10TH CLASS MATH

CHAPTER 3

SOLUTION NOTES

Exercise 3.1

Question No.1

Express the following as ratio $a:b$ and as a fraction in its simplest form.

i. $Rs. 750\ ,\ Rs. 1250$

$$750:\ 1250$$

$$\frac{750}{10}:\frac{1250}{10} \quad divide\ by\ 10$$

$$\frac{75}{5}:\frac{125}{5} \quad divide\ by\ 5$$

$$\frac{15}{5}:\frac{25}{5} \quad divide\ by\ 5$$

$$3:5$$

$$\frac{3}{5}$$

ii. $450cm, 3m$

$$450cm\ :\ 3\times 100cm \qquad \because 1m = 100cm$$

$$\frac{450}{10}=\frac{300}{10} \quad divide\ by\ 10$$

$$\frac{45}{5}:\frac{30}{5} \quad divide\ by\ 5$$

$$\frac{9}{3}:\frac{6}{3} \quad divide\ by\ 3$$

iii. $4kg, 2kg\ 750gm$

$$4\times 1000gm:\quad 2\times 1000gm+750gm$$

$$\because\ 1kg = 1000gm$$

$$4000gm:\quad 2000gm+750gm$$

$$4000gm: 2750gm$$

$$2000gm:\ 1375gm \quad divide\ by\ 2$$

$$16\ gm:\quad 11gm \quad divide\ by\ 125$$

$$\frac{16gm}{11gm}=\frac{16}{11}$$

iv.

$$27min. 30\ sec, 1\ hour$$

$$1650\sec\ : 3600sec$$

$$165\ \ sec: 360sec$$

$$11sec:\ \ 24sec$$

$$\frac{11sec}{24sec}$$

$$\frac{11}{24}\ \ Ans.$$

v. $75^0,\ 225^0$

$$75^0 \quad : \quad 225^o$$

$$3^0 \quad : \quad 9^0 \quad Dividing\ by\ 25$$

$$1^0 \quad : \quad 3^0 \quad Dividing\ by\ 3$$

$$\frac{1}{3}=1:3$$

Question No.2

In a class of 60 students, 25 students are girls and remaining students are boys compute the ratio of

$$\boldsymbol{Total\ students = 60}$$

$$\boldsymbol{Girls = 25}$$

$$\boldsymbol{boys = 60-25 = 35}$$

i. **Boys to total students**

$$\boldsymbol{Boys \quad : \quad Total\ students}$$

$$\boldsymbol{35:\ \ 60 \quad divide\ by\ 5}$$

$$\boldsymbol{7:\ \ 12}$$

ii. **Boys to girls**

$$\boldsymbol{Boys\ :\ Girls}$$

$$\boldsymbol{35 \quad : \quad 25\ \ Divide\ by\ 5}$$

$$\boldsymbol{7 \quad : \quad 5}$$

Question No.3

$\boldsymbol{if\ 3(4x-5y) = 2x-7y,\ \ find\ the\ ratio\ \ x:y}$

Solution:

$$\boldsymbol{3(4x-5y) = 2x-7y}$$

$$\boldsymbol{12x-15y = 2x-7y}$$

$$\boldsymbol{\Rightarrow 12x-2x = -7y+15y}$$

$$\boldsymbol{\Rightarrow 10x = 8y}$$

$$\boldsymbol{\Rightarrow 5x = 4y \quad divide\ by\ 2}$$

$$\boldsymbol{\frac{x}{y}=\frac{4}{5}}$$

$$\boldsymbol{x:y = 4:5}$$

Question No.4

Find the value of p, $\boldsymbol{if\ the\ ratio\ \ 2p+5:\ 3p+4\ \ and\ 3:4\ \ are\ equal.}$

Solution:

$\frac{2p+5}{3p+4}=\frac{3}{4}$

$4(2p+5)=3(3p+4)$

$8p+20=9p+12$

$20-12=9p-8p$

$8=p$

Question No.5

$\boldsymbol{if\ the\ ratio\ 3x+1\ \ :6+4x\ \ and\ 2:5\ \ are\ equal. find\ the\ value\ of\ x.}$

Solution:

$$\frac{3x+1}{6+4x}=\frac{2}{5}$$
$$5(3x+1)=2(6+4x)$$
$$15x+5=12+8x$$
$$15x-8x=12-5$$
$$7x=7$$
$$x=\frac{7}{7}$$
$$x=1$$

Question No.6

Two numbers are in ratio $5:8$. $\boldsymbol{if\ 9\ is\ added\ to\ each\ number, we\ get\ a\ new\ ratio\ 8:11}$ find the numbers.

Solution:

Let a number x

According to the given condition

$\frac{5x+9}{8x+9}=\frac{8}{11}$

$\Rightarrow 11(5x+9)=8(8x+9)$

$\Rightarrow 55x+99=64x+72$

$\Rightarrow 99-72=64x-55x$

$\Rightarrow 27=9x$

$\Rightarrow x=3$

$Now\ 5x=5(3)$

$8x=8(3)=24$

Hence required numbers are 15 and 24.

Question No.7

If 10 is added in each number of the ratio $4:13$ $\boldsymbol{we\ get}$ $1:2$.

What are the numbers?

Solution:

Let a number x

According to the given Condition

$$\frac{4x+10}{13x+10}=\frac{1}{2}$$
$$\Rightarrow 2(4x+10)=1(13x+10)$$
$$\Rightarrow 8x+20=13x+10$$
$$\Rightarrow 20-10=13x-8x$$
$$10=5x$$
$$x=\frac{10}{5}=2$$
$$x=2$$

Now $1st\ number=4x=4(2)=8$

$2nd\ number=13x=13(2)=26$

Question No.8

Find the cost of 8kg of mangoes, if 5kg of mangoes cost $Rs.250$

Solution:

Let $the\ cost\ of\ 8kg\ of\ mangoes\ be\ x-rupess.$

$$8kg\ :5kg\ \ ::Rs.x:Rs.250$$
$$8kg:5kg\ \ \ =\ Rs.x:Rs.250$$
$$product\ \ of\ extremes=product\ of\ means$$
$$8\times 250=5x$$
$$\frac{8\times 250}{5}=x$$
$$x=400Rs.$$

Question No.9

If $a:b=7:6$, $find\ the\ value\ of\ 3a+5b:7b-5a$

Solution:

$$As\ given\ that\quad a:b=7:6\quad or$$
$$\frac{a}{b}=\frac{7}{6}$$

Now

$$3a+5b:7b:5a=\frac{3a+5b}{7b-5a}$$
$$Dividing\ numerators\ and\ denominator\ by\ b$$
$$\frac{\frac{3a+5b}{b}}{\frac{7b-5a}{b}}=\frac{3\left(\frac{a}{b}\right)+5\left(\frac{b}{b}\right)}{7\left(\frac{b}{b}\right)-5\left(\frac{a}{b}\right)}=\frac{3\left(\frac{a}{b}\right)+5}{7-5\left(\frac{a}{b}\right)}$$
$$\because\ \frac{a}{b}=\frac{7}{6}$$
$$so\ \frac{3\left(\frac{7}{6}\right)+5}{7-5\left(\frac{7}{6}\right)}=\frac{\frac{21}{6}+5}{7-\frac{35}{6}}=\frac{\frac{21+30}{6}}{\frac{42-35}{6}}$$
$$\frac{\frac{51}{6}}{\frac{7}{6}}=\frac{51}{6}\times\frac{6}{7}=\frac{51}{7}=51:7$$

Question No.10 Complete the following

i. $\boldsymbol{if\ \frac{24}{7}=\frac{6}{x}\ \ then\ 4x=7}$

ii. $\boldsymbol{if\ \frac{5a}{3x}=\frac{15b}{y}\ \ then\ ay=9bx}$

iii. $\boldsymbol{if\ \frac{9pb}{2lm}=\frac{18p}{5m},\ then\ 5q=4l}$

Question No.11 find x in the following proportions.

i. $\boldsymbol{3x-2\ \ :\ 4\ \ ::\ \ 2x+3\ \ :\ \ 7}$

$$Product\ of\ extremes$$
$$=\ Product\ of\ means$$
$$7(3x-2)=4(2x+3)$$
$$21x-14\ \ =\ \ 8x+12$$
$$21x-8x=12+14$$
$$13x=\ \ 26$$
$$x=\frac{26}{13}=2$$

ii. $\frac{3x-1}{7}\ :\ \frac{3}{5}\ ::\ \frac{2x}{3}\ :\ \frac{7}{5}$

$$Product\ of\ extremes = Product\ of\ means$$

$$\left(\frac{3x-1}{7}\right)\frac{7}{5} = \frac{3}{5}\left(\frac{2x}{3}\right)$$

$$\frac{3x-1}{5} = \frac{2x}{5}$$

$$3x-1 = 2x$$

$$3x-2x = 1$$

$$x = 1$$

iii. $\frac{x-3}{2} : \frac{5}{x-1} :: \frac{x-1}{3} : \frac{4}{x+4}$

Solution:

$$Product\ of\ extremes = product\ of\ means$$

$$\left(\frac{x-3}{2}\right)\frac{4}{x+4} = \frac{5}{x-1}\left(\frac{x-1}{3}\right)$$

$$\frac{2x-6}{x+4} = \frac{5}{3}$$

$$3(2x-6) = 5(x+4)$$

$$6x-18 = 5x+20$$

$$6x-5x = 20+18$$

$$x = 38$$

iv. $P^2+pq+q^2 :: \frac{p^3-q^3}{p+q} : (p-q)^2$

$$product\ of\ extremes = product\ of\ means$$

$$(p^2+pq+q^2)(p-q)^2 = x \times \frac{p^3-q^3}{p+q}$$

$$(p^2+pq+q^2)(p-q)(p-q) = x \times \frac{p^3-q^3}{p+q}$$

$$(p^3-q^3)(p-q) = x \times \frac{p^3-q^3}{p+q}$$

$$(p^3-q^3)(p-q) \times \frac{p+q}{p^3-q^3} = x$$

$$x = (p-q)(p+q)$$

$$x = p^2-q^2$$

v. $8-x : 11-x :: 16-x : 25-x$

$$product\ of\ ectremes = product\ of\ means$$

$$(8-x)(25-x) = (11-x)(16-x)$$

$$200-8x-25x+x^2 = 176-11x-16x+x^2$$

$$200-33x+x^2 = 176-27x+x^2$$

$$-33x+x^2+27x-x^2 = 176-200$$

$$-6x = -24$$

$$x = 4$$

Exercise 3.2

Question No.1

if y varies directly as x, and $y = 8$ when $x = 2$ Find

i. y in terms of x

$$y \propto x$$

$$y = kx \rightarrow (i)$$

$$8 = k \times 2$$

$$k = \frac{8}{2} = 4$$

So equation (i) becomes

$$y = 4x$$

ii. y when $x = 5$

$$y = 4x$$

$$y = 4 \times 5$$

$$y = 20$$

iii. x when $y = 28$

$$y = 4x$$

$$28 = 4x$$

$$\frac{28}{4} = x$$

$$x = 7$$

Question No2.

if $y \propto x$, and $y = 7$ when $x = 3$ find

i. y in terms of x

$$y \propto x$$

$$y = kx \rightarrow (i)$$

$$7 = k \times 3$$

$$k = \frac{7}{3}$$

So equation (i) becomes

$$y = \frac{7}{3} \times x = \frac{7x}{3}$$

ii. x when $y = 35$ and y when $x = 18$

When $y = 35$

$$y = \frac{7}{3} \times x$$

$$35 = \frac{7}{3} \times x$$

$$\frac{35 \times 3}{7} = x$$

$$5 \times 3 = x$$

$$15 = x$$
Put $x = 18$ in (i)we get
$$y = kx$$
$$y = \frac{7}{3} \times 18$$
$$y = 42$$

Question No.3 if $R \propto T$ and $R = 5$ when $T = 8$ find the equation connection R and T. Also find R when $T = 64$ and find T when $R = 21$

solution:
$$R \propto T$$
$$R = kT$$
when $R = 5$ and $T = 8$
$$5 = k8$$
$$k = \frac{5}{8}$$
So,
$$R = \frac{5}{8}T$$
Put $T = 64$
$$R = \frac{5}{8}T$$
Put$T = 64$
$$R = \frac{5}{8} \times 64 = 40$$
$$R = 40$$
when $R = 20$
$$20 = \frac{5}{8} \times T$$
$$20 \times \frac{8}{5} = T$$
$$4 \times 8 = T$$
$$T = 32$$

Question No.5

if $V \propto R^3$, and $V = 5$ when $R = 3$ find when $V = 625$
$$V \propto R^3$$
$$V = kR^3$$
$$5 = k(3)^3$$
$$5 = 27k$$
$$\frac{5}{27} = k$$
So,

eq (i)becomes $V = \frac{5}{27}R^3$

Put $V = 625$
$$625 = \frac{5}{27} \times R^3$$
$$R^3 = 625 \times \frac{27}{5}$$
$$R^3 = 125 \times 27$$
Taking cubes on both sides
$$R^{3 \times \frac{1}{3}} = (5^3)^{\frac{1}{3}} \times (3^3)^{\frac{1}{3}}$$
$$R = 5 \times 3 = 15$$
$$R = 15$$

Question N0.6 if w varies dirctly as $u^3$and $w = 81$ when $u = 3$ Find w when $u = 5$
$$w \propto u^3$$
$$w = ku^3 \rightarrow (i)$$
when $w = 81$ and $u = 3$

so eq (i)becomes
$$81 = k(3)^3$$
$$81 = k\,27$$
$$\frac{81}{27} = k$$
$$k = 3$$
So,
$$w = 3u^3$$
Put $u = 5$ in (i)
$$w = 3 \times 5^3$$
$$3 \times 125$$
$$w = 375$$

Question N0.7 if y varies inversly as x and $y = 7$ when $x = 2$ find y when $x = 126$

Solution:
$$y \propto \frac{1}{x}$$
$$y = \frac{k}{x} \rightarrow (i)$$
when $y = 7$ and $x = 2$

so (i)eq becomes
$$7 = \frac{k}{2}$$
$$7 \times 2 = k$$
$$14 = k$$
So,
$$y = \frac{14}{x}$$
when $x = 126$
$$y = \frac{14}{126} = \frac{1}{9}$$
$$y = \frac{1}{9}$$

Question No.8 if $y \propto \frac{1}{x}$ and $y = 4$ when $x = 3$, find x when $y = 24$
$$y \propto \frac{1}{x}$$
$$y = \frac{k}{x}$$
when $y = 4$ and $x = 3$
$$y = \frac{k}{x}$$
$$4 = \frac{k}{3}$$
$$12 = k$$
$$\text{or } k = 12$$
So,
$$y = \frac{12}{x}$$
When $y = 24$

$$24 = \frac{12}{x}$$
$$x = \frac{12}{24}$$
$$x = \frac{1}{2}$$

Question No.9 if $w \propto \frac{1}{z}$ and $w = 5$ when $z = 7$, find w when $z = \frac{175}{4}$

Solution:

$$w \propto \frac{1}{z}$$
$$w = \frac{k}{z} \rightarrow (i)$$

When $w = 5$ and $z = 7$

so eq (i) becomes

$$5 = \frac{k}{7}$$
$$5 \times 7 = k$$
$$k = 35$$

So,

$$w = \frac{35}{z}$$

when $z = \frac{175}{4}$

$$w = \frac{35}{\frac{175}{4}} = 35 \times \frac{4}{175} = \frac{4}{5}$$
$$w = \frac{4}{5}$$

Question No.10 if $A \propto \frac{1}{r^2}$ and $A = 2$ when $r = 3$ find r when $A = 72$

$$A \propto \frac{1}{r^2}$$
$$A = \frac{k}{r^2}$$

when $A = 2$ and $r = 3$

$$A = \frac{k}{r^3}$$
$$2 = \frac{k}{(3)^2}$$
$$2 = \frac{k}{9}$$
$$2 \times 9 = k$$
$$k = 18$$

So,

$$A = \frac{18}{r^2}$$

When $A = 72$

$$A = \frac{k}{r^2}$$
$$72 = \frac{18}{r^2}$$
$$r^2 = \frac{18}{72}$$
$$r^2 = \frac{1}{4}$$

Taking squaring root on both sides

$$r = \pm\frac{1}{2}$$

Question No.11 if $a \propto \frac{1}{b^2}$ and $a = 3$ when $b = 4$, find a when $b = 8$

Solution:

$$a \propto \frac{1}{b^2}$$
$$a = \frac{k}{b^2} \rightarrow (i)$$

when $a = 3$ and $b = 4$

$$3 = \frac{k}{(4)^2} = \frac{k}{16}$$
$$3 \times 16 = k$$
$$48 = k$$
$$\text{or } k = 48$$

So, (i)becomes

$$a = \frac{48}{b^2}$$

When $b = 8$

$$a = \frac{k}{b^2}$$
$$a = \frac{48}{8^2}$$
$$a = \frac{48}{64}$$
$$a = \frac{3}{4}$$

Question No.12 if $v \propto \frac{1}{r^3}$ and $V = 5$ when $r = 3$ find V when $r = 6$ anr find r when $v = 320$

$$V \propto \frac{1}{r^3}$$
$$V = \frac{k}{r^3}$$

When $V = 5$ and $r = 3$

$$5 = \frac{k}{3^3}$$
$$5 = \frac{k}{27}$$
$$5 \times 27 = k$$
$$135 = k$$
$$\text{or } k = 135$$

When $r = 6$

$$V = \frac{135}{6^3}$$
$$V = \frac{135}{216}$$
$$V = \frac{5}{8}$$
$$\text{when } V = 320$$

When $V = 320$

$$320 = \frac{135}{r^3}$$
$$r^3 = \frac{135}{320}$$
$$r^3 = \frac{27}{64}$$

Taking cube on both sides

$$r^{3\times\frac{1}{3}} = \frac{(3)^{3\times\frac{1}{3}}}{(4)^{3\times\frac{1}{3}}}$$
$$r = \frac{3}{4}$$

Question No.13 if $m \propto \frac{1}{n^3}$ and $m = 2$ when $n = 4$, find m when $n = 6$ and n when $m = 432$

Solution:

$$m \propto \frac{1}{n^3}$$
$$m = \frac{k}{n^3}$$

When $m = 2$ and $n = 4$

$$m = \frac{k}{n^3}$$

When $m = 2$ and $n = 4$

$$m = \frac{k}{n^3}$$
$$2 = \frac{k}{(4)^3}$$
$$2 = \frac{k}{64}$$
$$2 \times 64 = k$$
$$128 = k$$

So,

$$m = \frac{128}{n^3}$$

When $n = 6$

$$m = \frac{128}{6^3}$$
$$m = \frac{128}{216}$$
$$m = \frac{16}{27}$$

When $m = 432$

$$m = \frac{k}{n^3}$$
$$432 = \frac{128}{n^3}$$
$$n^3 = \frac{128}{432}$$
$$n^3 = \frac{8}{27}$$

Taking cube on both sides by

$$n^{3\times\frac{1}{3}} = \frac{(2)^{3\times\frac{1}{3}}}{(3)^{3\times\frac{1}{3}}}$$
$$n = \frac{2}{3}$$

Exercise 3.3

Question No.1 find third proportional to

i. $6, 12$

let x be the third proportional to

$$6 : 12 :: 12 : x$$

product of Extremes = product of Means

$$6x = 12 \times 12$$
$$x = \frac{12 \times 12}{6}$$
$$x = 24$$

ii. $a^3, 3a^2$

let x be the third proportional to

$$a^3 : 3a^2 :: 3a^2 : x$$

product of Extremes = product of Means

$$a^3 \times x = 3a^2 \times 3a^2$$
$$x = \frac{3a^2 \times 3a^2}{a^3} = \frac{9a^4}{a^3}$$
$$x = 9a$$

iii. $a^2 - b^2, \; a - b$

let x be the third proportional to

$$a^2 - b^2 : a - b :: a - b : x$$

product of Extremes = product of Means

$$a^2 - b^2 \times x = a - b \times a - b$$
$$x = \frac{a - b \times a - b}{a^2 - b^2}$$
$$x = \frac{a - b \times a - b}{(a - b)(a + b)}$$
$$x = \frac{a - b}{a + b}$$

iv. $(x - y)^2, x^3 - y^3$

let x be the third proportional to

$$(x - y)^2 : x^3 - y^3 :: x^3 - y^3 : c$$

product of Extremes = product of Means

$$(x - y)^2 \times c = x^3 - y^3 \times x^3 - y^3$$
$$c = \frac{x^3 - y^3 \times x^3 - y^3}{(x - y)^2}$$

$$c = \frac{(x-y)(x^2+xy+y^2)(x-y)(x^2+xy+y^2)}{(x-y)(x-y)}$$

$$x = (x^2+xy+y^2)(x^2+xy+y^2)$$

v. $(x+y)^2, x^3-xy-2y^2$

let x be the third proportional to

$$(x+y)^2 : (x^2-xy-2y^2) :: (x^2-xy-2y^2) : c$$

product of Extremes = product of Means

$$(x-y)^2 \times c = (x^2-xy-2y^2) \times (x^2-xy-2y^2)$$

$$c = \frac{(x^2-xy-2y^2)\times(x^2-xy-2y^2)}{(x+y)^2}$$

$$c = \frac{(x^2-xy-2y^2)^2}{(x+y)^2}$$

$$c = \frac{(x^2-2xy+xy-2y^2)^2}{(x+y)^2}$$

$$c = \frac{\left(x(x-2y)+y(x-2y)\right)^2}{(x+y)^2}$$

$$c = \frac{\left((x+y)(x-2y)\right)^2}{(x+y)^2}$$

$$c = \frac{(x+y)^2(x-2y)^2}{(x+y)^2}$$

$$c = (x-2y)^2$$

$$c = (x-2y)(x-2y)$$

vi. $\frac{p^2-q^2}{p^3+q^3}, \frac{p-q}{p^2-pq+q^2}$

let x be the third proportional to

$$\frac{p^2-q^2}{p^3+q^3} : \frac{p-q}{p^2-pq+q^2} :: \frac{p-q}{p^2-pq+q^2} : c$$

product of Extremes = product of Means

$$\frac{p^2-q^2}{p^3+q^3} \times c = \frac{p-q}{p^2-pq+q^2} \times \frac{p-q}{p^2-pq+q^2}$$

$$c = \frac{p-q}{p^2-pq+q^2} \times \frac{p-q}{p^2-pq+q^2} \times \frac{p^3+q^3}{p^2-q^2}$$

$$c = \frac{p-q}{p^2-pq+q^2} \times \frac{p-q}{p^2-pq+q^2} \times \frac{(p+q)(p^2-pq+q^2)}{(p-q)(p+q)}$$

$$c = \frac{p-q}{p^2-pq+q^2}$$

Question No.2 find a fourth proportional to

i. 5,8,15

let x be the fourth proportional

$$5 : 8 :: 15 : x$$

product of Extremes = Product of means

$$5x = 8 \times 15$$

$$x = \frac{8 \times 15}{5}$$

$$x = 8 \times 3$$

$$x = 24$$

ii. $4x^2, 2x^3, 18x^5$

let x be the fourth proportional

$$4x^2 : 2x^3 :: 18x^5 : c$$

product of Extremes = Product of means

$$4x^4 \times c = 2x^3 \times 18x^5$$

$$c = \frac{2x^3 \times 18x^5}{4x^4}$$

$$c = \frac{36x^8}{4x^4}$$

$$c = 9x^4$$

iii. $15a^5b^6, 10a^2b^5, 21a^3b^3$

let x be the fourth proportional

$$15a^5b^6 : 10a^2b^5 :: 21a^3b^3 : c$$

product of Extremes = Product of means

$$15a^5b^6 \times c = 10a^2b^5 \times 21a^3b^3$$

$$c = \frac{10a^2b^5 \times 21a^3b^3}{15a^5b^6}$$

$$c = \frac{2a^2b^5 \times 7a^3b^3}{a^5b^6}$$

$$c = 14\,a^2b^5 \times a^3b^3 \times a^{-5}b^{-6}$$

$$c = 14a^{2+3-5}\,b^{5+3-6}$$

$$c = 14b^2$$

iv. $x^2-11x+24, x-3, 5x^4-40x^3$

let x be the fourth proportional

$$x^2-11x+24 : x-3 :: 5x^4-40x^3 : c$$

product of Extremes = Product of means

$$x^2-11x+24 \times c = (x-3) \times (5x^4-40x^3)$$

$$c = \frac{(x-3)\times(5x^4-40x^3)}{x^2-11x+24}$$

$$c = \frac{(x-3)5x^3(x-8)}{x^2-8x-3x+24}$$

$$c = \frac{5x^3(x-3)(x-8)}{x(x-8)-3(x-8)}$$

$$c = \frac{5x^3(x-3)(x-8)}{(x-3)(x-8)}$$

$$c = 5x^3$$

v. $p^3+q^3, p^2-q^2, p^2-pq+q^2$

let x be the fourth proportional

$$p^3+q^3 : p^2-q^2 :: p^2-pq+q^2 : c$$

product of Extremes = Product of means

$$p^3+q^3 \times c = p^2-q^2 \times p^2-pq+q^2$$

$$c = \frac{p^2-q^2 \times p^2-pq+q^2}{p^3+q^3}$$

$$c = \frac{(p-q)(p+q)(p^2-pq+q^2)}{p^3+q^3}$$

$$c = \frac{(p-q)(p^3+q^3)}{p^3+q^3}$$

$$c = (p-q)$$

vi. $(p^2-q^2)(p^2+pq+q^2), p^3+q^3, p^3-q^3$

let x be the fourth proportional

$$(p^2-q^2)(p^2+pq+q^2) : p^3+q^3 :: p^3-q^3 : c$$

$$(p^2-q^2)(p^2+pq+q^2) \times c = p^3+q^3 \times p^3-q^3$$

$$c = \frac{p^3+q^3 \times p^3-q^3}{(p^2-q^2)(p^2+pq+q^2)}$$

$$c = \frac{(p+q)(p^2-pq+q^2)(p-q)(p^2+pq+q^2)}{(p+q)(p-q)(p^2+pq+q^2)}$$

$$c = p^2 - pq + q^2$$

Question No.3 find a mean proportional between

i. ***20,45***

let x be the mean proportional

$$20 : x :: x : 45$$

$$\text{product of means} = \text{product of extremes}$$

$$x^2 = 20 \times 45$$

$$x^2 = 900$$

Taking squaring root on both sides

$$\sqrt{x^2} = \sqrt{(30)^2}$$

$$x = \pm 30$$

ii. ***20***$x^3y^5,\ 5x^7y$

let x be the mean proportional

$$20x^3y^5 : c :: c : 5x^7y$$

$$\text{product of means} = \text{product of extremes}$$

$$c^2 = 20x^3y^5 \times 5x^7y$$

$$c^2 = 100x^{10}y^6$$

Taking squaring root on both sides

$$\sqrt{c^2} = \sqrt{(10)^2x^{10}y^6}$$

$$x = \pm 10x^5y^3$$

iii. $15p^4qr^3, 135q^5r^7$

let x be the mean proportional

$$15p^4qr^3 : x :: x : 135q^5r^7$$

$$\text{product of means} = \text{product of extremes}$$

$$x^2 = 15p^4qr^3 \times 135q^5r^7$$

$$x^2 = 2025p^4q^6r^{10}$$

Taking squaring root on both sides

$$\sqrt{x^2} = \sqrt{(45)^2p^4q^6r^{10}}$$

$$x = \pm 45p^2q^3r^5$$

iv. $x^2 - y^2, \frac{x-y}{x+y}$

let x be the mean proportional

$$x^2 - y^2 : c :: c : \frac{x-y}{x+y}$$

$$\text{product of means} = \text{product of extremes}$$

$$c^2 = x^2 - y^2 \times \frac{x-y}{x+y}$$

$$c^2 = (x-y)(x+y) \times \frac{x-y}{x+y}$$

$$c^2 = (x-y)(x-y)$$

$$c^2 = (x-y)^2$$

Taking squaring root on both sides

$$\sqrt{c^2} = \sqrt{(x-y)^2}$$

$$x = \pm(x-y)$$

Question No.4 find the valves of the letter involved in the following continued proportional

i. $5, p, 45$

$$5 : p :: p : 45$$

$$\text{product of means} = \text{product of extremes}$$

$$p^2 = 5 \times 45$$

$$p^2 = 225$$

Squaring root on both sides

$$\sqrt{p^2} = \sqrt{(15)^2}$$

$$p = \pm 15$$

ii. $8, x, 18$

$$8 : x :: x : 18$$

$$\text{product of means} = \text{product of extremes}$$

$$x^2 = 8 \times 18$$

$$x^2 = 144$$

Squaring root on both sides

$$\sqrt{x^2} = \sqrt{(12)^2}$$

$$x = \pm 12$$

iii. $12, 3p - 6, 27$

$$12 : 3p - 6 :: 3p - 6 : 27$$

$$\text{product of means} = \text{product of extremes}$$

$$(3p-6)^2 = 12 \times 27$$

$$(3p-6)^2 = 324$$

Squaring root on both sides

$$\sqrt{(3p-6)^2} = \sqrt{(18)^2}$$

$$3p - 6 = \pm 18$$

$3p - 6 = +18$	$3p - 6 = -18$
$3p = 18 + 6$	$3p = -18 + 6$
$3p = 24$	$3p = -12$
$p = \frac{24}{3} = 8$	$p = -\frac{12}{3}$
$p = 8$	$p = -4$

iv. $7, m - 3,\ 28$

$$7 : m - 3 :: 3m - 3 : 28$$

$$\text{product of means} = \text{product of extremes}$$

$$(m-3)^2 = 7 \times 28$$

$$(m-3)^2 = 196$$

Squaring root on both sides

$$\sqrt{(m-3)^2} = \sqrt{(14)^2}$$

$$m - 3 = \pm 14$$

$m - 3 = +14$	$m - 3 = 14$
$m = 14 + 3$	$m = -14 + 3$
$m = 14$	$m = -11$

Exercise 3.4

Question No.1 prove that $a:b=c:d \quad if$

i. $\frac{4a+5b}{4a-5b} = \frac{4c+5d}{4c-5d}$

Solution:

$By\ componendo-dividendo$

$$\frac{(4a+5b)+(4a-5b)}{(4a+5b)-(4a-5b)} = \frac{(4c+5d)+(4c-5d)}{(4c-5d)-(4c-5d)}$$

$$\frac{4a+5b+4a-5b}{4a+5b-4a+5b} = \frac{4c+5d+4c-5d}{4c+5d-4c+5d}$$

$$\frac{4a+4a}{5b+5b} = \frac{4c+4c}{5d+5d}$$

$$\frac{8a}{10b} = \frac{8c}{10d}$$

Multiplying by $\frac{10}{8}$

$$\frac{a}{b} = \frac{c}{d}$$

$$a:b=c:d$$

ii. $\frac{2a+9b}{2a-9b} = \frac{2c+9d}{2c-9d}$

$By\ componendo-dividendo$

$$\frac{(2a+9b)+(2a-9b)}{(2a+9b)-(2a-9b)} = \frac{(2c+9d)+(2c-9d)}{(2c+9d)-(2c-9d)}$$

$$\frac{2a+9b+2a-9b}{2a+9b-2a+9b} = \frac{2c+9d+2c-9d}{2c+9d-2c+9d}$$

$$\frac{2a+2a}{9b+9b} = \frac{2c+2c}{9d+9d}$$

$$\frac{4a}{18b} = \frac{4c}{18d}$$

Multiplying by $\frac{18}{4}$

$$\frac{a}{b} = \frac{c}{d}$$

$$a:b=c:d$$

iii. $\frac{ac^2+bd^2}{ac^2-bd^2} = \frac{c^3+d^3}{c^3-d^3}$

$By\ componendo-dividendo$

$$\frac{(ac^2+bd^2)+(ac^2-bd^2)}{(ac^2+bd^2)-(ac^2-bd^2)} = \frac{(c^3+d^3)+(c^3-d^3)}{(c^3+d^3)-(c^3-d^3)}$$

$$\frac{ac^2+bd^2+ac^2-bd^2}{ac^2+bd^2-ac^2+bd^2} = \frac{c^3+d^3+c^3-d^3}{c^3+d^3-c^3+d^3}$$

$$\frac{2ac^2}{2bd^2} = \frac{2c^3}{2d^3}$$

$$\frac{ac^2}{bd^2} = \frac{c^3}{d^3}$$

Multiplying by $\frac{d^2}{c^2}$

$$\frac{a}{b} = \frac{c}{d}$$

$$a:b=c:d$$

iv. $\frac{a^2c+b^2d}{a^2c-b^2d} = \frac{ac^2+bd^2}{ac^2-bd^2}$

$By\ componendo-dividendo$

$$\frac{(a^2c+b^2d)+(a^2c-b^2d)}{(a^2c+b^2d)-(a^2c-b^2d)} = \frac{(ac^2+bd^2)+(ac^2-bd^2)}{(ac^2+bd^2)-(ac^2-bd^2)}$$

$$\frac{a^2c+b^2d+a^2c-b^2d}{a^2c+b^2d-a^2c+b^2d} = \frac{ac^2+bd^2+ac^2-bd^2}{ac^2+bd^2-ac^2+bd^2}$$

$$\frac{2a^2c}{2b^2d} = \frac{2ac^2}{2bd^2}$$

$$\frac{a^2c}{b^2d} = \frac{ac^2}{bd^2}$$

Multiplying by $\frac{bd}{ac}$

$$\frac{a}{b} = \frac{c}{d}$$

$$a:b=c;d$$

v. $\frac{pa+qb}{pa-qb} = \frac{pc+qd}{pc-qd}$

$By\ componendo-dividendo$

$$\frac{(pa+qb)+(pa-qb)}{(pa+qb)-(pa-qb)} = \frac{(pc+qd)+(pc-qd)}{(pc+qd)-(pc-qd)}$$

$$\frac{pa+qb+pa-qb}{pa+qb-pa+qb} = \frac{pc+qd+pc-qd}{pc+qd-pc+qd}$$

$$\frac{2pa}{2qb} = \frac{2pc}{2qd}$$

$$\frac{pa}{qb} = \frac{pc}{qd}$$

Multiplying by $\frac{q}{p}$

$$\frac{a}{b}=\frac{c}{d}$$
$$a:b=c:d$$

vi. $\frac{a+b+c+d}{a+b-c-d}=\frac{a-b+c-d}{a-b-c+d}$

By componendo – dividendo

$$\frac{(a+b+c+d)+(a+b-c-d)}{(a+b+c+d)-(a+b-c-d)}=\frac{(a-b+c-d)+(a-b-c+d)}{(a-b+c-d)-(a-b-c+d)}$$
$$\frac{a+b+c+d+a+b-c-d}{a+b+c+d-a-b+c+d}=\frac{a-b+c-d+a-b-c+d}{a-b+c-d-a+b+c+d}$$
$$\frac{2a+2b}{2c+2d}=\frac{2a-2b}{2c-2d}$$
$$\frac{a+b}{c+d}=\frac{a-b}{c-d}$$

by comoponendo – dividendo

$$\frac{(a+b)+(a-b)}{(a+b)-(a-b)}=\frac{(c+d)+(c-d)}{(c+d)-(c-d)}$$
$$\frac{2a}{2b}=\frac{2c}{2d}$$
$$\frac{a}{b}=\frac{c}{d}$$
$$a:\ b=\ c:dd$$

vii. $\frac{2a+3b+2c+3d}{2a+3b-2c-3d}=\frac{2a-3b+2c-3d}{2a-3b-2c+3d}$

Solution:

$$\frac{(2a+3b+2c+3d)+(2a+3b-2c-3d)}{(2a+3b+2c+3d)-(2a+3b-2c-3d)}=\frac{(2a-3b+2c-3d)+(2a-3b-2c+3d)}{(2a-3b+2c-3d)-(2a-3b-2c+3d)}$$
$$\frac{2a+3b+2c+3d+2a+3b-2c-3d}{2a+3b+2c+3d-2a+3b+2c+3d}=\frac{2a-3b+2c-3d+2a-3b-2c+3d}{2a-3b+2c-3d-2a+3b+2c-3d}$$
$$\frac{4a+6b}{4c+6d}=\frac{4a-6b}{4c-6d}$$
$$\frac{4a+6b}{4a-6b}=\frac{4c+6d}{4c-6d}$$

by componendo – dividendo

$$\frac{(4a+6b)+(4a-6b)}{(4a+6b)-(4a-6b)}=\frac{(4c+6d)+(4c-6d)}{(4c+6d)-(4c-6d)}$$
$$\frac{4a+6b+4a-6b}{4a+6b-4a+6b}=\frac{4c+6d+4c-6d}{4c+6d-4c+6d}$$
$$\frac{8a}{12b}=\frac{8c}{12d}$$
$$\frac{2a}{3b}=\frac{2c}{3d}$$

Multiplying by $\frac{3}{2}$

$$\frac{a}{b}=\frac{c}{d}$$
$$a:\ b=\ c:dd$$

viii. $\frac{a^2+b^2}{a^2-b^2}=\frac{ac+bd}{ac-bd}$

by componendo – dividendo

$$\frac{(a^2+b^2)+(a^2-b^2)}{(a^2+b^2)-(a^2-b^2)}=\frac{(ac+bd)+(ac-bd)}{(ac+bd)-(ac-bd)}$$
$$\frac{a^2+b^2+a^2-b^2}{a^2+b^2-a^2+b^2}=\frac{ac+bd+ac-bd}{ac+bd-ac+bd}$$
$$\frac{2a^2}{2b^2}=\frac{2ac}{2bd}$$
$$\frac{a^2}{b^2}=\frac{ac}{bd}$$

Multiplying by $\frac{b}{a}$

$$\frac{a}{b}=\frac{c}{d}$$
$$a:\ b=\ c:dd$$

Question No.2 using theorem of componendo – dividendo

i. ***Find the value of*** $\frac{x+2y}{x-2y}+\frac{x+2z}{x-2z}$ if $x=\frac{4yz}{y+z}$

$$x=\frac{4yz}{y+z}\rightarrow (i)$$

From eq. (i)

$$x=\frac{2y+2z}{y+z}$$
$$\frac{x}{2y}=\frac{2z}{y+z}$$

By applying componendo – dividendo ***theorem***

$$\frac{x+2y}{x-2y}=\frac{2z+y+z}{2z-y-z}$$
$$\frac{x+2y}{x-2y}=\frac{y+3z}{z-y}\rightarrow (ii)$$

From eq (i)

$$x=\frac{2y\times 2z}{y+z}$$
$$\frac{x}{2z}=\frac{2y}{y+z}$$

By applying componendo – dividendo ***theorem***

$$\frac{x+2z}{x-2z}=\frac{2y+y+z}{2y-y-z}$$
$$\frac{x+2z}{x-2z}=\frac{z+3y}{y-z}\rightarrow (iii)$$

Adding equations (ii) and (iii)

$$\frac{x+2y}{x-2y}+\frac{x+2z}{x-2z}=\frac{y+3z}{z-y}+\frac{z+3y}{y-z}$$
$$=-\frac{y+3z}{y-z}+\frac{z+3y}{y-z}$$
$$=\frac{z+3y}{y-z}-\frac{y+3z}{y-z}$$
$$=\frac{z+3y-y-3z}{y-z}$$
$$=\frac{2y-2z}{y-z}$$

$$=\frac{2(y-z)}{y-z}=2$$

ii. find the value of $\frac{m+5n}{m-5n}+\frac{m+5p}{m-5p}$

If $m=\frac{10np}{n+p}$

Solution:

$$m=\frac{10np}{n+p}\rightarrow(i)$$

From eq(i)

$$m=\frac{5n\times 2p}{n+p}$$
$$\frac{m}{5n}=\frac{2p}{n+p}$$

By applying componendo − dividendo ***theorem***

$$\frac{m+5n}{m-5n}=\frac{2p+n+p}{2p-n-p}$$
$$\frac{m+5n}{m-5n}=\frac{3p+n}{p-n}\rightarrow(ii)$$

From eq.(i)

$$m=\frac{2n\times 5p}{n+p}$$
$$\frac{m}{5p}=\frac{2n}{n+p}$$

By applying componendo − dividendo ***theorem***

$$\frac{m+5p}{m-5p}=\frac{2n+n+p}{2n-n-p}$$
$$\frac{m+5p}{m-5p}=\frac{3n+p}{n-p}\rightarrow(iii)$$

Adding (ii) and (iii)

$$\frac{m+5n}{m-5n}+\frac{m+5p}{m-5p}=\frac{3p+n}{p-n}+\frac{3n+p}{n-p}$$
$$=-\frac{3p+n}{n-p}+\frac{3n+p}{n-p}$$
$$=\frac{3n+p}{n-p}-\frac{3p+n}{n-p}$$
$$=\frac{3n+p-3p-n}{n-p}$$
$$=\frac{2n-2p}{n-p}$$
$$\frac{2(n-p)}{n-p}=2$$

iii. ***Find the value*** of $\frac{x-6a}{x+6a}-\frac{x+6a}{x-6a}$, if $x=\frac{12ab}{a-b}$

Solution:

$$x=\frac{12ab}{a-b}\rightarrow(i)$$

From equation (i)

$$x=\frac{6a\times 2b}{a-b}$$
$$\frac{x}{6a}=\frac{2b}{a-b}$$

By applying componendo − dividendo ***theorem***

$$\frac{x+6a}{x-6a}=\frac{2b+a-b}{2b+a-b}$$
$$\frac{x+6a}{x-6a}=\frac{a+b}{3b-a}$$
$$\frac{x-6a}{x+6a}=\frac{3b-a}{a+b}\rightarrow(ii)$$

From eq.(i)

$$x=\frac{6b\times 2a}{a-b}$$
$$\frac{x}{6b}=\frac{2a}{a-b}$$

By applying componendo − dividendo ***theorem***

$$\frac{x+6b}{x-6b}=\frac{2a+a-b}{2a-a+b}$$
$$\frac{x+6b}{x-6b}=\frac{3a-b}{a+b}\rightarrow(iii)$$

Subtracting equation (iii) from (ii)

$$\frac{x-6a}{x+6a}-\frac{x+6b}{x-6b}=\frac{3b-a}{a+b}-\frac{3a-b}{a+b}$$
$$=\frac{3b-a-3a+b}{a+b}$$
$$=\frac{-4a+4b}{a+b}$$
$$=\frac{4(b-a)}{a+b}$$

iv. ***Find the value of*** $\frac{x-3y}{x+3y}-\frac{x+3z}{x-3z}$, if $x=\frac{3yz}{y-z}$

$$x=\frac{3yz}{y-z}\rightarrow(i)$$

From equation (i)

$$x=\frac{3y\times z}{y-z}$$
$$\frac{x}{3y}=\frac{z}{y-z}$$

By applying componendo − dividendo ***theorem***

$$\frac{x+3y}{x-3y}=\frac{z+y-z}{z-y+z}$$
$$\frac{x+3y}{x-3y}=\frac{y}{2z-y}$$
$$\frac{x-3y}{x+3y}=\frac{2z-y}{y}\rightarrow(ii)$$

From equation (i)

$$x=\frac{3z\times y}{y-z}$$
$$\frac{x}{3z}=\frac{y}{y-z}$$

By applying componendo − dividendo ***theorem***

$$\frac{x+3z}{x-3z}=\frac{y+y-z}{y-y+z}$$

$$\frac{x+3z}{x-3z}=\frac{2y-z}{z}$$
$$\frac{x+3z}{x-3z}=\frac{2y-z}{z}\rightarrow \text{(iii)}$$
Subtracting equation (iii) from eq. (ii)
$$\frac{x-3y}{x+3y}-\frac{x+3z}{x-3z}=\frac{2z-y}{y}-\frac{2y-z}{z}$$
$$=\frac{z(2z-y)-y(2y-z)}{yz}$$
$$=\frac{2z^2-zy-2y^2+yz}{yz}$$
$$\frac{2(z^2-y^2)}{yz}$$

v. ***Find the value of*** $\frac{s-3p}{s+3p}+\frac{s+3q}{s-3q}$, if $s=\frac{6pq}{p-q}$
$$s=\frac{6pq}{p-q}\rightarrow \text{(i)}$$
From eq. (i)
$$s=\frac{3p\times 2p}{p-q}$$
$$\frac{s}{3p}=\frac{2q}{p-q}$$
By applying componendo − dividendo ***theorem***
$$\frac{s+3p}{s-3p}=\frac{2q+p-q}{2q-p+q}$$
$$\frac{s+3p}{s-3p}=\frac{q+p}{3q-p}$$
$$\frac{s-3p}{s+3p}=\frac{3q-p}{p+q}\rightarrow \text{(ii)}$$
From eq.(i)
$$s=\frac{2p\times 3q}{p-q}$$
$$\frac{s}{3q}=\frac{2p}{p-q}$$
$$\frac{s+3q}{s-3q}=\frac{2p+p-q}{2p-p+q}$$
$$\frac{s+3q}{s-3q}=\frac{3p-q}{p+q}\rightarrow \text{(iii)}$$
Adding equation (ii) and (i)
$$\frac{s-3p}{s+3p}+\frac{s+3q}{s-3q}=\frac{3q-p}{p+q}+\frac{3p-q}{p+q}$$
$$=\frac{3q-p+3p-q}{p+q}$$
$$=\frac{2q+2p}{p+q}$$
$$=2\frac{p+q}{p+q}$$
$$=2$$

vi. ***Solve*** $\frac{(x-2)^2-(x-4)^2}{(x-2)^2+(x-4)^2}=\frac{12}{13}$
$$\frac{(x-2)^2-(x-4)^2}{(x-2)^2+(x-4)^2}=\frac{12}{13}$$
By applying componendo − dividendo ***theorem***
$$\frac{(x-2)^2-(x-4)^2+(x-2)^2+(x-4)^2}{(x-2)^2-(x-4)^2-(x-2)^2-(x-4)^2}$$
$$=\frac{12+13}{12-13}$$
$$\frac{2(x-2)^2}{-2(x-4)^2}=\frac{25}{-1}$$
$$\frac{(x-2)^2}{(x-4)^2}=25$$
Taking root on both side
$$\frac{x-2}{x-4}=\pm 5$$

$$\frac{x-2}{x-4}=5$$
$$x-2=5(x-4)$$
$$x-2=5x-20$$
$$x-5x=-20+2$$
$$-4x=-18$$
$$4x=18$$
$$x=\frac{18}{4}$$
$$x=\frac{9}{2}$$

$$\frac{x-2}{x-4}=-5$$
$$x-2$$
$$=-5(x-4)$$
$$x-2$$
$$=-5x+20$$
$$x+5x=20+2$$
$$6x=22$$
$$x=\frac{22}{6}$$
$$x=\frac{11}{3}$$

$$S.S=\left\{\frac{9}{2},\frac{11}{3}\right\}$$

vii. ***Solve*** $\frac{\sqrt{x^2+2}+\sqrt{x^2-2}}{\sqrt{x^2+2}-\sqrt{x^2-2}}=2$

Solution:
$$\frac{\sqrt{x^2+2}+\sqrt{x^2-2}}{\sqrt{x^2+2}-\sqrt{x^2-2}}=2$$
By applying componendo − dividendo ***theorem***
$$\frac{\sqrt{x^2+2}+\sqrt{x^2-2}+\sqrt{x^2+2}-\sqrt{x^2-2}}{\sqrt{x^2+2}+\sqrt{x^2-2}-\sqrt{x^2+2}+\sqrt{x^2-2}}$$
$$=\frac{2+1}{2-1}$$
$$\frac{2\sqrt{x^2+2}}{2\sqrt{x^2-2}}=\frac{3}{1}$$
$$\frac{\sqrt{x^2+2}}{\sqrt{x^2-2}}=3$$
Taking square on both sides
$$\frac{x^2+3}{x^2-2}=9$$
$$x^2+2=9(x^2-2)$$
$$x^2+2=9x^2-18$$
$$2+18=9x^2-x^2$$
$$20=8x^2$$
$$\text{or } 8x^2=20$$
$$x^2=\frac{20}{8}$$
$$x^2=\frac{5}{2}$$

$$x = \pm\sqrt{\frac{5}{2}}$$

If we check the given equation for this value doesn't satisfy the equation so the given solution is extraneous.

So,

$$S.S = \{\ \}$$

viii. $\frac{\sqrt{x^2+8p^2}-\sqrt{x^2-p^2}}{\sqrt{x^2+8p^2}+\sqrt{x^2-p^2}} = \frac{1}{3}$

Solution:

$$\frac{\sqrt{x^2+8p^2}-\sqrt{x^2-p^2}}{\sqrt{x^2+8p^2}+\sqrt{x^2-p^2}} = \frac{1}{3}$$

By applying componendo − dividendo ***theorem***

$$\frac{\sqrt{x^2+8p^2}-\sqrt{x^2-p^2}+\sqrt{x^2+8p^2}+\sqrt{x^2-p^2}}{\sqrt{x^2+8p^2}-\sqrt{x^2-p^2}-\sqrt{x^2+8p^2}-\sqrt{x^2-p^2}} = \frac{1+3}{1-3}$$

$$\frac{2\sqrt{x^2+8p^2}}{-2\sqrt{x^2-p^2}} = \frac{4}{-2}$$

$$\frac{\sqrt{x^2+8p^2}}{\sqrt{x^2-p^2}} = 2$$

Taking square on both sides

$$\frac{x^2+8p^2}{x^2-p^2} = 4$$

$$x^2+8p^2 = 4(x^2-p^2)$$

$$x^2+8p^2 = 4x^2-4p^2$$

$$x^2-4x^2 = -4p^2-8p^2$$

$$-3x^2 = -12p^2$$

$$x^2 = \frac{12}{3}p^2$$

$$x^2 = 4p^2$$

Taking squaring root both sides

$$x = \pm 2p$$

$$S.S = \{2p, -2p\}$$

ix. Solve $\frac{(x+5)^3-(x-3)^3}{(x+5)^3+(x-3)^3} = \frac{13}{14}$

Solution:

$$\frac{(x+5)^3-(x-3)^3}{(x+5)^3+(x-3)^3} = \frac{13}{14}$$

By applying componendo − dividendo ***theorem***

$$\frac{(x+5)^3-(x-3)^3+(x+5)^3+(x-3)^3}{(x+5)^3-(x-3)^3-(x+5)^3-(x-3)^3} = \frac{13+14}{13-14}$$

$$\frac{2(x+5)^3}{-2(x-3)^3} = \frac{27}{-1}$$

$$\frac{(x+5)^3}{(x-3)^2} = 27$$

Taking cube roots on both side

$$\frac{x+5}{x-3} = 3$$

$$x+5 = 3(x-3)$$

$$x+5 = 3x-9$$

$$x-3x = -9-5$$

$$-2x = -14$$

$$x = \frac{14}{2}$$

$$x = 7$$

$$S.S = \{7\}$$

Exercise 3.5

Question No.1 if S varies directly as u^2 ***and inversely as*** v and $s = 7$ ***when*** $u = 3, v = 2$. ***Find the value of S when u=6***

Solution:

$$s \propto u^2$$

$$s \propto \frac{1}{v}$$

$$s = k\frac{u^2}{v} \rightarrow \text{(i)}$$

$$\text{put } s = 7, u = 3, v = 2$$

$$7 = k\frac{3^2}{2}$$

$$7 = k\frac{9}{2}$$

$$\frac{7 \times 2}{9} = k$$

$$\frac{14}{9} = k$$

$$\text{or } k = \frac{14}{9}$$

So, equation (i) becomes

$$S = \frac{14u^2}{9v} \rightarrow \text{(ii)}$$

$$\text{Put } u = 6 \text{ and } v = 10 \text{ in equation (ii)}$$

$$S = \frac{14(6)^2}{9(10)}$$

$$S = \frac{14 \times 36}{9 \times 10}$$

$$S = \frac{28}{5}$$

Question No.2 if w varies jointly as x, y^2 ***and z and w=5 when x=2,y=3, z=10. Find w when x=4,y=7 and z=3.***

Solution:

$$w \propto x$$
$$w \propto y^2$$
$$w \propto z$$
$$w = kxy^2z \rightarrow (i)$$

Put $w = 5, x = 2, y = 3, z = 10$

$$5 = k(2)(3)^2(10)$$
$$5 = k(180)$$
$$k = \frac{5}{180}$$
$$k = \frac{1}{36}$$

so, Equation (i) becomes

$$w = \frac{xy^2z}{36} \rightarrow (ii)$$

Put $x = 4, y = 7$ and $z = 3$ in equation (ii)

$$S = \frac{4(7)^2(3)}{36}$$
$$S = \frac{49}{3}$$

Question No.3 if Y varies directly as x^3 and inversly as z^3 and t, and $y = 16$ ***When*** $x = 4, z = 2, t = 3$. find the value of y when $x = 2, z = 3$ and $t = 4$

Solution:

$$y \propto x^3$$
$$y \propto \frac{1}{z^2}$$
$$y \propto \frac{1}{t}$$
$$y = k\frac{x^3}{z^2t} \rightarrow (i)$$

Put $y = 16, x = 4, z = 2, t = 3$

$$16 = k\frac{4^3}{2^2(3)}$$
$$16 = k\frac{64}{12}$$
$$\frac{16 \times 12}{64} = k$$
$$\frac{16}{4} = k$$
$$3 = k$$
$$\text{or } k = 3$$

So, equation (i) becomes

$$y = \frac{3x^3}{z^2t} \rightarrow (ii)$$

Put $x = 2, z = 3$ and $t = 4$ ***in equation(ii)***

$$y = \frac{3(2)^3}{(3)^2(4)}$$
$$y = \frac{3(8)}{9(4)}$$

Divide by 3

$$y = \frac{8}{12}$$

Divide by 4 $\quad y = \frac{2}{3}$

Question No.4 if u varies directly as x^2 and inversly as the product yz^3, and $u = 2$ ***when x=8, y=7. Find the value of u when x=6, y=3 , z=2***

Solution:

$$u \propto x^2$$
$$u \propto \frac{1}{yz^3}$$
$$u = k\frac{x^2}{yz^3} \rightarrow (i)$$

put $u = 2, x = 8, y = 7, z = 2$

$$2 = k\frac{8^2}{7(2)^3}$$
$$2 = k\frac{64}{7 \times 8}$$
$$2 = k\frac{64}{56}$$
$$\frac{2 \times 56}{64} = k$$

divide by 8

$$\frac{2 \times 7}{8} = k$$
$$\frac{14}{8} = k$$

Divide by 2

$$\frac{7}{4} = k$$

So, Equation (i)

$$u = \frac{7x^2}{4yz^3} \rightarrow (ii)$$

put $x = 6, y = 3$ and $z = 2$ in equation (ii)

$$u = \frac{7(6)^2}{4(3)(2)^3}$$
$$u = \frac{7 \times 36}{4(3)(8)}$$
$$u = \frac{7 \times 9}{3(8)} \quad (\div \text{by} 4)$$
$$u = \frac{7 \times 3}{8} \quad (\div \text{ by } 3)$$
$$u = \frac{21}{8}$$

Question No.5 if v varies directly as the product xy^3 and inversly as z^2 and $v = 27$ ***when*** $x = 7, y = 6, z = 7$. find the ***value of v when*** $x = 6, y = 2, z = 3$

Solution:

$$v \propto xy^3$$
$$v \propto \frac{1}{z^2}$$

$$u = k\frac{xy^3}{z^2} \rightarrow (i)$$

put $v = 27, x = 7, y = 6, z = 7$

$$27 = k\frac{(7)(6)^3}{(7)^2}$$

$$27 = k\frac{216}{7} \quad (\div \text{ by } 7)$$

$$\frac{27 \times 7}{216} = k$$

$$\frac{7}{8} = k \qquad \text{by}(27 \div 216 = 8)$$

$$\text{or } k = \frac{7}{8}$$

so, Eqution (i)becomes

$$u = \frac{7xy^3}{8z^2} \rightarrow (ii)$$

put $x = 6, y = 2, z = 3$ in equation (ii)

$$u = \frac{7(6)(2)^3}{8(9)}$$

$$u = \frac{7 \times 6 \times 8}{8 \times 9}$$

$$u = \frac{7 \times 6}{9}$$

$$u = \frac{7 \times 2}{3}$$

$$u = \frac{14}{3}$$

Question No.6 if w varies inversely as the cube of u, and w=5 when u=3. Find w when u=6

Solution:

$$w \propto \frac{1}{u^3}$$

$$w = \frac{k}{u^3} \rightarrow (i)$$

Put w=5 and u=3

$$5 = \frac{k}{3^3}$$

$$5 = \frac{k}{27}$$

$$(5 \times 27) = k$$

$$135 = k$$

$$\text{or } 135 = k$$

So, equation (i) becomes

$$w = \frac{135}{u^3} \rightarrow (ii)$$

Put u=6 in equation (ii)

$$w = \frac{135}{(6)^3}$$

$$w = \frac{135}{216}$$

$$w = \frac{5}{8}$$

Exercise 3.6

Question No1 if $a : b = c : d$, , then show that

i. $\frac{4a-9b}{4a+9b} = \frac{4c-9d}{4c+9d}$

Solution:

As $a : b = c : d$

let $\frac{a}{b} = \frac{c}{d} = k$

then $a = bk$ and $c = dk$

$$\frac{4a - 9b}{4a + 9b} = \frac{4c - 9d}{4c + 9d}$$

Putting the valves

$$\frac{4(bk) - 9b}{4(bk) + 9b} = \frac{4(dk) - 9d}{4(dk) + 9d}$$

Putting the valves

$$\frac{4bk - 9b}{4bk + 9b} = \frac{4dk - 9d}{4dk + 9d}$$

$$\frac{b(4k - 9)}{b(4k + 9)} = \frac{d(4k - 9)}{d(4k + 9)}$$

$$\frac{(4k - 9)}{(4k + 9)} = \frac{(4k - 9)}{(4k + 9)}$$

$$L.H.S = R.H.S$$

ii. $\frac{6a-5b}{6a+5b} = \frac{6c-5d}{6c+5d}$

Solution:

As $a : b = c : d$

let $\frac{a}{b} = \frac{c}{d} = k$

then $a = bk$ and $c = dk$

$$\frac{6a - 5b}{6a + 5b} = \frac{6c - 5d}{6c + 5d}$$

Putting these valves

$$\frac{6(bk)-5b}{6(bk)+5b}=\frac{6(dk)-5d}{6(dk)+5d}$$
$$\frac{6bk-5b}{6bk+5b}=\frac{6dk-5d}{6dk+5d}$$
$$\frac{b(6k-5)}{b(6k+5)}=\frac{d(6k-5)}{d(6k+5)}$$
$$\frac{6k-5}{6k+5}=\frac{6k-5}{6k+5}$$
$$L.H.S=R.H.S$$

iii. $\frac{a}{b}=\sqrt{\frac{a^2+c^2}{b^2+d^2}}$

Solution:

As $a : b = c : d$

let $\frac{a}{b}=\frac{c}{d}=k$

then $a=bk$ and $c=dk$

$$\frac{a}{b}=\sqrt{\frac{a^2+c^2}{b^2+d^2}}$$

Putting these valves

$$\frac{bk}{b}=\sqrt{\frac{(bk)^2+(dk)^2}{b^2+d^2}}$$
$$\frac{bk}{b}=\sqrt{\frac{b^2k^2+k^2d^2}{b^2+d^2}}$$
$$k=\sqrt{\frac{k^2(b^2+d^2)}{b^2+d^2}}$$
$$k=\sqrt{k^2}$$
$$k=k$$
$$L.H.S=R.H.S$$

iv. $a^6+c^6 : b^6+d^6 = a^3c^2 : b^3d^3$

$$\frac{a^6+c^6}{b^6+d^6}=\frac{a^3c^3}{b^3d^3}$$

As $a : b = c : d$

let $\frac{a}{b}=\frac{c}{d}=k$

then $a=bk$ and $c=dk$

$$\frac{a^6+c^6}{b^6+d^6}=\frac{a^3c^3}{b^3d^3}$$

Putting these valves

$$\frac{(bk)^6+(dk)^6}{b^6+d^6}=\frac{(bk)^3(dk)^3}{b^3d^3}$$
$$\frac{b^6k^6+d^6k^6}{b^6+d^6}=\frac{b^3k^3d^3k^3}{b^3d^3}$$
$$\frac{k^6(b^6+d^6)}{b^6+d^6}=\frac{k^6(b^3d^3)}{b^3d^3}$$
$$k^6=k^6$$
$$L.H.S=R.H.S$$

v. $p(a+b)+qb :=(c+d)+qd = a : c$

$$\frac{p(a+b)+qb}{p(c+d)+qd}=\frac{a}{c}$$

As $a : b = c : d$

let $\frac{a}{b}=\frac{c}{d}=k$

then $a=bk$ and $c=dk$

putting these valves

$$\frac{p(bk+b)+qb}{p(dk+d)+qd}=\frac{bk}{dk}$$
$$\frac{(pkb+pb)+qb}{(pdk+pd)+qd}=\frac{bk}{dk}$$
$$\frac{b[p(k+1)+q]}{d[p(k+1)+q]}=\frac{b}{d}$$
$$\frac{b}{d}=\frac{d}{d}$$
$$L.H.S=R.H.S$$

vi. $a^2+b^2 : \frac{a^3}{a+b} = c^2+d^2 : \frac{c^3}{c+d}$

Solution:

$$\frac{a^2+b^2}{\frac{a^3}{a+b}}=\frac{c^2+d^2}{\frac{c^3}{c+d}}$$

As $a : b = c : d$

let $\frac{a}{b}=\frac{c}{d}=k$

then $a=bk$ and $c=dk$

putting these valves

$$\frac{(bk)^2+b^2}{\frac{(bk)^3}{bk+b}}=\frac{(dk)^2+d^2}{\frac{(dk)^3}{kd+d}}$$
$$\frac{b^2k^2+b^2}{\frac{b^3k^3}{bk+b}}=\frac{d^2k^2+d^2}{\frac{d^3k^3}{kd+d}}$$
$$\frac{b^2(k^2+1)}{\frac{b^3k^3}{b(k+1)}}=\frac{d^2(k^2+1)}{\frac{d^3k^3}{d(k+1)}}$$
$$\frac{b^2(k^2+1)}{\frac{b^2k^3}{(k+1)}}=\frac{d^2(k^2+1)}{\frac{d^2k^3}{k+1}}$$
$$\frac{(k^2+1)(k+1)}{k^3}=\frac{(k^2+1)(k+1)}{k^3}$$
$$L.H.S=R.H.S$$

vii. $\frac{a}{a-b} : \frac{a+b}{b} = \frac{c}{c-d} : \frac{c+d}{d}$

Solution:

$$\frac{a}{a-b} : \frac{a+b}{b} = \frac{c}{c-d} : \frac{c+d}{d}$$
$$\frac{\frac{a}{a-b}}{\frac{a+b}{b}}=\frac{\frac{c}{c-d}}{\frac{c+d}{d}}$$

As $a : b = c : d$

let $\frac{a}{b}=\frac{c}{d}=k$

then $a=bk$ and $c=dk$

putting these valves

$$\frac{a}{a-b}\times\frac{b}{a+b}=\frac{c}{c-d}\times\frac{d}{c+d}$$
$$\frac{bk}{bk-b}\times\frac{b}{bk+b}=\frac{dk}{dk-d}\times\frac{d}{dk+d}$$
$$\frac{bk}{b(k-1)}\times\frac{b}{b(k+1)}=\frac{dk}{d(k-1)}\times\frac{d}{d(k+1)}$$
$$\frac{k}{(k-1)(k+1)}=\frac{k}{(k-1)(k+1)}$$
$$L.H.S=R.H.S$$

Question No.2

if $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}(a,b,c,d,e,f,\neq 0)$ ***show that***

i. $\frac{a}{b}=\sqrt{\frac{a^2+c^2+e^2}{b^2+d^2+f^2}}$

As $a : b = c : d = e : f$

let $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=k$

then $a=bk$, $c=dk$ and $e=fk$

putting these valves

$$\frac{a}{b}=\sqrt{\frac{a^2+c^2+e^2}{b^2+d^2+f^2}}$$
$$\frac{bk}{b}=\sqrt{\frac{(bk)^2+(dk)^2+(fk)^2}{b^2+d^2+f^2}}$$
$$k=\sqrt{\frac{b^2k^2+d^2k^2+f^2k^2}{b^2+d^2+f^2}}$$
$$k=\sqrt{\frac{k^2(b^2+d^2+f^2)}{b^2+d^2+f^2}}$$
$$k=\sqrt{k^2}$$
$$k=k$$
$$L.H.S=R.H.S$$

ii. $\frac{ac+ce+ea}{bd+df+fb}=\left[\frac{ace}{bdf}\right]^{\frac{2}{3}}$

Solution:

$$\frac{ac+ce+ea}{bd+df+fb}=\left[\frac{ace}{bdf}\right]^{\frac{2}{3}}$$

As $a : b = c : d = e : f$

let $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=k$

then $a=bk$, $c=dk$ and $e=fk$

putting these valves

$$\frac{bkdk+dkfk+fkbk}{bd+df+fb}=\left[\frac{bkdkfk}{bdf}\right]^{\frac{2}{3}}$$
$$\frac{k^2bd+k^2df+k^2fb}{bd+df+fb}=\left[\frac{k^3bdf}{bdf}\right]^{\frac{2}{3}}$$
$$\frac{k^2(bd+df+fb)}{bd+df+fb}=[k^3]^{\left(\frac{2}{3}\right)}$$
$$k^2=k^2$$
$$L.H.S=R.H.S$$

iii. $\frac{ac}{bd}+\frac{ce}{df}+\frac{ea}{fb}=\frac{a^2}{b^2}+\frac{c^2}{d^2}+\frac{e^2}{f^2}$

Solution:

As $a : b = c : d = e : f$

let $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=k$

then $a=bk$, $c=dk$ and $e=fk$

putting these valves

$$\frac{ac}{bd}+\frac{ce}{df}+\frac{ea}{fb}=\frac{a^2}{b^2}+\frac{c^2}{d^2}+\frac{e^2}{f^2}$$
$$\frac{bkdk}{bd}+\frac{dkfk}{df}+\frac{fkbk}{fb}=\frac{b^2k^2}{b^2}+\frac{d^2k^2}{d^2}+\frac{f^2k^2}{f^2}$$
$$\frac{k^2bd}{bd}+\frac{k^2df}{df}+\frac{k^2fb}{fb}=k^2+k^2+k^2$$
$$k^2+k^2+k^2=k^2+k^2+k^2$$

Exercise 3.7

Question No1. The surface area A of a cube varies directly as the square of the length l of an edge and A=27 square units when $l=3units$

Find $(i) A$ when $l=4units$

(ii) l when $A=12\ sq.units$

Solution:

$$A\propto l^2$$
$$A=kl^2\rightarrow(i)$$

When $A=27\ square\ units, l=3\ units$

$$27=k(3)^2$$
$$27=k(9)$$
$$\frac{27}{9}=k$$
$$3=k$$
$$or\quad k=3$$

So equation (i) becomes

i. $when\ l=4\ units$

$$A=3(4)^2$$
$$A=3(16)$$
$$A=48sq.units$$

ii. $when\ A=12\ sq.units$

$$12=3l^2$$
$$\frac{12}{3}=l^2$$
$$4=l^2$$
$$l=2\quad taking\ squaring\ root\ on\ both\ sides$$

Question No.2

The surface area

S of a surface varies dircetly as the square of radius r, and $S=16\pi$ when r=2.

Find r when $S=36\pi$

Solution:

$$S\propto r^2$$
$$S=kr^2\rightarrow(i)$$
$$S=16\pi\ when\ r=2$$

$$16\pi = k(2)^2$$
$$16\pi = 4k$$
$$\frac{16\pi}{4} = k$$
$$4\pi = k$$
$$or \quad k = 4\pi$$

So, equation (i) becomes

$$S = 4\pi r^2$$

When $S = 36\pi\ units$

$$36\pi = 4\pi r^2$$
$$r^2 = 9$$
$$r = 3$$

Question No.3 in Hook's law the force F applied to stretch a spring varies directly as the amount of elongation S and $F = 32\ lb\ when\ S = 1.6\ inch. find\ (i)\ \ when\ F = 50lb$

Solution:

$$F \propto S$$
$$F = kS \rightarrow (i)$$
$$F = 32lb\ when\ S = 1.6$$
$$(32) = k(1.6)$$
$$k = \frac{32}{1.6}$$
$$k = \frac{32}{16} \times 10$$
$$k = 20$$

So equation (i) becomes

$$F = 20S$$

i. $when\ F = 50\ lb$

$$50lb = 20S$$
$$S = 2.5\ in$$

ii. $when\ S = 0.8\ in$

$$F = 20(0.8)$$
$$F = 16\ lb$$

Question No.4 The intensity I of light from a given source varies inversely as the square of the distance d from it. If the intensity is 20 candlepower at a distance of 12ft. from the source, find the intensity at a point 8ft. from the source.

Solution:

$$I \propto \frac{1}{d^2}$$
$$I = \frac{k}{d^2} \rightarrow (i)$$
$$I = 20\ when\ d = 12ft$$
$$(20) = \frac{k}{(12)^2}$$
$$20 = \frac{k}{144}$$
$$k = 20 \times 144$$
$$k = 2880$$

So equation $(i) becomes$

$$I = \frac{2880}{d^2}$$

When $d = 8ft$

$$I = \frac{2880}{8^2}$$
$$I = \frac{2880}{64}$$
$$I = 45cp$$

Question No.5 The pressure p in a body of fluid varies directly as the depth d. if the pressure exerted on the bottom of a tank by a column of fluid 5ft. high is 2.25 $lb\backslash sq.in$. In how deep must fluid be to exert a pressure of 9 $lb\backslash sq.in$?

Solution:

$$P \propto d$$
$$p = kd \rightarrow (i)$$
$$d = 5ft\ when\ P = 2.25lb\backslash sq.in$$
$$2.25 = k(5)$$
$$k = \frac{2.25}{5}$$
$$k = \frac{225}{5 \times 100}$$
$$k = \frac{45}{100}$$
$$k = 0.45$$

So equation (i)

$$P = o.45d$$
$$when\ P = 9\ lb\backslash sq.in$$
$$9 = 0.45d$$
$$\frac{9}{0.45} = d$$
$$\frac{9 \times 100}{45} = d$$
$$\frac{900}{45} = d$$
$$20 = d$$

Question No.6 $Labour$ costs c varies jointly as the number of workers n and the average number of days d, if the cost of 800 workers for 13 days is Rs. 286000, then find the $labour$ cost of 600 workers for 18days.

Solution:

$$\boldsymbol{c} \propto nd$$
$$\boldsymbol{c} = knd \rightarrow (i)$$
$$c = Rs.286000, n = 800, d = 13$$
$$286000 = k(800)(13)$$
$$k = \frac{286000}{10400}$$
$$k = 27.5$$

So equation (i) becomes

$$c = 27.5$$

When $n = 600, d = 18$

$$c = 27.5(600)(18)$$
$$c = 297000$$

Question No. 7 The supporting load c of a pillar varies as the fourth power of its diameter d and inversely as the square of its length l. A pillar

of diameter 6 *inch and of height* 30 **feet will support a load of 63 tons. How high a 4 inch pillar must be to support a load of 28 tons.?**

Solution:

$$c \propto \frac{d^4}{l^2}$$

$$c = \frac{kd^2}{l^2} \to (\boldsymbol{i})$$

$$c = 63\ tons, d = 6inch\ , I = 30feet$$

$$63 = \frac{k(6)^4}{(\mathbf{30})^2}$$

$$k = 43.75$$

So equation (i) becomes

$$c = \frac{43.75d^2}{l^2}$$

When $d = 4inch\ , c = 28\ tons$

$$28 = \frac{43.75(4)^4}{l^2}$$

$$l^2 = \frac{43.75(4)^4}{28}$$

$$l^2 = 400$$

$$l^2 = 20feet$$

Question No.8

The times T required for an elevator to lift a weight varies jointly as the weight w and the lifting depth d varies inverlsy as the power P of the motor. If 25 sec. are required for a 4-hp motor to lift 500lb through 40ft, what power is required to lift 800lb. through 120 ft in 40 sec?

Solution:

$T \propto \frac{wd}{p}$

$$T = \frac{kwd}{p} \to (i)$$

$$T = 25sec, p = 4 - hp, w = 500lb, \quad d = 40ft$$

$$25 = \frac{k(500)(40)}{4}$$

$$K = \frac{25 \times 4}{500 \times 40}$$

So equation (i) becomes

$$T = \frac{0.005wd}{p}$$

When $w = 800lb, d = 120\ ft\ , T = 40\ sec.$

$$40 = \frac{0.005 \times 800 \times 120}{P}$$

$$P = \frac{0.005 \times 800 \times 120}{40}$$

$$p = 12hp$$

Question No.9

The kinetic energy $(K.E)$of a body varies jointly as the mass "m" of the body and the square of its velocity "V" if the energy is 4320ft/lb when the mass is 45lb and the velocity is 24ft/sec. Determine the kinetic energy of a 3000 lb automobile travelling 44ft/sec.

Solution:

$$K.E \propto mv^2$$

$$K.E = kmv^2 \to (i)$$

$K.E = 4320ft$/lb, m=45 lb, v=24ft/sec.

$$4320 = k(45)(24)^2$$

$$k = \frac{4320}{45 \times 476}$$

$$k = \frac{4320}{25920}$$

$$k = \frac{1}{6}$$

So equation (i) becomes

$$K.E = \frac{mv^2}{6}$$

When $m = 3000lb, v = 44\ ft\ /\ sec.$

$$K.E = \frac{(3000)(44)^2}{6}$$

$$K.E = 500 \times 1936$$

$$K.E = 96800\ ft/\ \text{lb}$$