

10th CLASS

MATH

CHAPTER 4

SOLUTION

NOTES

Exercise 4.1

Resolve into partial fractions.

Question N0.1

$$\frac{7x - 9}{(x + 1)(x - 3)}$$

Solution:

$$\text{Let } \frac{7x - 9}{(x+1)(x-3)} = \frac{A}{(x+1)} + \frac{B}{(x-3)} \rightarrow (i)$$

Multiplying equation (i) by $(x + 1)(x - 3)$

$$7x - 9 = A(x - 3) + B(x + 1) \rightarrow (ii)$$

For determine the value of A and B

$\Rightarrow x - 3 = 0 \Rightarrow x = 3$ and $x + 1 = 0 \Rightarrow x = -1$
putting $x = 3$ and $x = -1$ put in eq(ii) we get

$\begin{aligned} \text{For } x = 3 & \\ 7(3) - 9 &= B(3 + 1) \\ 21 - 9 &= 4B \\ 12 &= 4B \end{aligned}$	$\begin{aligned} \text{For } x = -1 & \\ 7(-1) - 9 &= A(-1 - 3) \\ -7 - 9 &= -4A \\ -16 &= -4A \end{aligned}$
$\Rightarrow B = 3$	$\Rightarrow A = 4$

Putting the value of A and B in equation (i) We
get the required partial fractions

$$\frac{4}{(x + 1)} + \frac{3}{(x - 3)}$$

Thus

$$\frac{7x - 9}{(x + 1)(x - 3)} = \frac{4}{(x + 1)} + \frac{3}{(x - 3)}$$

Question No.2

$$\frac{x - 11}{(x - 4)(x + 3)}$$

Solution:

$$\text{Let } \frac{x - 11}{(x-4)(x+3)} = \frac{A}{(x-4)} + \frac{B}{(x+3)} \rightarrow (i)$$

Multiplying equation (i) by $(x - 4)(x + 3)$ on
Both sides, we get

$$x - 11 = A(x + 3) + B(x - 4) \rightarrow (ii)$$

$$\Rightarrow x + 3 = 0 \Rightarrow x = -3 \text{ and } x - 4 = 0 \Rightarrow x = 4$$

putting $x = -3$ and $x = 4$ put in eq(ii) we get

$\begin{aligned} \text{For } x = -3 & \\ -3 - 11 &= B(-3 - 4) \\ -14 &= -7B \end{aligned}$	$\begin{aligned} \text{For } x = 4 & \\ 4 - 11 &= A(4 + 3) \\ -7 &= 7A \end{aligned}$
$\Rightarrow B = 2$	$\Rightarrow A = -1$

Putting the value of A and B in equation (i) We
get the required partial fractions

$$\frac{-1}{(x - 4)} + \frac{2}{(x + 3)}$$

Hence the required partial fraction are

$$\frac{x-11}{(x-4)(x+3)} = \frac{-1}{(x-4)} + \frac{2}{(x+3)}$$

Question No.3

$$\frac{3x-1}{x^2-1}$$

Solution:

$$\frac{3x-1}{x^2-1} = \frac{3x-1}{(x-1)(x+1)}$$

$$\text{Let } \frac{3x-1}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1} \rightarrow (i)$$

Multiplying both sides by $(x-1)(x+1)$ we get $3x-1 = A(x+1) + B(x-1) \rightarrow (ii)$ Let $x+1=0$ i.e. $x=-1$ and $x-1=0$ i.e. $x=1$
putting $x=-1$ and $x=1$ put in eq(ii) we get

$$\text{For } x=1$$

$$3(1)-1=A(1+1) \\ 3-1=2A \\ 2=2A$$

$$\Rightarrow A=1$$

$$\text{For } x=-1$$

$$3(-1)-1=B(-1-1) \\ -3-1=-2B \\ -4=-2B$$

$$\Rightarrow B=2$$

Hence the required partial fraction are

$$\frac{3x-1}{(x-1)(x+1)} = \frac{1}{x-1} + \frac{2}{x+1}$$

Question No.4

$$\frac{x-5}{x^2+2x-3}$$

Solution:

$$\begin{aligned} \frac{x-5}{x^2+2x-3} &= \frac{x-5}{x^2+3x-x-3} \\ &= \frac{x-5}{x(x+3)-1(x+3)} = \frac{x-5}{(x-1)(x+3)} \\ &= \frac{x-5}{(x-1)(x+3)} = \frac{A}{x-1} + \frac{B}{x+3} \rightarrow (ii) \end{aligned}$$

Multiplying both sides by $(x-1)(x+3)$, we get
 $x-5 = A(x+3) + B(x-1) \rightarrow (ii)$ Let $x+3=0 \Rightarrow x=-3$ and $x-1=0 \Rightarrow x=1$
putting $x=-3$ and $x=1$ in equation(ii) we get

$$\text{For } x=-3$$

$$-3-5=+B(-3-1) \\ -8=-4B \\ B=\frac{-8}{-4}$$

$$\Rightarrow B=2$$

$$\text{For } x=1$$

$$1-5=A(1+3) \\ -4=4A \\ A=\frac{-4}{4}$$

$$\Rightarrow A=-1$$

Hence the required partial fractions are

$$\frac{x-5}{(x-1)(x+3)} = \frac{-1}{x-1} + \frac{2}{x+3}$$

Question No.5

$$\frac{3x+3}{(x-1)(x+2)}$$

Solution:

$$\text{Let } \frac{3x+3}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2} \rightarrow (i)$$

Multiplying both sides by $(x-1)(x+2)$ we get
 $3x+3 = A(x+2) + B(x-1) \rightarrow (ii)$ Let $x-1=0$ i.e. $x=1$ and $x+2=0$ i.e. $x=-2$ Putting $x=1$ and $x=-2$ in equation (ii)

$$\text{For } x=1$$

$$3(1)+3=A(1+2) \\ 3+3=3A \\ 6=3A \\ \frac{6}{3}=A$$

$$\Rightarrow A=2$$

$$\text{For } x=-2$$

$$3(-2)+3=B(-2-1) \\ -6+3=-B \\ -6+3=-3B \\ \frac{-6+3}{-3}=B \\ B=\frac{-3}{-3}$$

$$\Rightarrow A=1$$

Hence the required partial fractions are

$$\frac{3x+3}{(x-1)(x+2)} = \frac{2}{x-1} + \frac{1}{x+2}$$

Question No.6

$$\frac{7x-25}{(x-4)(x-3)}$$

Solution:

$$\text{Let } \frac{7x-25}{(x-4)(x-3)} = \frac{A}{x-4} + \frac{B}{x-3}$$

Multiplying both sides by $(x-4)(x-3)$, we get
 $7x-25 = A(x-3) + B(x-4) \rightarrow (ii)$ Let $x-3=0$ i.e. $x=3$ and $x-4=0$ i.e. $x=4$
Putting $x=3$ and $x=4$ in equation (ii) we get

$$\text{For } x=3$$

$$7(3)-25=B(3-4) \\ 21-25=-B \\ -4=-B$$

$$\Rightarrow B=4$$

$$\text{For } x=4$$

$$7(4)-25=A(4-3) \\ 28-25=1A \\ 3=A$$

$$\Rightarrow A=3$$

Hence the required partial fractions are

$$\frac{7x-25}{(x-4)(x-3)} = \frac{3}{x-4} + \frac{4}{x-3}$$

Question No.7

$$\frac{x^2+2x+1}{(x-2)(x+3)}$$

Solution:

$$\frac{x^2+2x+1}{(x-2)(x+3)} \text{ is an important fraction.}$$

First we resolve it into proper fraction.

By long division we get

$$\begin{array}{r} 1 \\ x^2 + x - 6 \sqrt{x^2 + 2x + 1} \\ \underline{+ x^2 + x - 6} \\ \underline{\underline{+ x + 7}} \end{array}$$

$$\text{We have } \frac{x^2+2x+1}{x^2+x-6} = 1 + \frac{x+7}{x^2+x-6}$$

$$\text{Let } \frac{x^2+2x+1}{(x-2)(x+3)} = \frac{A}{x-2} + \frac{B}{x+3} \rightarrow (i)$$

Multiplying both sides by $(x-2)(x+3)$ we get
 $x+7 = A(x+3) + B(x-2) \rightarrow (ii)$ Let $x+3=0$ i.e. $x=-3$
and $x-2=0$ i.e. $x=2$

$$\begin{aligned} \text{For } x = -3 \\ -3 + 7 = B(-3 - 2) \\ 4 = -5B \\ \Rightarrow B = -\frac{4}{5} \end{aligned}$$

$$\begin{aligned} \text{For } x = 2 \\ 2 + 7 = A(2 + 3) \\ 9 = 5A \\ \Rightarrow A = \frac{9}{5} \end{aligned}$$

Hence the required partial fractions are

$$\frac{x^2 + 2x + 1}{x^2 + x - 6} = 1 + \frac{9}{5(x-2)} - \frac{4}{5(x+3)}$$

Question No.8

$$\frac{6x^3 + 5x^2 - 7}{3x^2 - 2x - 1}$$

Solution:

$\frac{6x^3 + 5x^2 - 7}{3x^2 - 2x - 1}$ is an improper fraction.

First we resolve it into proper fraction.

$$\begin{aligned} & \frac{2x+3}{3x^2-2x-1} \sqrt{6x^3+5x^2-7} \\ & \frac{\pm 6x^3 \pm 4x^2 \mp 2x}{9x^2+2x-7} \\ & \frac{\pm 9x^2 \mp 6x \mp 3}{8x-4} \\ & \frac{6x^3+5x^2-7}{3x^2-2x-1} = (2x+3) + \frac{8x-4}{(3x+1)(x-1)} \end{aligned}$$

$$\text{Now, Let } \frac{8x-4}{(3x+1)(x-1)} = \frac{A}{3x+1} + \frac{B}{x-1}$$

Multiplying both sides by $(3x+1)(x-1)$, we get

$$8x-4 = A(x-1) + B(3x+1) \rightarrow (ii)$$

$$\text{Let } x-1=0 \text{ i.e. } x=1$$

$$\text{and } 3x+1=0 \text{ i.e. } x=-\frac{1}{3}$$

putting $x=1$ and $x=-\frac{1}{3}$ in equation (ii), we get

we get

$$\begin{aligned} \text{For } x=1 \\ 8(1)-4=B[3(1)+1] \\ 8-4=4B \\ 4=4B \end{aligned}$$

$$4B=4 \\ B=\frac{4}{4}$$

$$\Rightarrow B=1$$

$$\begin{aligned} \text{For } x=-\frac{1}{3} \\ 8\left(-\frac{1}{3}\right)-4=A\left(-\frac{1}{3}\right) \\ -\frac{8}{3}-4=A\left(-\frac{1}{3}\right) \end{aligned}$$

$$\begin{aligned} -\frac{8}{3}-4 &= A\left(\frac{-1-3}{3}\right) \\ -\frac{8}{3}-4 &= A(-4) \\ -\frac{20}{3} &= \frac{A(-4)}{3} \\ \Rightarrow A &= 5 \end{aligned}$$

Hence the required fraction when $D(x)$ consists of repeated linear factors are

$$\frac{6x^3 + 5x^2 - 7}{3x^2 - 2x - 1} = 2x + 3 + \frac{5}{3x+1} + \frac{1}{x-1}$$

Exercise 4.2

Resolve into partial fractions:

Question No.1

$$\frac{x^2 - 3x + 1}{(x-1)^2(x-2)}$$

Solution:

$$\text{Let } \frac{x^2 - 3x + 1}{(x-1)^2(x-2)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x-2} \rightarrow (i)$$

Multiplying both sides by $(x-1)^2(x-2)$ we get

$$\begin{aligned} x^2 - 3x + 1 &= A(x-1)(x-2) + B(x-2) + C(x-1)^2 \\ &\rightarrow (ii) \end{aligned}$$

$$x^2 - 3x + 1 = A(x^2 - 3x + 2) + B(x-2) + C(x^2 - 2x + 1)$$

Putting $x-1=0$ i.e. $x=1$ in (ii) we get

$$(1)^2 - 3(1) + 1 = (1-2)$$

$$1-3+1=B(-1)$$

$$-1=-B$$

$$\Rightarrow B=1$$

Putting $x-2=0$ i.e. $x=2$ in (ii) we get

$$(2)^2 - 3 + 1 = C(2-1)^2$$

$$4-6+1=C$$

$$-1=C \Rightarrow -1$$

$$\Rightarrow C=-1$$

Equating the coefficient of x^2 in (ii) we get

$$1=A+C$$

$$1=A-1$$

$$\Rightarrow A=1+1$$

$$\Rightarrow A=2$$

Hence the required partial fractions are

$$\frac{x^2 - 3x + 1}{(x-1)^2(x-2)} = \frac{2}{x-1} + \frac{1}{(x-1)^2} + \frac{1}{x-2}$$

Question No.2

$$\frac{x^2 + 7x + 11}{(x+2)^2(x+3)}$$

Solution:

$$\text{Let } \frac{x^2+7x+11}{(x+2)^2(x+3)} = \frac{A}{x+2} + \frac{B}{(x+2)^2} + \frac{C}{x+3} \rightarrow (i)$$

Multiplying both sides by $(x+2)^2(x+3)$

$$\begin{aligned} x^2 + 7x + 11 &= A(x+2)(x+3) + B(x+3) + C(x+2)^2 \\ &\Rightarrow x^2 + 7x + 11 = A(x^2 + 5x + 6) + B(x+3) + C(x^2 + 4x + 4) \end{aligned}$$

$$\Rightarrow x^2 + 7x + 11 = A(x^2 + 5x + 6) + B(x + 3) + C(x^2 + 4x + 4) \rightarrow (ii)$$

Putting $x + 2 = 0$ i.e. $x = -2$ in (ii) we get

$$(-2)^2 + 7(-2) + 11 = B(-2 + 3)$$

$$4 - 14 + 11 = B$$

$$\Rightarrow B = 1$$

Putting $x + 3 = 0$ i.e. $x = -3$ in (ii) we get

$$(-3)^2 + 7(-3) + 11 = C(-3 + 2)^2$$

$$9 - 21 + 11 = C(-1)^2$$

$$20 - 21 + 11 = C(1)$$

$$-1 = C$$

$$\Rightarrow C = -1$$

Equating coefficient of x^2 in (ii) we get

$$A + C = 1$$

$$A - 1 = 1$$

$$A = 1 + 1$$

$$\Rightarrow A = 2$$

Hence the required partial fractions are

$$\frac{x^2 + 7x + 11}{(x + 2)^2(x + 3)} = \frac{2}{x + 2} + \frac{1}{(x + 2)^2} - \frac{1}{x + 3}$$

Question No.3

$$\frac{9}{(x - 1)(x + 2)^2}$$

Solution:

$$\text{Let } \frac{9}{(x - 1)(x + 2)^2} = \frac{A}{x - 1} + \frac{B}{x + 2} + \frac{C}{(x + 2)^2} \rightarrow (i)$$

Multiplying both sides by $(x - 1)(x + 2)^2$ we get

$$9 = A(x + 2)^2 + B(x - 1)(x + 2) + C(x - 1) \rightarrow (ii)$$

Putting $x - 1 = 0$ i.e. $x = 1$ in (ii) we get

$$9 = A(1 + 2)^2$$

$$9 = A(3)^2$$

$$9 = 9A$$

$$\Rightarrow A = 1$$

Putting $x + 2 = 0$ i.e. $x = -2$ in (ii) we get

$$9 = C(-2 - 1)$$

$$9 = -3C$$

$$\Rightarrow C = -3$$

Equating the coefficient of x^2 in (ii) we get

$$A + B = 0$$

$$B = -A$$

$$\Rightarrow B = -1$$

Hence the partial fractions are

$$\frac{9}{(x - 1)(x + 2)^2} = \frac{1}{x - 1} - \frac{1}{x + 2} + \frac{3}{(x + 2)^2}$$

Question No.4

$$\frac{x^4 + 1}{x^2(x - 1)}$$

Solution:

$$\frac{x^4 + 1}{x^2(x - 1)} = \frac{x^4 + 1}{x^3 - x^2} \text{ is an improper fraction.}$$

First we resolve it into proper fraction.

$$x^3 - x^2 \sqrt{x^4 + 1}$$

$$\frac{\pm x^4}{x^3 + 1} \quad \mp x^3$$

$$\frac{\pm x^3}{x^3 + 1} \quad \mp x^2$$

$$\frac{x^4 + 1}{x^2(x - 1)} = (x + 1) + \frac{x^2 + 1}{x^2(x - 1)} \rightarrow (i)$$

$$\text{Let } \frac{x^2 + 1}{x^2(x - 1)} = (x + 1) + \frac{x^2 + 1}{x^2(x - 1)} \rightarrow (ii)$$

Multiplying both sides by $x^2(x - 1)$ we get

$$x^2 + 1 = A(x)(x - 1) + B(x - 1) + cx^2 \rightarrow (iii)$$

Putting $x = 0$ in (iii) we get

$$0 + 1 = B(0 - 1)$$

$$1 = -B$$

$$\Rightarrow B = 1$$

Putting $x - 1 = 0$ i.e. $x = 1$ in (iii) we get

$$(1)^2 + 1 = C(1)^2$$

$$1 + 1 = C(1)$$

$$2 = C$$

$$\Rightarrow C = 2$$

Equating the coefficient of x^2 in (iii) we get

$$A + C = 1$$

$$A + 2 = 1$$

$$A = 1 - 2$$

$$\Rightarrow A = -1$$

Putting the value of A, B, C in equation (ii)

Thus required partial fraction are

$$\frac{x^4 + 1}{x^2(x - 1)} = (x + 1) - \frac{1}{x} - \frac{1}{x^2} + \frac{2}{x - 1}$$

Question No.5

$$\frac{7x + 4}{(3x + 2)(x + 1)^2}$$

Solution:

$$\text{Let } \frac{7x + 4}{(3x + 2)(x + 1)^2} = \frac{A}{3x + 2} + \frac{B}{x + 1} + \frac{C}{(x + 1)^2} \rightarrow (i)$$

Multiplying both sides by $(3x + 2)(x + 1)^2$ we get

$$7x + 4 = A(x + 1)^2 + B(3x + 2)(x + 1) + C(3x + 2) \rightarrow (ii)$$

Putting $3x + 2 = 0$ i.e. $x = -\frac{2}{3}$ in (ii) we get

$$7\left(-\frac{2}{3}\right) + 4 = A\left(-\frac{2}{3} + 1\right)^2$$

$$-\frac{14}{3} + 4 = A\left(\frac{-2 + 3}{3}\right)^2$$

$$\frac{-14 + 12}{3} = A\left(\frac{1}{3}\right)^2$$

$$-\frac{2}{3} = \frac{1}{9}A$$

$$-18 = 3A$$

$$A = -\frac{18}{3}$$

$$\Rightarrow A = -6$$

Putting $x + 1 = 0$ i.e. $x = -1$ in (ii) we get

$$7(-1) + 4 = C(3(-1) + (+2))$$

$$-7 + 4 = -C$$

$$\Rightarrow -3 = -C$$

$$\Rightarrow C = 3$$

Equating the value of A, B and C in equation (i) we get required partial fractions.

$$\frac{7x+4}{(3x+2)(x+1)^2} = \frac{-6}{3x+2} + \frac{2}{x+1} + \frac{3}{(x+1)^2}$$

Question No.6

$$\frac{1}{(x-1)^2(x+1)}$$

Solution:

$$\text{Let } \frac{1}{(x-1)^2(x+1)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+1} \rightarrow (ii)$$

Multiplying both sides by $(x-1)(x-1)^2$ we get

$$1 = A(x-1)(x+1) + B(x+1) + C(x-1)^2 \rightarrow (ii)$$

Putting $x-1=0$ i.e. $x=1$ in (ii) we get

$$1 = B(1+1)$$

$$1 = 2B$$

$$\Rightarrow B = \frac{1}{2}$$

Putting $x+1=0$ i.e. $x=-1$ in (ii) we get

$$1 = C(-1-1)^2$$

$$1 = C(-1-1)^2$$

$$1 = C(-2)^2$$

$$1 = 4C$$

$$\Rightarrow C = \frac{1}{4}$$

Equating the coefficient of x^2 in (ii) we get

$$A + C = 0$$

$$A = -C$$

$$A = -\left(\frac{1}{4}\right)$$

$$\Rightarrow A = -\frac{1}{4}$$

Putting the value of

A, B, and C in equation (i) we got required partial fractions

$$\begin{aligned} \frac{1}{(x-1)^2(x+1)} &= \frac{-1}{4(x-1)} + \frac{1}{2(x-1)^2} + \frac{1}{4(x+1)} \\ &= \frac{-1}{4(x-1)} + \frac{1}{2(x-1)^2} + \frac{1}{4(x+1)} \end{aligned}$$

Question No.7

$$\frac{3x^2 + 15x + 16}{(x+2)^2}$$

Solution:

$$\frac{3x^2 + 15x + 16}{(x+2)^2} = \frac{3x^2 + 15x + 16}{x^2 + 4x + 4}$$

The given fraction is improper fraction.

By long division,

$$\begin{array}{r} 3 \\ x^2 + 4x + 4 \sqrt{3x^2 + 15x + 16} \\ \underline{\pm 3x^2 \pm 12x \pm 12} \\ \hline 3x + 4 \\ 3x^2 + 15x + 16 \\ \hline (x+2)^2 = 3 + \frac{3x+4}{x^2 + 4x + 4} \rightarrow (i) \end{array}$$

$$\text{Let } \frac{3x+4}{(x+2)^2} = \frac{A}{x+2} + \frac{B}{(x+2)^2} \rightarrow (ii)$$

Multiplying both sides by $(x+2)^2$ we get

$$3x+4 = A(x+2) + B \rightarrow (iii)$$

Putting $x+2=0$ i.e. $x=-2$ in (iii) we get

$$3(-2)+4=B$$

$$-6+4=B$$

$$\Rightarrow B = -2$$

Equating the coefficient of "x" we get

$$3=A$$

$$\Rightarrow A = 3$$

Putting the value of A and B in equation (ii) and using eq.(i) we get required partial fractions.

$$\frac{3x^2 + 15x + 16}{(x+2)^2} = 3 + \frac{3}{x+2} - \frac{2}{(x+2)^2}$$

Question No.8

$$\frac{1}{(x^2 - 1)(x+1)}$$

Solution:

$$\begin{aligned} \frac{1}{(x^2 - 1)(x+1)} &= \frac{1}{(x-1)(x+1)(x+1)} \\ &= \frac{1}{(x-1)(x+1)^2} \end{aligned}$$

$$\text{Let } \frac{1}{(x-1)(x+1)^2} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$

Multiplying both sides by $(x-1)(x+1)^2$ we get

$$1 = A(x+1)^2 + B(x+1)(x-1) + C(x-1) \rightarrow (ii)$$

Putting $x-1=0$ i.e. $x=1$ in (ii) we get

$$1 = A(1+1)^2$$

$$1 = A(2)^2$$

$$1 = 4A$$

$$\Rightarrow A = \frac{1}{4}$$

Putting $x+1=0$ i.e. $x=-1$ in (ii) we get

$$1 = C(-1-1)$$

$$1 = -2C$$

$$\Rightarrow C = \frac{-1}{2}$$

Equating the coefficient of x^2 in equation (ii)

We get $A + B = 0$

$$B = -A$$

$$B = -\left(\frac{1}{4}\right)$$

$$\Rightarrow B = -\frac{1}{4}$$

Putting the value of A and B in equation (ii)

We get required partial fractions.

$$\begin{aligned} \frac{1}{(x-1)(x+1)^2} &= \frac{1}{4(x-1)} - \frac{1}{4(x+1)} - \frac{1}{2(x+1)^2} \end{aligned}$$

$$\frac{3x+7}{(x^2+1)(x+3)}$$

Solution:

$$\text{Let } \frac{3x+7}{(x^2+1)(x+3)} = \frac{Ax+B}{x^2+1} + \frac{C}{x+3} \rightarrow (ii)$$

Multiplying both sides by $x^2+1)(x+3)$

$$3x+7 = (Ax+B)(x+3) + C(x^2+1)$$

$$3x+7 = Ax(x+3) + B(x+3) + C(x^2+1) \rightarrow (ii)$$

Putting $x+3=0$ i.e. $x=-3$ in (ii) we get

$$3(-3)+7 = C[(-3)^2+1]$$

$$-9+7 = C(9+1)$$

$$-2 = 10C$$

$$\Rightarrow C = -\frac{2}{10}$$

Now equating the coefficient of x^2 and x in equation (iii) we get.

Exercise 4.3

Resolve into partial fraction.

Question No.1

$$\frac{3x-11}{(x+3)(x^2+1)}$$

Solution:

$$\text{Let } \frac{3x-11}{(x+3)(x^2+1)} = \frac{A}{x+3} + \frac{Bx+C}{x^2+1}$$

Multiplying both sides $(x-3)(x^2+1)$, we get

$$3x-11 = A(x^2+1) + (Bx+C)(x+3) \rightarrow (i)$$

$$3x-11 = A(x^2+1) + Bx(x+3) + C(x+3) \rightarrow (ii)$$

Putting $x+3=0$ i.e. $x=-3$, we get

$$3(-3)-11 = A[(-3)^2+1]$$

$$-9-11 = A(9+1)$$

$$-20 = 10A$$

$$A = \frac{-20}{10}$$

$$\Rightarrow A = -2$$

Now equating the coefficient of x^2 and x we get from equation (iii)

$$A+B=0$$

$$-2+B=0$$

$$B=2$$

$$\Rightarrow B=1$$

$$3B+C=3$$

$$3(2)+C=3$$

$$6+C=3$$

$$C=3-6$$

$$\Rightarrow C=-3$$

Putting the value

of A, B and C in equation (i) we get

Required partial fractions.

$$\frac{3x-11}{(x+3)(x^2+1)} = \frac{-2}{x+3} + \frac{2x-3}{x^2+1}$$

Question No.2

$$A+C=0$$

$$A+\left(\frac{-1}{5}\right)=0$$

$$A-\frac{1}{5}=0$$

$$\Rightarrow A=\frac{1}{5}$$

$$3A+B=3$$

$$3\left(\frac{1}{5}\right)+B=3$$

$$B=3-\frac{3}{5}$$

$$B=3-\frac{3}{5}$$

$$B=\frac{15-3}{5}$$

$$B=\frac{12}{5}$$

$$\Rightarrow C=-3$$

Putting the value of

A, B and C in equation (i) we get required partial fraction.

$$\frac{3x+7}{(x^2+1)(x+3)} = \frac{x+12}{5(x^2+1)} - \frac{1}{5(x+3)}$$

Question No.3

$$\frac{1}{(x+1)(x^2+1)}$$

Solution:

$$\text{Let } \frac{1}{(x+1)(x^2+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1} \rightarrow (ii)$$

Multiplying both sides by $(x+1)(x^2+1)$ we get

$$1 = A(x^2+1) + (Bx+C)(x+1)$$

$$1 = A(x^2+1) + Bx(x+1) + C(x+1) \rightarrow (ii)$$

putting $x+1=0$ i.e. $x=-1$ in (ii) we get

$$1 = A[(-1)^2+1]$$

$$1 = A(1+1)$$

$$1 = 2A$$

$$\Rightarrow A = \frac{1}{2}$$

Equating the coefficients of

x^2 and x in equation (ii)

We get

$\begin{aligned} A + B &= 0 \\ \frac{1}{2} + B &= 0 \\ B &= -\frac{1}{2} \\ \Rightarrow B &= -\frac{1}{2} \end{aligned}$	$\begin{aligned} B + C &= 0 \\ -\frac{1}{2} + C &= 0 \\ C &= \frac{1}{2} \\ \Rightarrow C &= \frac{1}{2} \end{aligned}$
--	---

Putting the value of A, B and C in equations (i) we get

$$\Rightarrow A = \frac{-2}{13} \quad \text{required partial fractions.}$$

$$\frac{1}{(x+1)(x^2+1)} = \frac{1}{2(x+1)} + \frac{x-1}{2(x^2+1)}$$

Question No.4

$$\frac{9x-7}{(x+3)(x^2+1)}$$

Solution:

$$\text{Let } \frac{9x-7}{(x+3)(x^2+1)} = \frac{A}{x+3} + \frac{Bx+C}{x^2+1} \rightarrow (i)$$

Multiplying both sides by $(x+3)(x^2+1)$ we get

$$9x-7 = A(x^2+1) + (Bx+C)(x^2+1)$$

$$9x-7 = A(x^2+1) + Bx(x+3) + C(x+3) \rightarrow (ii)$$

Putting $x+3=0$ i.e. $x=-3$ in (ii) we get

$$9(-3)-7 = A[(-3)^2+1]$$

$$-27-7 = A(9+1)$$

$$-34 = 10A$$

$$\Rightarrow A = \frac{-34}{10}$$

$$\Rightarrow A = \frac{-17}{5}$$

Equating coefficient of

x^2 and x in equation (ii) we get

$\begin{aligned} A + B &= 0 \\ \frac{-17}{5} + B &= 0 \\ B &= \frac{17}{5} \\ \Rightarrow B &= \frac{-17}{5} \end{aligned}$	$\begin{aligned} 3B + C &= 9 \\ 3\left(\frac{17}{5}\right) + C &= 9 \\ \frac{51}{5} + C &= 9 \\ C &= 9 - \frac{51}{5} \\ C &= \frac{45-51}{5} \\ \Rightarrow C &= \frac{-6}{5} \end{aligned}$
---	---

Putting the value

of A, B and C in equation (i) we get

required partial fraction.

$$\frac{9x-7}{(x+3)(x^2+1)} = \frac{-17}{5(x+3)} + \frac{17x-6}{5(x^2+1)}$$

Question No.5

$$\frac{3x+7}{(x+3)(x^2+4)}$$

Solution:

$$\text{Let } \frac{3x+7}{(x+3)(x^2+4)} = \frac{A}{x+3} + \frac{Bx+C}{x^2+4} \rightarrow (i)$$

Multiplying both sides by $(x+3)(x^2+4)$ we get

$$3x+7 = A(x^2+4) + (Bx+C)(x+3)$$

$$3x+7 = A(x^2+4) + Bx(x+3) + C(x+3) \rightarrow (ii)$$

Putting $x+3=0$ i.e. $x=-3$ in (ii) we get

$$3(-3)+7 = A((-3)^2+4)$$

$$-9+7 = A(9+4)$$

$$-2 = 13A$$

Equating the coefficient of x^2 and x in equation (ii) we get

$\begin{aligned} A + B &= 0 \\ \frac{-2}{13} + B &= 0 \\ B &= \frac{2}{13} \\ \Rightarrow B &= \frac{2}{13} \end{aligned}$	$\begin{aligned} 3B + C &= 3 \\ 3\left(\frac{2}{13}\right) + C &= 3 \\ \frac{6}{13} + C &= 3 \\ C &= 3 - \frac{6}{13} \\ C &= \frac{39-6}{13} \\ \Rightarrow C &= \frac{33}{13} \end{aligned}$
--	--

Putting the value of A, B and C in equation (i) we get
Required partial fractions.

$$\frac{3x+7}{(x+3)(x^2+4)} = \frac{-2}{13(x+3)} + \frac{2x+33}{13(x^2+4)}$$

Question No.6

$$\frac{x^2}{(x+2)(x^2+4)}$$

Solution:

$$\text{Let } \frac{x^2}{(x+2)(x^2+4)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+4} \rightarrow (i)$$

Multiplying both sides by $(x+2)(x^2+4)$ we get

$$x^2 = A(x^2+4) + (Bx+C)(x+2)$$

$$x^2 = A(x^2+4) + Bx(x+2) + C(x+2) \rightarrow (ii)$$

Putting $x+2=0$ i.e. $x=-2$ in (ii) we get

$$(-2)^2 = A[(-2)^2+4]$$

$$4 = A(4+A)$$

$$4 = 8 + A$$

$$\Rightarrow A = \frac{1}{2}$$

Equating the coefficients of

x^3 and x in equation (ii)

We get

$\begin{aligned} A + B &= 1 \\ \frac{1}{2} + B &= 1 \\ B &= 1 - \frac{1}{2} \\ \Rightarrow B &= \boxed{\frac{1}{2}} \end{aligned}$	$\begin{aligned} 2B + C &= 0 \\ 2\left(\frac{1}{2}\right) + C &= 0 \\ 1 + C &= 0 \\ C &= 0 - 1 = -1 \\ \Rightarrow C &= \boxed{-1} \end{aligned}$
--	---

Putting the value of A, B and C in equation (i) we get
Required partial fractions.

$$\frac{x^2}{(x+2)(x^2+4)} = \frac{1}{2(x+2)} + \frac{x-2}{2(x^2+4)}$$

Question No.7

$$\frac{1}{x^3+1} \quad \left[\text{Hint: } \frac{1}{x^3+1} = \frac{1}{(x+1)(x^2-x+1)} \right]$$

Solution:

$$\text{Let } \frac{1}{(x+1)(x^2-x+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2-x+1} \rightarrow (i)$$

Multiplying both sides by $(x-1)(x^2-x+1)$, we get
 $1 = A(x^2-x+1) + (Bx+C)(x+1)$
 $1 = A(x^2-x+1) + Bx(x+1) + c(x+1) \rightarrow (ii)$

Putting $x+1=0$ i.e. $x=-1$ in (ii) we get

$$\begin{aligned} 1 &= A[(-1)^2 - (-1) + 1] \\ 1 &= A[(-1)^2 - 1(-1) + 1] \\ 1 &= A(1+1+1) \\ 1 &= 3A \end{aligned}$$

$$\Rightarrow \boxed{A = \frac{1}{3}}$$

Comparing the coefficients of x^2 and x in equation (ii) we get

$\begin{aligned} A + B &= 0 \\ \frac{1}{3} + B &= 0 \\ B &= \frac{-1}{3} \\ \Rightarrow B &= \boxed{\frac{-1}{3}} \end{aligned}$	$\begin{aligned} -A + B + C &= 0 \\ \left(-\frac{1}{3}\right) - \frac{1}{3} + C &= 0 \\ -\frac{2}{3} + C &= 0 \\ \Rightarrow C &= \boxed{\frac{2}{3}} \end{aligned}$
--	--

Putting the value of A, B and C in equation (i) we get
Required partial fractions.

$$\frac{1}{(x+1)(x^2-x+1)} = \frac{1}{3(x+1)} - \frac{x-2}{3(x^2-x+1)}$$

Question No.8

$\frac{x^2+1}{x^3+1}$	Solution: $\frac{x^2+1}{[x^3+1]} = \frac{x^2+1}{(x+1)(x^2-x+1)}$ $\text{Let } \frac{x^2+1}{(x+1)(x^2-x+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2-x+1} \rightarrow (i)$
-----------------------	---

$$\begin{aligned} \text{Multiplying both sides by } (x+1)(x^2-x+1) &\text{ we get} \\ x^2+1 &= A(x^2-x+1) + (Bx+C)(x+1) \\ x^2+1 &= A(x^2-x+1) + Bx(x+1) + C(x+1) \rightarrow (ii) \end{aligned}$$

Putting $x+1=0$ i.e. $x=-1$ in (ii) we get

$$\begin{aligned} (-1)^2+1 &= A[(-1)^2 - (-1) + 1] \\ 1+1 &= A(1+1+1) \\ 2 &= 3A \end{aligned}$$

$$\Rightarrow \boxed{A = \frac{2}{3}}$$

Equating the coefficients of x^2 and x in equation (ii) we get

$\begin{aligned} A+B &= 1 \\ \frac{2}{3} + B &= 1 \\ B &= 1 - \frac{2}{3} \\ \Rightarrow B &= \boxed{\frac{1}{3}} \end{aligned}$	$\begin{aligned} -A+B+C &= 0 \\ \left(-\frac{2}{3}\right) + \frac{1}{3} + C &= 0 \\ -\frac{1}{3} + C &= 0 \\ \Rightarrow C &= \boxed{\frac{1}{3}} \end{aligned}$
--	--

Putting the value of A, B and C in equation (i) we get
required partial fractions.

$$\frac{x^2+1}{(x+1)(x^2-x+1)} = \frac{2}{3(x+1)} + \frac{x+1}{3(x^2-x+1)}$$

Exercise 4.4

Resolve into Partial Fractions.

Question No.1

$$\frac{x^3}{(x^2 + 4)^2}$$

Solution:

$$\text{Let } \frac{x^3}{(x^2 + 4)^2} = \frac{Ax + B}{x^2 + 4} + \frac{Cx + D}{(x^2 + 4)^2} \rightarrow (i)$$

Multiplying both sides by $(x^2 + 4)^2$, we get

$$x^3 = (Ax + B)(x^2 + 4) + (Cx + D)$$

$$x^3 = Ax(x^2 + 4) + B(x^2 + 4) + (Cx + D) \rightarrow (iii)$$

Equating the coefficients of x^3, x^2, x and constant, we get

$$\text{Coefficients of } x^3: A = 1$$

$$\text{Coefficients of } x^2: B = 0$$

$$\text{Coefficients of } x: 4A + C = 0$$

$$\Rightarrow C = -4$$

$$\text{Constants: } 4B + D = 0$$

$$4(0) + D = 0$$

$$\Rightarrow D = 0$$

Putting the value of A, B , and C in equation (i)

We get required partial fractions.

$$\frac{x^3}{(x^2 + 4)^2} = \frac{x}{x^2 + 4} - \frac{4x}{(x^2 + 4)^2}$$

Question No.2

$$\frac{x^4 + 3x^2 + x + 1}{(x + 1)(x^2 + 1)^2}$$

Solution:

$$\text{Let } \frac{x^4 + 3x^2 + x + 1}{(x + 1)(x^2 + 1)^2} = \frac{A}{x + 1} + \frac{Bx + C}{x^2 + 1} + \frac{Dx + E}{(x^2 + 1)^2} \rightarrow (i)$$

Multiplying both sides by $(x + 1)(x^2 + 1)^2$ we get

$$x^4 + 3x^2 + x + 1$$

$$= A(x^2 + 1)^2 + (Bx + C)(x + 1)(x^2 + 1) + (Dx + E)(x + 1) \rightarrow (ii)$$

$$x^4 + 3x^2 + x + 1 = A(x^4 + 2x^2 + 1)$$

$$+ Bx(x^3 + x^2 + x + 1) + C(x^3 + x^2 + x + 1)$$

$$+ Dx(x + 1) + E(x + 1)$$

$$x^4 + 3x^2 + x + 1 = A(x^4 + 2x^2 + 1)$$

$$+ B(x^4 + x^3 + x^2 + x) + C(x^3 + x^2 + x + 1)$$

$$+ D(x^2 + x) + E(x + 1) \rightarrow (iii)$$

Putting $x + 1 = 0$ i.e. $x = -1$ in eq(ii) we get

$$(-1)^4 + 3(-1)^2 + (-1) + 1 = A[(-1)^2 + 1]^2$$

$$1 + 3(1) - 1 + 1 = A(1 + 1)^2$$

$$4 = 4A$$

$$\Rightarrow A = 1$$

Now equating the coefficients of x^4, x^3, x^2, x and constants, we get from equation (iii)

$$\text{coefficients of } x^4: A + B = 0$$

$$1 + B = 1$$

$$B = 1 - 1$$

$$\Rightarrow B = 0$$

Coefficients of $x^3: B + C = 0$

$$0 + C = 0$$

$$\Rightarrow C = 0$$

Coefficients of $x^2: 2A + B + C + D = 3$

$$2(1) + 0 + 0 + D = 3$$

$$D = 3 - 2$$

$$D = 1$$

Coefficients of $x: B + C + D + E = 1$

$$0 + 0 + 1 + E = 1$$

$$E = 1 - 1$$

$$\Rightarrow E = 0$$

Putting the value of A, B, C and D in equation (i)

We get required partial fractions.

$$\text{Let } \frac{x^4 + 3x^2 + x + 1}{(x + 1)(x^2 + 1)^2} = \frac{1}{x + 1} + \frac{x}{(x^2 + 1)^2}$$

Question No.3

$$\frac{x^2}{(x + 1)(x^2 + 1)^2}$$

Solution:

$$\text{Let } \frac{x^2}{(x + 1)(x^2 + 1)^2} = \frac{A}{x + 1} + \frac{Bx + C}{x^2 + 1} + \frac{Dx + E}{(x^2 + 1)^2} \rightarrow (i)$$

Multiplying both sides by $(x + 1)(x^2 + 1)^2$ we get

$$x^2 = A(x^2 + 1)^2 + (Bx + C)(x + 1)(x^2 + 1) + (Dx + E)(x + 1) \rightarrow (ii)$$

$$x^2 = A(x^4 + 2x^2 + 1) + Bx(x^3 + x^2 + x + 1) +$$

$$C(x^3 + x^2 + x + 1) + Dx(x + 1) + E(x + 1)$$

$$x^2 = A(x^4 + 2x^2 + 1) + B(x^4 + x^3 + x^2 + x)$$

$$+ C(x^3 + x^2 + x + 1) + D(x^2 + x) + E(x + 1) \rightarrow (iii)$$

Putting $x + 1 = 0$ i.e. $x = -1$ in eq(ii) we get

$$(-1)^2 = A[(-1)^2 + 1]^2$$

$$1 = A(1 + 1)^2$$

$$1 = 4A$$

$$\Rightarrow A = \frac{1}{4}$$

Now equating the coefficients of x^4, x^3, x^2, x and constants, we get from equation (iii)

coefficients of $x^4: A + B = 0$

$$\frac{1}{4} + B = 1$$

$$\Rightarrow B = -\frac{1}{4}$$

Coefficients of $x^3: B + C = 0$

$$-\frac{1}{4} + C = 0$$

$$\Rightarrow C = \frac{1}{4}$$

Coefficients of $x^2: 2A + B + C + D = 1$

$$\begin{aligned} 2\left(\frac{1}{4}\right) - \frac{1}{4} + \frac{1}{4} + D &= 1 \\ \frac{1}{2} + D &= 1 \\ D &= 1 - \frac{1}{2} \\ \Rightarrow D &= \frac{2-1}{2} \\ \Rightarrow D &= \boxed{\frac{1}{2}} \end{aligned}$$

Coefficients of x : $B + C + D + E = 0$

$$\begin{aligned} -\frac{1}{4} + \frac{1}{2} + \frac{1}{2} + E &= 0 \\ \frac{1}{2} + E &= 0 \\ \Rightarrow E &= \boxed{-\frac{1}{2}} \end{aligned}$$

Putting the value of A, B, C and D in equation (i)
We get required partial fractions.

$$\begin{aligned} \frac{x^2}{(x+1)(x^2+1)^2} &= \frac{A}{4(x+1)} + \frac{x-1}{4(x^2+1)} + \frac{x-1}{2(x^2+1)^2} \end{aligned}$$

Question No.4

$$\frac{x^2}{(x-1)(x^2+1)^2}$$

Solution:

$$\frac{x^2}{(x-1)(x^2+1)^2} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1} + \frac{Dx+E}{(x^2+1)^2} \rightarrow (i)$$

Multiplying both sides by $(x+1)(x^2+1)^2$ we get

$$\begin{aligned} x^2 &= A(x^2+1)^2 + (Bx+C)(x-1)(x^2+1) \\ &\quad + (Dx+E)(x-1) \rightarrow (ii) \\ x^2 &= A(x^4+2x^2+1) + Bx(x-1)(x^2+1) + \\ &C(x-1)(x^2+1) + Dx(x-1) + E(x-1) \\ x^2 &= A(x^4+2x^2+1) + B(x^4-x^3+x^2-x) \\ &+ C(x^3-x^2+x-1) + D(x^2-x) + E(x-1) \rightarrow (iii) \end{aligned}$$

Putting $x-1=0$ i.e $x=1$ in eq(ii) we get

$$\begin{aligned} (1)^2 &= A[(1)^2+1]^2 \\ 1 &= A(1+1)^2 \\ 1 &= 4A \\ \Rightarrow A &= \boxed{\frac{1}{4}} \end{aligned}$$

Now equating the coefficients of x^4, x^3, x^2, x and constants, we get from equation (iii)

coefficients of x^4 : $A+B=0$

$$\begin{aligned} \frac{1}{4} + B &= 1 \\ \Rightarrow B &= \boxed{-\frac{1}{4}} \end{aligned}$$

Coefficients of x^3 : $B+C=0$

$$-\left(-\frac{1}{4}\right) + C = 0$$

$$\Rightarrow C = \boxed{-\frac{1}{4}}$$

Coefficients of x^2 : $2A+B-C+D=1$

$$\begin{aligned} 2\left(\frac{1}{4}\right) - \frac{1}{4} - \left(-\frac{1}{4}\right) + D &= 1 \\ \frac{1}{2} - \frac{1}{4} + \frac{1}{4} + D &= 1 \\ D &= 1 - \frac{1}{2} \\ \Rightarrow D &= \frac{2-1}{2} \\ \Rightarrow D &= \boxed{\frac{1}{2}} \end{aligned}$$

Coefficients of x : $-B+C-D+E=0$

$$\begin{aligned} -\left(-\frac{1}{4}\right) - \frac{1}{4} - \frac{1}{2} + E &= 0 \\ \frac{1}{4} - \frac{1}{4} - \frac{1}{2} + E &= 0 \\ -\frac{1}{2} + E &= 0 \\ \Rightarrow E &= \boxed{\frac{1}{2}} \end{aligned}$$

Putting the value of A, B, C and D in equation (i)

We get required partial fractions.

$$\begin{aligned} \frac{x^2}{(x-1)(x^2+1)^2} &= \frac{1}{4(x+1)} + \frac{x+1}{4(x^2+1)} + \frac{x+1}{2(x^2+1)^2} \end{aligned}$$

Question No.5

$$\frac{x^4}{(x^2+2)^2}$$

Solution:

$$\frac{x^4}{(x^2+2)^2} = \frac{x^4}{x^4+4x^2+4} \text{ is an improper fraction}$$

First we resolve it into proper fraction

$$\begin{aligned} \frac{1}{x^4+4x^2+4\sqrt{x^4}} &= \frac{\pm x^4 \pm 4x^2 \pm 4}{-4x^2-4} \\ \frac{x^4}{(x^2+2)^2} &= 1 + \frac{-4x^2-4}{(x^2+2)^2} \end{aligned}$$

$$\text{Let } \frac{-4x^2-4}{(x^2+2)^2} = \frac{Ax+B}{x^2+2} + \frac{Cx+D}{(x^2+2)^2} \rightarrow (i)$$

Multiplying both sides by $(x^2+2)^2$ we get

$$\begin{aligned} -4x^2-4 &= (Ax+B)(x^2+2) + (Cx+D) \\ -4x^2-4 &= A(x^3+2x) + B(x^2+2) + Cx+D \\ \rightarrow (ii) & \end{aligned}$$

Equating the coefficients of x^3, x^2, x and constants

In equation (ii) we get

Coefficients of x^2 : $B=-4$

Coefficients of x : $2A+C=0$

$$2(0)+C=0$$

Constants : $2B+D=-4$

$$2(-4)+D=-4$$

$$-8 + D = -4$$

$$D = 8 - 4$$

$$D = 4$$

Putting the value of A, B, C and D in equation (i) we get required partial fractions.

$$\frac{x^4}{(x^2 + 2)^2} = 1 + \frac{-4}{x^2 + 2} + \frac{4}{(x^2 + 2)^2}$$

Question No.6

$$\frac{x^5}{(x^2 + 1)^2}$$

Solution:

$$\frac{x^5}{(x^2 + 1)^2} = \frac{x^5}{x^4 + 2x^2 + 1} \text{ is an improper fraction.}$$

First we resolve it into proper fraction.

$$\begin{array}{r} x \\ x^4 + 2x^2 + 1 \sqrt{x^5} \\ \hline \pm x^5 \pm 2x^3 \pm x \\ \hline -2x^3 - x \\ \hline x^5 \\ (x^2 + 1)^2 = x + \frac{-2x^3 - x}{(x^2 + 1)^2} \end{array}$$

$$\text{Let } \frac{-2x^3 - x}{(x^2 + 1)^2} = \frac{Ax + B}{x^2 + 1} + \frac{Cx + D}{(x^2 + 1)^2} \rightarrow (i)$$

Multiplying both sides by $(x^2 + 1)^2$ we get

$$-2x^3 - x = (Ax + B)(x^2 + 1) + (Cx + D)$$

$$-2x - x = A(x^3 + x) + B(x^2 + 1) + Cx + D$$

Equating the coefficients of x^3, x^2, x and constants

We get

Coefficients of x^3 : $A = -2$

Coefficients of x^2 : $B \equiv 0$

Coefficients of x : $A + C = -1$

$$-2 + C \equiv -1$$

$$C = -1 + 2$$

$$\Rightarrow \boxed{C = 1}$$

Constants: $R + D = 0$

$$0 + D = 0$$

$$\Rightarrow \boxed{D = 0}$$

Hence the required partial fractions are

$$\frac{x^5}{(x^2 + 1)^2} = x + \frac{-2x}{x^2 + 1} + \frac{x}{(x^2 + 1)^2}$$

$$\frac{x^5}{(x^2 + 1)^2} = x - \frac{2x}{x^2 + 1} + \frac{x}{(x^2 + 1)^2}$$