation to the top of antenna is 21.8^0. find the height of the house.

Solution:



From the figure

Distance of point from house $m\overline{BC} = x$

Height of house $= m\overline{AC} = h = ?$

Height of antenna $= m\overline{AD} = 8feet$

Angle of elevation of top of house $= 17^0$

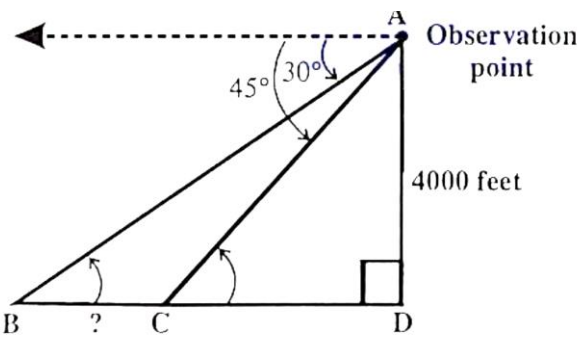
Angle of elevation of top of antenna$= 21.8^0$

In right angled ΔABC
$$tan17^0 = \frac{m\overline{AC}}{m\overline{BC}}$$
$$tan17^0 = \frac{h}{x}$$
$$x = \frac{1}{tan17^0} \times h$$
$x = 3.271 \times h \rightarrow (i)$

Now in right angle ΔDBC
$$tan21.8 = \frac{m\overline{CD}}{m\overline{BC}}$$
$$tan21.8 = \frac{m\overline{AD} + m\overline{AC}}{m\overline{BC}}$$
$$tan21.8 = \frac{8+h}{x}$$
$$0.40 = \frac{8+h}{3.271h} \quad from\ (i)$$
$$0.40 \times 3.271h = 8 + h$$
$$1.3084h - h = 8$$
$$(1.3084 - 1)h = 8$$
$$0.3084h = 8$$
$$h = \frac{8}{0.3084}$$
$$h = \frac{8}{0.3084} = 25.94feet$$

question No.10 from an observation point, the angles of depression of two boats in line with this point are found to 30^0 and 45^0 find the distance between the two bosts if the point of observation is 4000 feet high.

Solution:



From the figure

Height of observation point $= m\overline{AD} = 4000feet$

Distance between boats $= m\overline{BC} = ?$

Angle of depression of points $B\ and\ C\ are$ $30^0\ and\ 45^0$ respectively from point A.

As the alternate angles of parallel lines are equal, so
$$m\angle B = 30^0 \quad and\ m\angle C = 45^0$$

Now in right angled ΔACD

$$tan45^0 = \frac{m\overline{AD}}{m\overline{CD}}$$

$$1 = \frac{4000}{m\overline{CD}}$$

$$m\overline{CD} = 4000feet$$

Now in right angled ΔACD

$$tan30^0 = \frac{m\overline{AD}}{m\overline{BD}}$$

$$\frac{1}{\sqrt{3}} = \frac{4000}{m\overline{BC} + m\overline{CD}}$$

$$\frac{1}{\sqrt{3}} = \frac{4000}{m\overline{BC} + 4000}$$

$$m\overline{BC} + 4000 = 4000\sqrt{3}$$

$$m\overline{BC} = 4000\sqrt{3} - 4000$$

$$m\overline{BC} = 6928.20 - 4000$$

$$m\overline{BC} = 2928.20feet$$

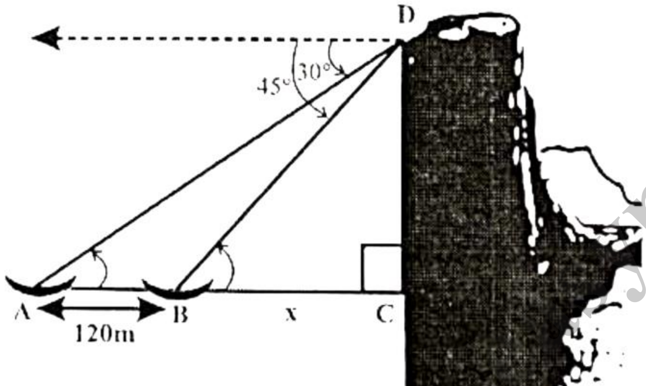
So, the distance between boats is 2928.2 feet.

Question No.11 Two ships, which are in lines with the base of a vertical cliff are 120 meters apart. The angles of depression from the top of the cliff to the ship are 30^0 and 45^0 as shown in the diagram.

(a) Calculate the distance BC

(b) Calculate the height CD of the cliff.

Solution:



From the figure

Height of cliff $= \overline{CD} = h = ?$

Distance$= \overline{BC} = x = ?$

Distance between boats $= \overline{AB} = 120m$

Angles of depression from point D to point A and B are 30^0 and 45^0 respectively.

As the altitude angles of parallel lines are equal, so $m\angle A = 30^0$ and $m\angle B = 45^0$

In right angled ΔBCD

$$tan45^0 = \frac{m\overline{CD}}{m\overline{BC}}$$

$$l = \frac{h}{x}$$

$$x = h \rightarrow (i)$$

Now in right angled ΔACD

$$tan30^0 = \frac{m\overline{CD}}{m\overline{AC}}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{m\overline{BC} + m\overline{BC}}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{120 + x}$$

$$120 + x = \sqrt{3}h$$

$$120 + h = \sqrt{3}h$$

$$120 = \sqrt{3}h - h$$

$$120 = h(\sqrt{3} - 1)$$

$$120 = h(1.7321 - 1)$$

$$120 = (0.7321h)$$

$$\frac{120}{0.7321} = h$$

$$h = 163.91m$$

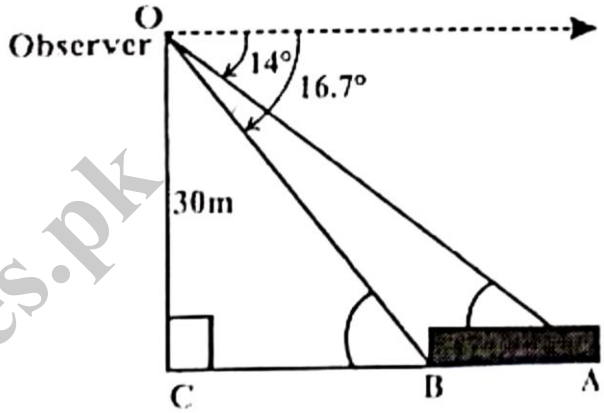
As $x = h$ $, so$

$$x = 163.91m \ \ or\ 164m$$

Height of cliff= $m\overline{CD} = 164m$

Question No.12 Suppose that we are standing on a bridge 30 meter above a river watching a log (piece of wood) floating towards us. If the angle with the horizontal to the front of the log is 16.7^0 and angle with the horizontal to the back of the log is 14^0, *how long is the log*?

Solution:



Height of the observer position $= m\overline{OC} = 30m$

Length of log wood= $m\overline{AB} = x = ?$

Angles of depression from point O of the points A and B are 14^0 and $=16.7^0$ respectively

In right angled ΔOBC

$$tan16.7^0 = \frac{m\overline{OC}}{m\overline{BC}}$$

$$0.30 = \frac{30}{m\overline{BC}}$$

$$m\overline{BC} = \frac{30}{0.30}$$

$$m\overline{BC} = 100m$$

Now in right angled ΔOAC

$$tan14^0 = \frac{m\overline{OC}}{m\overline{AC}}$$

$$0.249 = \frac{30}{m\overline{AB} + m\overline{BC}}$$

$$0.249 = \frac{30}{(x + 100)}$$

$$0.249(x + 100) = 30$$

$$x + 100 = \frac{30}{0.249}$$

$$x + 100 = 120.482$$

$$x = 120.482 - 100$$

$$x = 20.482m$$

So the length of log is 20.48222m.