10TH CLASS MATH

CHAPTER 2

SOLUTION NOTES

Exercise 2.1

1. Find the discriminant of the following given quadratic equation.

(i) $2x^2+3x-1$

Solution:

$$2x^2+3-1=0$$
$$compare\ it\ with$$
$$ax^2+bx+c=0$$
$$\Rightarrow a=2,\ b=3\ ,c=-1$$
$$Disc.=b^2-4ac$$
$$=(3)^2-4(2)(-1)$$
$$=9+8$$
$$=17$$

(ii) $6x^3-8x+3=0$

Solution:

$$6x^3-8x+3=0$$
$$Compare\ it\ with$$
$$ax^2+bx+c=0$$
$$\Rightarrow a=6,\ b=-8\ c=3$$
$$Disc.=b^2-4ac$$
$$=(-8)^2-4(6)(3)$$
$$=64-72$$
$$=-8$$

(iii) $9x^2-30x+25=0$

Solution:

$$9x^2-30x+25=0$$
$$Compare\ it\ with$$
$$ax^2+bx+c=0$$
$$\Rightarrow a=9,b=-30,c=25$$
$$Disc.=b^2-4ac$$
$$=(-30)^2-4(9)(25)$$
$$=900-900$$
$$=0$$

(iv) $4x^2-7x-2=0$

Solution:

$$4x^2-7x-2=0$$
$$Compare\ it\ with$$
$$ax^2+bx+c=0$$
$$\Rightarrow a=4,b=-7,c=-2$$
$$Disc.=b^2-4ac$$
$$=(-7)^2-4(4)(-2)$$
$$=49+32$$
$$=81$$

2. Find the nature of the roots of the follow given quadratic and verify the result by solving equations:

(i) $x^2+23x+120=0$

Solution:

$$x^2+23x+120=0$$
$$Compare\ it\ with$$
$$ax^2+bx+c=0$$
$$\Rightarrow a=1,b=-23,c=120$$
$$Disc.=b^2-4ac$$
$$=(-23)^2-4(1)(120)$$
$$=529-480$$
$$=49$$
$$=(7)^2>0$$

As the disc.is possible and is perfect square. Therefore the roots are rational (real) and unequal, verification by solving the equation.

$$x^2-23x+120=0$$
$$x^2-15x-8x+120=0$$
$$x(x-15)-8(x-15)=0$$
$$(x-15)(x-8)=0$$
$$Either\quad x-8=0\quad or\ x-15=0$$
$$x=8\qquad x=15$$

Thus, the roots are rational (real) and unequal.

(ii) $\mathbf{2x^2+3x+7=0}$

Solution:

$$2x^2+3x+7=0$$

Compare it with

⇨ $a=2, b=3, c=7$

$$Disc.=b^2-4ac$$
$$=(3)^2-4(2)(7)$$
$$=9-56$$
$$=-47<0$$

As the Disc. is negative.

Therefore the roots are imaginary and unequal.

Verification by solving the equation.

$$2x^2+3x+7=0$$

Using quadratic formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-3\pm\sqrt{(3)^2-4(2)(7)}}{2a}$$
$$=\frac{-3\pm\sqrt{9-56}}{4}$$

Thus, the roots are imaginary and unequal

(iii) $\mathbf{16x^2-24x+9=0}$

Solution:

$$16x^2-24x+9=0$$

Compare it with

$$ax^2+bx+c=0$$

⇨ $a=16, b=-24, c=9$

$$Disc.=b^2-4ac$$
$$=(-24)^2-4(16)(9)$$
$$=576-576$$
$$=0$$

As the Disc. is zero

Therefore the roots of the equation are real and equal.

Verification by solving the equations.

$$16x^2-24x+9=0$$

using quadratic formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-(-24)\pm\sqrt{(-24)^2-4(16)(9)}}{2(16)}$$
$$=\frac{24\pm\sqrt{576-576}}{32}$$
$$=\frac{24\pm\sqrt{0}}{32}$$
$$=\frac{24}{32}=\frac{3}{4}$$

Thus the roots are real and unequal.

iv) $\mathbf{3x^2+7x-13=0}$

Solution:

$$3x^2+7x-13=0$$

Compare it with

$$ax^2+bx+c=0$$

⇨ $a=3,\ b=7,\ c=-13$

$$Disc.=b^2-4ac$$
$$=(7)^2-4(3)(-13)$$
$$=49+156$$
$$=205>0$$

As the Disc. Is positive and is not perfect square.

Therefore the roots are irrational (real) and unequal.

Verification by solving the equation.

$$3x^2+7x-13=0$$

Using quadratic formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-7\pm\sqrt{(7)^2-4(3)(-13)}}{2a}$$
$$=\frac{-7\pm\sqrt{49+156}}{6}$$
$$=\frac{-7\pm\sqrt{205}}{6}$$

Thus, the roots are irrational (real) and unequal.

3. For what value of A, the expression

$$\boldsymbol{k^2x^2+2(k+1)x+4\ is\ square.}$$

Solution:

$$k^2x^2+2(k+1)x+4=0$$

Compare it with

$$ax^2+bx+c=0$$

⇨ $a=k^2,\ \ b=2(k-1), c=4$

$$Disc.=b^2-4ac$$
$$=\big(2(k-1)\big)^2-4(k^2)(4)$$
$$=4(k^2-2k+1)-16k^2$$
$$=4k^2-8k+4-16k^2$$
$$=-12k^2-8k+4=0$$

As the disc. Of the given expression is a perfect square. Therefore the roots are rational and equal.

So, $Disc.=0$

$$-12k^2+8k+4=0$$
$$-(12k^2+8k+4)=0$$
$$⇨\ 12k^2-8k-4=0$$
$$12k^2-12k+4k-4=0$$
$$12k(k-1)+4(k-1)=0$$
$$(12k+4k)(k-1)=0$$

$Either\quad 12k+4=0 \quad or\ k-1=0$

$12k=-4 \quad or\quad k=1$

$$k=-\frac{4}{12}$$
$$k=-\frac{1}{3}$$

5. Find the value of k, if the roots of the following equations are equal.

(i) $\mathbf{(2k+1)x^2+3Kx+3=0}$

⇨ $a=2k+1, b=3k,\ c=3$

As the roots are equal, So

$$Disc.=0$$
$$b^2-4ac=0$$
$$(3k^2)-4(2k+1)(3)=0$$

$$9k^2-12(2k+1)=0$$
$$9k^2-24k-12=0$$
$$3(3k^2-8k-4)=0$$
$$\Rightarrow 3k^2-8k-4=0$$
$$3k^2-6k-2k+4=0$$
$$3k(k-2)-2(k-2)=0$$
$$(3k-2)(k-2)=0$$
$$Either\ \ 3k-2=0 \quad or \quad k-2=0$$
$$3k=2 \quad or\ k=2$$
$$k=\frac{2}{3} \quad or \quad k=2$$

(ii) $\boldsymbol{x^3+2(k+2)x+(3k+4)=0}$

Solution:

$$x^3+2(k+2)x+(3k+4)=0$$

$\Rightarrow a=1\ \ b=2(k+2)\ \ c=3k+4$

As the roots are equal

$$Disc.=0$$
$$b^2-4ac=0$$
$$[2(k+2)]^2-4(1)(3k+4)=0$$
$$4(k+2)^2-4(3k+4)=0$$
$$4(k^2+4k+4)-12k-16=0$$
$$4k^2+4k+4-12k-16=0$$
$$4k^2+4k=0$$
$$4k(k+1)=0$$
$$Either \quad 4k=0 \quad (k+1)=0$$
$$k=0 \quad or \quad k=-1$$

(iii) $\boldsymbol{(3k+2)x^2-5(k+1)x+(2k+3)=0}$

Solution:

$$(3k+2)x^2-5(k+1)x+(2k+3)=0$$

$\Rightarrow a=3k+2, \quad b=-5(k+1), \quad c=(2k+3)$

$$As\ the\ roots\ are\ equal, So$$
$$Disc.=0$$
$$b^2-4ac=0$$
$$[-5(k+1)]^2-4(3k+2)(2k+3)=0$$
$$25(k^2+2k+1)-4(6k^2+13k+6)=0$$
$$25k^2+50k+25-24k^2-52k-24=0$$
$$k^2-2k+1=0$$
$$(k-1)^2=0$$

6. Show that the equation $\boldsymbol{x^2+(mx+c)^2=a^2}$ *has*

Equal roots,

$$\boldsymbol{if\ c^2=a62\ (1+m^2)}$$

Solution:

$$x^2+(mx+c)^2=a^2$$
$$x^2+m^2x^2+2mcx+c^2=a^2$$
$$(1+m^2)x62+2mcx+c^2-a^2=0$$
$$a=1+m^2, \quad b=2mc, \quad c=c^2=a^2$$
$$As\ the\ roots\ are\ equal, So$$
$$Disc.=0$$
$$b^2-4ac=0$$
$$(2mc)^2-4(1+m^2)(c^2-a^2)=0$$
$$4m^2c^2-4(c^2-a^2+m^2c^2-a^2m^2)=0$$
$$4m^2c^2-4c^2+4a^2-4m^2c^2+4a^2m^2=0$$
$$-4c^2+4a^2+4ac^2+4a^2m^2=0$$
$$-4(c^2-a^2-a^2m^2)=0$$
$$c^2-a^2-a62\ m^2=0$$
$$c^2=a^2+a^2m^2$$
$$c^2=a^2(a+m^2)$$

Hence proved.

7. Find the condition that the roots of the equation

$\boldsymbol{(my+c)^2-4ax=0\ are\ equal}.$

Solution:

$(my+c)^2-4ax=0$

$m^2x^2+2mcx+c^2-4ax=0$

$m^2x^2+2mcx-4ax+c^2=0$

$m^2x^2+2(mc-2a)x+c^2=0$

$\Rightarrow a=m^2, \ b=2(mc-2a), \ c=c^2$

$As\ the\ roots\ are\ equal \quad Disc.=0$

$b^2-4ac=0$

$[2(mc-2a)]^2-4(m^2)(c^2)=0$

$4(mc-2a)^2-4m^2c^2=0$

$4(m^2c^2-4amc+4a^2)-4m^2c^2=0$

$4(\ m^2c^2-amc+a^2-m^2c^2)=0$

$\Rightarrow 4a^2-4amc=0$

$\Rightarrow$ 4a(a-mc)=0

$\Rightarrow a-mc=0$

$\Rightarrow a=mc$

Which is required condition.

8. If the roots of the equation

$\boldsymbol{(c^2-ab)x^2-2(a^2-bc)x+(b^3-ac)=0}$ ***are equal***

$\boldsymbol{,then\ a=0\ or\ a^3+b^3+c^3=3abc}$

Solution:

$(c^2-ab)x^2-2(a^2-bc)x+(b^3-ac)=0$

$\Rightarrow a=c^3-ab, b=-2(a^2-bc), c=b^2-ac$

As the roots are equal so

$$Disc.=0$$
$$b^2-4ac=0$$
$$[-2(a^2-bc)]^2-4(c^2-ab)(b^2-ac)=0$$
$$4(a^2-bc)^2-4(c^2-ab)(b^2-ac)=0$$
$$4[a^4-2a^2bc+b^2c^2)-(b^2c^2-ac^3+ab^3+a^2bc)]=0$$
$$a^4-2a^2bc+b^2c^2-b^2c^2+ac^3+ab^3+a^2bc=0$$

$\Rightarrow a^4+ab^3+ac^3-3a^2bc=0$

$$=c^2-2ac+a^2-ab+4b^2+4ac-4bc$$
$$k-1=0\Rightarrow k=1$$
$$a(a^3+b^3+c^3-3abc)=0$$

Either $a=0 \quad or \quad a^3+b^3+c^3-3abc=0$

$Hence\ proved \quad a^3+b^3+c^3-3abc$

8. Show that the roots of the following equations are rational.

(i) $\boldsymbol{a(b-c)x^2+b(c-a)x+c(a-b)}=0$

Solution:

$\Rightarrow a=a(b-c), \ b=b(c-a), \ \ c=c(a-b)$

$$Disc.=b^2-4ac$$
$$=[b(c-a)]^2-4[a(b-c)][c(a-b)]$$
$$=b^2(c-a)^2-4ac(b-c)(a-b)$$
$$=b^2(c^2-2ac+a^2)-4ac(ab-b^2-ac+bc)$$

$$= b^2c^2 - 2ab^2a^2 - 4a^2cb + 4acb^2 + 4a^2c^2 - 4abc^2$$
$$= a^2b^2 + b^2c^2 + 44a^2c^2 + 2ab^2c - 4a^2bc - 4abc^2$$
$$= (ab)^2 + (bc)62 + (-2ac)^2 + 2(ab)(bc) + 2(bc)(-2ac) + 2(-2ac)(ab)$$
$$= (ab + bc - 2ac)^2$$

Hence the roots are rational.

(ii) $\boldsymbol{(a+2b)x^2 + 2(a+b+c)x + (a+2c) = 0}$

Solution:

⇨ $a = (a+2b),\ \ b = 2(a+b+c)\ ,\ \ c = (a+2c)$

$$Disc. = b^2 - 4ac$$
$$= [2(a+b+c)]^2 - 4(a+2b)(a+2c)$$
$$= 4(a+b+c)^2 - 4(a^2 + 2ac + 2ab + 4bc)$$
$$= 4[a^2 + b^2 + c^2 + 2ab + 2bc + 2ca - a^2 - 2ac - 4bc]$$
$$= 4(b-c)^2$$

Hence the roots are rational.

9. For all valves of k, prove that the roots of the equation.

$$x^2 - 2\left(k + \frac{1}{k}\right)x + 3 = 0$$

Solution:

⇨ $a = 1,\ \ b = -2\left(k + \frac{1}{k}\right),\ \ c = 3$

$$Disc. = b^2 - 4ac$$
$$= \left[-2\left(k + \frac{1}{k}\right)\right]^2 - 4(1)(3)$$
$$= 4\left(k + \frac{1}{k}\right)^2 - 12$$
$$= 4\left[\left(k + \frac{1}{k}\right)^2 - 3\right]$$
$$= \left[k^2 + \frac{1}{k^2} + 2 - 3\right]$$
$$= \left[k^2 + \frac{1}{k^2} - 1\right] > 0$$

Hence the roots are real.

10. Show that the roots of the equation.

$$\boldsymbol{(b-c)x^2 + (c-a)x + (a-b)^2 = 0}$$

Solution:

⇨ $a = (b-c),\ \ b = (c-a),\ \ c = (a-b)$

$$Disc. = b^2 - 4ac$$
$$= (c-a)^2 - 4(b-c)(a-b)$$
$$= (c^2 - 2ac + a^2) - 4(ab - b^2 - ac + bc)$$
$$= a^2 + 4b^2 + c^2 - 4ab - 4bc + 2ac$$
$$= (a)^2 + (-2b)^2 + (c)^2 + 2(a)(-2b) + 2(-2b)(c) + 2(a)(c)$$
$$= (a - 2b + c)^2 > 0$$

hence the roots of the equation are real.

2.2 Cube Roots Of Unity And Their Properties

2.2.1 The cube roots of unity:

let a number x be the cube root of unity.

$$i.e \quad x = (1)^{\frac{1}{3}}$$
$$or\ \ x^3 = 1$$
$$x^3 - 1 = 0$$
$$(x^3) - (1)^3 = 0$$
$$(x-1)(x^2 + x + 1) = 0$$
$$[using\ a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$
$$Either\ x - 1 = 0 \quad or\ x^2 + x + 1 = 0$$

⇨ $x = 1 \quad or \quad x = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(1)}}{2(1)}$

⇨ $= \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm \sqrt{-3}}{2} = \frac{-1 \pm i\sqrt{3}}{2}$

∵ three cube roots of unity are

$$1, \quad \frac{-1 + i\sqrt{3}}{2}, \quad \frac{-1 - i\sqrt{3}}{2}$$

2.2.2 Recognize complex cube roots of unity as $\boldsymbol{\omega}$ and $\boldsymbol{\omega^2}$

$\boldsymbol{\omega\ pronoun\ as\ omega}$

2.2.3 Properties of cube roots of unity

(a) Prove that each of the complex cube roots of unity is the square of the other.

Proof:

The complex cube roots of unity are

$$\frac{-1 + i\sqrt{3}}{2}, \quad \frac{-1 - i\sqrt{3}}{2}$$

We prove that

$$\left(\frac{-1 + i\sqrt{3}}{2}\right)^2 = \frac{-1 - \sqrt{3}}{2}$$

$$\left(\frac{-1 + i\sqrt{3}}{2}\right)^2 = \frac{1 + (-3) - 2\sqrt{-3}}{4}$$
$$= \frac{-2 - 2\sqrt{3}}{4}$$
$$= \frac{2(-1 - \sqrt{3})}{4}$$
$$= \frac{-1 - \sqrt{3}}{2}$$

And

The complex cube roots of unity are

$$\frac{-1 + i\sqrt{3}}{2}, \quad \frac{-1 - i\sqrt{3}}{2}$$

We prove that

$$\left(\frac{-1 - i\sqrt{3}}{2}\right)^2 = \frac{-1 + \sqrt{3}}{2}$$

$$\left(\frac{-1 - i\sqrt{3}}{2}\right)^2 = \frac{1 + (-3) + 2\sqrt{-3}}{4}$$
$$= \frac{-2 + 2\sqrt{3}}{4}$$
$$= \frac{2(-1 + \sqrt{3})}{4}$$

$$= \frac{-1+\sqrt{3}}{2}$$

Thus , each of the complex cube root of unity is square of the other, that is

$$if\ \omega = \frac{-1+\sqrt{3}}{2}\ then\ \omega^2 = \frac{-1-\sqrt{3}}{2}$$

And if $\omega = \frac{-1-\sqrt{3}}{2}\ then\ \omega^2 = \frac{-1+\sqrt{3}}{2}$

(b) Prove that the product of three cube roots of unity is one.

Proof:

Three cube roots of unity are

$$1,\quad \frac{-1+\sqrt{3}}{2},\ \frac{-1-\sqrt{3}}{2}$$

The product of cube roots of unity

$$1\times\ \frac{-1+\sqrt{3}}{2},\times\ \frac{-1-\sqrt{3}}{2}$$

$$= \frac{(-1)^2-(\sqrt{-3})^2}{4} = \frac{1-(-3)}{4} = \frac{1+3}{4} = \frac{4}{4} = 1$$

$$i.e\ (1)(\omega)(\omega^2) = 11\quad or\ w^3 = 1$$

$$\omega^4 = \omega^3.\omega = 1.\omega = \omega$$

(c) Prove that each complex cube root of unity is reciprocal of the other.

Proof:

We know that $w^3 = 1 \Rightarrow \omega\omega^2 = 1\ so.$

$$\omega = \frac{1}{\omega^2}\quad or\ \omega^2 = \frac{1}{\omega}$$

(d) Prove that the sum of all the cube roots of unity is zero.

$$i.e\ 1+\omega+\omega^2 = 0$$

Proof:

Three cube roots of unity are

$$1,\quad \frac{-1+\sqrt{3}}{2},\ \frac{-1-\sqrt{3}}{2}$$

The product of cube roots of unity

$$= 1 + \frac{-1+\sqrt{3}}{2} + \frac{-1-\sqrt{3}}{2}$$

$$= \frac{2-1+i\sqrt{3}-1-\sqrt{3}}{2}$$

$$= \frac{0}{2} = 0$$

$$1+\omega+\omega^2 = 0$$

(i) $1+\omega^2 = -\omega$

(ii) $1+\omega = -\omega^2$

(iii) $\omega+\omega^2 = -1$

Exercise 2.2

Q.1 Find the cube root of $-1, 8, -27, 64.$

(i) Cube roots of -1

Solution : Let $x = (-1)^{\frac{1}{3}}$

$x^3 = -1$

$x^3 + 1 = 0$

$x^3 + (1)^3 = 0$

$\because (a^3+b^3) = (a+b)(a^2-ab+b^2)$

$(x+1)(x^2-(x)(1)+1^2) = 0$

$(x+1)(x^2-x+1) = 0$

$x+1 = 0\qquad (x^2-x+1) = 0$

$x = -1\qquad x^2-x+1 = 0$

Then we solve $x^2-x+1=0$ *by formula*

$ax^2+bx+c = 0$

$a=1,\ b=-1,\ c=1$

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-(-1)\pm\sqrt{(-1)^2-4(1)(1)}}{2\times1}$$

$$x = \frac{1\pm\sqrt{1-4}}{2}$$

$$x = \frac{1\pm\sqrt{-3}}{2}$$

Cube roots of -1

$$1, \frac{1-\sqrt{-3}}{2}\qquad 1, \frac{1+\sqrt{-3}}{2}$$

$$1, -\left(\frac{-1+\sqrt{-3}}{2}\right)\qquad -1\left(\frac{-1-\sqrt{-3}}{2}\right)$$

$1, -\omega\qquad 1, -\omega^2$

$x = -1\omega\qquad x = -1(\omega)^2$

$x = -\omega\qquad x = -\omega^2$

Cube roots of -1 *are* $-1, -\omega, -\omega^2$

(ii) Cube roots of 8

Solution: Let $x=(8)^{\frac{1}{3}}$

$x^3=8$

$x^3-8=0$

$x^3-2^3=0$

$\because (a^3-b^3)=(a-b)(a^2+ab+b^2)$

$(x-2)(x^2+(x)(2)+2^2)=0$

$(x-2)(x^2+2x+2^2)=0$

$(x-2)(x^2+2x+4)=0$

$x-2=0 \qquad x^2+2x+4=0$

$x=2 \qquad x^2+2x+4=0$

Then we solve $x^2+2x+4=0$ by formula

$a=1,\ b=2,\ c=4$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-2\pm\sqrt{(2)^2-4(1)(4)}}{2(1)}$$

$$x=\frac{-2\pm\sqrt{4-16}}{2}$$

$$x=\frac{-2\pm\sqrt{-12}}{2}$$

$$x=\frac{-2\pm\sqrt{4\times(-3)}}{2}$$

$$x=\frac{-2\pm\sqrt{4}\sqrt{-3}}{2}$$

$$x=\frac{-2\pm2\sqrt{-3}}{2}$$

$$x=\frac{2(-1\pm\sqrt{-3})}{2}$$

$$x=2\left(\frac{-1\pm\sqrt{-3}}{2}\right)$$

$$x=2\left(\frac{-1+\sqrt{-3}}{2}\right) \qquad x=2\left(\frac{-1-\sqrt{-3}}{2}\right)$$

$x=2\omega \qquad x=2\omega^2$

Cube roots of 8 are $2, 2\omega, 2\omega^2$

(iii) Cube roots of -27

Solution: Let $x=(-27)^{\frac{1}{3}}$

$x^3=-27$

$x^3+27=0$

$x^3+3^3=0$

$\because (a^3+b^3)=(a+b)(a^2-ab+b^2)$

$(x+3)(x^2-(x)(3)+3^2)$

$(x+3)(x^2-3x+9)=0$

$x+3=0 \qquad x^2-3x+9=0$

$x=-3 \qquad x^2-3x+9=0$

Then we solve $x^2-3x+9=0$ by formula

$a=1,\ b=-3,\ c=9$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-3)\pm\sqrt{(-3)^2-4(1)(9)}}{2(1)}$$

$$x=\frac{3\pm\sqrt{9-36}}{2}$$

$$x=\frac{3\pm\sqrt{-27}}{2}$$

$$x=\frac{3\pm\sqrt{9\times(-3)}}{2}$$

$$x=\frac{3\pm\sqrt{9}\sqrt{-3}}{2}$$

$$x=\frac{3\pm3\sqrt{-3}}{2}$$

$$x=\frac{-3(-1\pm\sqrt{-3})}{2}$$

$$x=-3\left(\frac{-1\pm\sqrt{-3}}{2}\right)$$

$$x=-3\left(\frac{-1+\sqrt{-3}}{2}\right) \qquad x=-3\left(\frac{-1-\sqrt{-3}}{2}\right)$$

$x=-3\omega \qquad x=-3\omega^2$

Cube roots of -27 are $-3, -3\omega, -3\omega^2$

(ii) Cube roots of 64

Solution: Let $x=(64)^{\frac{1}{3}}$

$x^3=64$

$x^3-64=0$

$x^3-4^3=0$

$\because (a^3-b^3)=(a-b)(a^2+ab+b^2)$

$(x-4)(x^2+(x)(4)+4^2)=0$

$(x-4)(x^2+4x+16)=0$

$x-4=0 \qquad x^2+4x+16=0$

$x=4 \qquad x^2+4x+16=0$

Then we solve $x^2+4x+16=0$ by formula

$a=1,\ b=4,\ c=16$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-4\pm\sqrt{(4)^2-4(1)(16)}}{2(1)}$$

$$x=\frac{-4\pm\sqrt{16-64}}{2}$$

$$x=\frac{-4\pm\sqrt{-48}}{2}$$

$$x=\frac{-4\pm\sqrt{16\times(-3)}}{2}$$

$$x=\frac{-4\pm4\sqrt{-3}}{2}$$

$$x=\frac{4(-1\pm\sqrt{-3})}{2}$$

$$x=4\left(\frac{-1\pm\sqrt{-3}}{2}\right)$$

$$x=4\left(\frac{-1+\sqrt{-3}}{2}\right) \qquad x=4\left(\frac{-1-\sqrt{-3}}{2}\right)$$

$x=4\omega \qquad x=4\omega^2$

Cube roots of 64 are $4, 4\omega, 4\omega^2$

Q.2 Evaluate

(i) $\left(1-\omega-\omega^2\right)^7$

Solution: $\left(1-\omega-\omega^2\right)^7$

$=\left[1-\left(\omega-\omega^2\right)\right]^7$

$=\left[1-(-1)\right]^7$

$=(1+1)^7$

$=2^7=128$

(ii) $\left(1-3\omega-3\omega^2\right)^5$

Solution: $\left(1-3\omega-3\omega^2\right)^5$

$=\left[1-3\left(\omega+\omega^2\right)\right]^5$

$=\left[1-3(-1)\right]^5$

$=(1+3)^5$

$=4^5=1025$

(iii) $\left(9+4\omega+4\omega^2\right)^3$

Solution: $\left(9+4\omega+4\omega^2\right)^3$

$=\left[9+4\left(\omega+\omega^2\right)\right]^3$

$=\left[9+4(-1)\right]^3 \quad \left(\because \omega+\omega^2=-1\right)$

$=(9-4)^3$

$=5^3=125$

(iv) $\left(2+2\omega+2\omega^2\right)\left(3-3\omega+3\omega^2\right)$

Solution: $\left(2+2\omega+2\omega^2\right)\left(3-3\omega+3\omega^2\right)$

$=\left(2(1+\omega)-2\omega^2\right)\left(3+3\omega^2-3\omega\right)$

$=\left[2(1+\omega)-2\omega^2\right]\left[3\left(1+\omega^2\right)-3\omega\right]$

$\left\{\because\ 1+\omega+\omega^2=0\right\}$

$\left\{1+\omega=-\omega^2 \quad 1+\omega^2=-\omega\right\}$

$=\left[2(-\omega)^2-2\omega^2\right]\left[3(-\omega)-3\omega\right]$

$=\left(-2\omega^2-2\omega^2\right)(-3\omega-3\omega)$

$=\left(-4\omega^2\right)(-6\omega)$

$=24\omega^3=24(1)=24$

(v) $\left(-1+\sqrt{-3}\right)^6+\left(-1-\sqrt{-3}\right)^6$

Solution: $\left(-1+\sqrt{-3}\right)^6+\left(-1-\sqrt{-3}\right)^6$

As $\dfrac{-1+\sqrt{-3}}{2}=\omega \qquad \dfrac{-1-\sqrt{-3}}{2}=\omega^2$

$-1+\sqrt{-3}=2\omega \qquad -1-\sqrt{-3}=2\omega^2$

Then it becomes

$=(2\omega)^6+\left(2\omega^2\right)^6$

$=2^6\omega^6+2^6\omega^{12}$

$=2^6\left[\left(\omega^3\right)^2+\left(\omega^3\right)^4\right]$

$=2^6\left[(1)^2+(1)^4\right]$

$=64(1+1)$

$=64(2)=128$

(vi) $\left(\dfrac{-1+\sqrt{-3}}{2}\right)^9+\left(\dfrac{-1-\sqrt{-3}}{2}\right)^9$

Solution: $\left(\dfrac{-1+\sqrt{-3}}{2}\right)^9+\left(\dfrac{-1-\sqrt{-3}}{2}\right)^9$

As $\dfrac{-1+\sqrt{-3}}{2}=\omega \qquad \dfrac{-1-\sqrt{-3}}{2}=\omega^2$

Then it becomes

$$=(\omega)^9+(\omega^2)^9$$
$$=\omega^9+\omega^{18}$$
$$=(\omega^3)^3+(\omega^3)^6$$
$$=(1)^3+(1)^6$$

(vii) $\omega^{37}+\omega^{38}-5$

Solution: $\omega^{37}+\omega^{38}-5$

$$=\omega^{36}\omega+\omega^{36}\omega^2-5$$
$$=(\omega^3)^{12}\omega+(\omega^3)^{12}\omega^2-5$$
$$=(1)^{12}\omega+(1)^{12}\omega^2-5$$
$$=1\omega+1\omega^2-5$$
$$=(\omega+\omega^2)-5$$
$$=(-1)-5$$
$$=-1-5=-6$$

(viii) $\omega^{-13}+\omega^{-17}$

Solution: $\omega^{-13}+\omega^{-17}$

$$=\frac{1}{\omega^{13}}+\frac{1}{\omega^{17}}$$
$$=\frac{1}{\omega^{12}\omega}+\frac{1}{\omega^{15}\omega^2}$$
$$=\frac{1}{(\omega^3)^4\omega}+\frac{1}{(\omega^3)^5\omega^2}$$
$$=\frac{1}{(1)^4\omega}+\frac{1}{(1)^5\omega^2}$$
$$=\frac{1}{\omega}+\frac{1}{\omega^2}$$
$$=\frac{\omega^2+\omega}{(\omega)(\omega^2)}=\frac{-1}{\omega^3}$$
$$=-\frac{1}{1}=-1$$

Q.3 Prove that

$$x^3+y^3=(x+y)(x+\omega y)(x+\omega^2 y)$$

Solution: Let,

$$R.H.S=(x+y)(x+\omega y)(x+\omega^2 y)$$
$$=(x+y)(x^2+\omega^2xy+\omega xy+\omega^3y^2)$$
$$=(x+y)\left[x^2+(\omega^2+\omega)xy+\omega^3y^2\right]$$
$$\because 1+\omega+\omega^2=0,\ \omega+\omega^2=-1,\ \omega^3=1$$
$$=(x+y)\left[x^2+(-1)xy+1y^2\right]$$
$$=(x+y)(x^2-xy+y^2)$$

As $(a^3+b^3)=(a+b)(a^2-ab+b^2)$ so,

$$=x^3+y^3=L.H.S$$

Q.4 Prove that

$$x^3+y^3+z^3-3xyz=(x+y+z)(x+\omega y+\omega^2z)(x+\omega^2y+\omega z)$$

Solution: Let R.H.S

$$=(x+y+z)(x+\omega y+\omega^2z)(x+\omega^2y+\omega z)$$
$$=(x+y+z)(x^2+\omega^2xy+\omega xz+\omega yx+\omega^3y^2+\omega^2yz+\omega^2xz+\omega^4zy+\omega^3z^2)$$
$$=(x+y+z)\left[(x^2+\omega^3y^2+\omega^3z^2+(\omega^2+\omega)xy+(\omega^2+\omega^4)yz+(\omega+\omega^2)zx)\right]$$
$$=(x+y+z)\left[x^2+1y^2+1z^2+(-1)xy+(\omega^2+\omega^3\omega)yz+(-1)zx\right]$$
$$=(x+y+z)\left[x^2+1y^2+1z^2+(-1)xy+(\omega^2+1\omega)yz+(-1)zx\right]$$
$$=(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$$
$$=x^3+y^3+z^3-3xyz=L.H.S$$

Using formula:

$$=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)=a^3+b^3+c^3-3abc$$

Question No 5

$$=(1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8)\text{..........}2n\text{ factors}=1$$

Solution: L H S

$$=(1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8)\text{..........}2n\text{ factors}$$
$$=(1+\omega)(1+\omega^2)(1+\omega\omega^3)(1+\omega^6\omega^2)\text{..........}2n\text{ factors}$$
$$=(1+\omega)(1+\omega^2)(1+\omega)(1+\omega^2)\text{..........}2n\text{ factors}\qquad\therefore\omega^3=1$$
$$=\left[(1+\omega)(1+\omega^2)\right]\left[(1+\omega)(1+\omega^2)\right]\text{.........}n\text{ factors}$$
$$=\left[(1+\omega)(1+\omega^2)\right]^n$$
$$=\left[(-\omega^2)(-\omega)\right]^n$$
$$=\left[\omega^3\right]^n$$
$$=[1]^n$$
$$=1$$
$$L.H.S=R.H.S$$

Exercise 2.3

Q.1 Without solving, find the sum and product of the roots of following quadratic equations:

$(i)\ x^2-5x+3=0$

$Solution: x^2-5x+3=0$

$ax^2+bx+c=0$

$a=1,\ b=-5,\ c=3$

$Sum\ of\ roots=S=\frac{-b}{a}=-\left(\frac{-5}{1}\right)=5$

$\text{Product of roots}=P=\frac{c}{a}=\frac{3}{1}=3$

$(ii)\ 3x^2+7x-11=0$

$Solution: 3x^2+7x-11=0$

$ax^2+bx+c=0$

$a=3,\ b=7,\ c=-11$

$Sum\ of\ roots=S=\frac{-b}{a}=\frac{-7}{3}$

$\text{Product of roots}=P=\frac{c}{a}=\frac{-11}{3}$

$(iii)\ px^2-qx+r=0$

$Solution: px^2-qx+r=0$

$ax^2+bx+c=0$

$a=p,\ b=-q,\ c=r$

$Sum\ of\ roots=S=\frac{-b}{a}=-\left(\frac{-q}{p}\right)=\frac{q}{p}$

$\text{Product of roots}=P=\frac{c}{a}=\frac{r}{p}$

$(iv)\ (a+b)x^2-ax+b=0$

$\text{Solution}: (a+b)x^2-ax+b=0$

$ax^2+bx+c=0$

$a=(a+b),\ b=-a,\ c=b$

$\text{Sum of roots}=S=\frac{-b}{a}=-\left(\frac{-a}{a+b}\right)=\frac{a}{a+b}$

$\text{Product of roots}=P=\frac{c}{a}=\frac{b}{a+b}$

$(v)\ (l+m)x^2+(m+n)x+n-l=0$

$Solution: (l+m)x^2+(m+n)x+n-l=0$

$ax^2+bx+c=0$

$a=(l+m),\ b=(m+n),\ c=n-l$

$Sum\ of\ roots=S=\frac{-b}{a}=\frac{-(m+n)}{l+m}$

$\text{Product of roots}=P=\frac{c}{a}=\frac{n-l}{l+m}$

$(vi)\ 7x^2-5mx+9n=0$

$Solution: 7x^2-5mx+9n=0$

$ax^2+bx+c=0$

$a=7,\ b=-5m,\ c=9n$

$Sum\ of\ roots=S=\frac{-b}{a}=-\left(\frac{-5m}{7}\right)=\frac{5m}{7}$

$\text{Product of roots}=P=\frac{c}{a}=\frac{9n}{7}$

Q.2 Find the value of k if.

(i) Sum of the roots of equation $2kx^2-3x+4k=0$ is twice the product of roots.

$Solution: 2kx^2-3x+4k=0$

$ax^2+bx+c=0$

$a=2k,\ b=-3,\ c=4k$

Let α,β be the roots of equation

$Sum\ of\ roots=\alpha+\beta=\frac{-(-3)}{2k}=\frac{3}{2k}$

$\text{Product of roots}=\alpha\beta=\frac{c}{a}=\frac{4k}{2k}=2$

According to given conditions:

$S=2P$

$\frac{3}{2k}=2(2)$

$\frac{3}{2k}=4$

$3=4(2k)$

$3=8k$

$\frac{3}{8}=k$

$k=\frac{3}{8}$

(ii) Sum of the roots of the equation $x^2+(3k-7)x+5k=0$ *is* $\frac{3}{2}$ *times the products of roots.*

Solution: $1x^2+(3k-7)x+5k=0$

$ax^2+bx+c=0$

$a=1,\ b=(3k-7),\ c=5k$

Let α,β *be the roots of equation*

$\text{Sum of roots}=\alpha+\beta=\frac{-b}{a}=\frac{-(3k-7)}{1}=-3k+7$

$\text{Product of roots}=\alpha\beta=\frac{c}{a}=\frac{5k}{1}=5k$

According to given conditions:

$S=\frac{3}{2}P$

$-3k+7=\frac{3}{2}(5k)$

$2(-3k+7)=3(5k)$

$-6k+14=15k$

$14=15k+6k$

$14=21k$

$k=\frac{14}{21}=\frac{2}{3}$

$k=\frac{2}{3}$

Q.3 Find k, if

(ii) Sum of the squares of the roots of the equation $4kx^2+3kx-8=0$ *is 2*

Solution: $x^2+3kx-8=0$

$ax^2+bx+c=0$

$a=4k,\ b=3k,\ c=-8$

$\text{Sum of roots}=\alpha+\beta=\frac{-b}{a}$

$=\frac{-3k}{4k}=\frac{-3}{4}$

$\text{Product of roots}=P=\alpha\beta=\frac{c}{a}$

$=\frac{-8}{4k}=\frac{-2}{k}$

Given that sum of square of roots is 2

$\alpha^2+\beta^2=2$

$\alpha^2+\beta^2+2\alpha\beta-2\alpha\beta=2$

$(\alpha+\beta)^2-2\alpha\beta=2$

Put $\alpha+\beta=\frac{-3}{4}\quad \alpha\beta=\frac{-2}{k}$

$\left(\frac{-3}{4}\right)^2-2\left(\frac{-2}{k}\right)=2$

$\frac{9}{16}+\frac{4}{k}=2\quad\Rightarrow\frac{9k+64}{16k}=2$

$9k+64=2(16k)$

$9k+64=32k$

$64=32k-9k$

$64=23k\ \Rightarrow k=\frac{64}{23}$

Hence $k=\frac{64}{23}$

(ii) Sum of the square of the roots of the equation $x^2-2kx+(2k+1)=0$ *is 6*

Solution: $x^2-2kx+(2k+1)=0$

$ax^2+bx+c=0$

$a=1,\ b=-2k,\ c=2k+1$

α,β *are the roots of equation*

$\text{Sum of roots}=\alpha+\beta=\frac{-b}{a}$

$=\frac{-(-2k)}{1}=2k$

$\text{Product of roots}=P=\alpha\beta=\frac{c}{a}$

$=\frac{2k+1}{1}=2k+1$

According to given condition:

$\alpha^2+\beta^2=6$

$\alpha^2+\beta^2+2\alpha\beta-2\alpha\beta=6$

$(\alpha+\beta)^2-2\alpha\beta=6$

Put $\alpha+\beta=2k\quad \alpha\beta=2k+1$

$(2k)^2-2(2k+1)=6$

$4k^2-4k-2=6$

$4k^2-4k-2-6=0$

$4k^2-4k-8=0$

$4(k^2-k-2)=0$

$\because k^2-k-2=0$

$k^2-2k+k-2=0$

$k(k-2)+1(k-2)=0$

$(k-2)(k+1)=0$

$k-2=0\qquad k+1=0$

$k=2 \qquad k=-1$

$k=-1,2$

Q.4 Find p if

(i) The roots of the equation $x^2-x+p^2=0$ differ by unity.

Solution: $x^2-x+p^2=0$

$ax^2+bx+c=0$

$a=1,\ b=-1,\ c=p^2$

Let the roots are 'α' and '$\alpha-1$'

Sum of roots $=S=\alpha+\alpha-1$

$=\frac{-b}{a}=-\left(\frac{-1}{1}\right)=1$

$2\alpha-1=1$

$2\alpha=1+1$

$2\alpha=2$

$\alpha=1$

Product of roots $=P=\alpha(\alpha-1)$

$=\frac{c}{a}=\frac{p^2}{1}=p^2$

$\alpha(\alpha-1)=p^2$

Putting value of $\alpha=1$

$1(1-1)=p^2$

$p^2=0 \Rightarrow p=0$

(ii) Find p if the roots of the equation $x^2+3x+p-2=0$ differ by 2.

Solution: $x^2+3x+p-2=0$

$ax^2+bx+c=0$

$a=1,\ b=3,\ c=p-2$

Let the roots are 'α' and '$\alpha-2$'

Sum of roots $=S=\alpha+\alpha-2$

$=\frac{-b}{a}=\left(\frac{-3}{1}\right)=-3$

$2\alpha-2=-3$

$2\alpha=-3+2$

$2\alpha=-1$

$\alpha=\frac{-1}{2}$

Product of roots $=P=\alpha(\alpha-2)$

$=\frac{c}{a}=\frac{p-2}{1}=p-2$

$\alpha(\alpha-2)=p-2$

Putting value of $\alpha=\frac{-1}{2}$

$\frac{-1}{2}\left(\frac{-1}{2}-2\right)=p-2$

$\frac{-1}{2}\left(\frac{-1-4}{2}\right)=p-2$

$\frac{-1}{2}\left(\frac{-5}{2}\right)=p-2$

$\frac{5}{4}=p-2$

$p=\frac{5}{4}+2$

$p=\frac{5+8}{4}$

$p=\frac{13}{4}$

Q.5 Find m if

(i) The roots of the equation $x^2-7x+3m-5=0$ satisfy the relation $3\alpha+2\beta=4$.

Solution: $x^2-7x+3m-5=0$

Let α,β *be the roots of given equation*

$1x^2-7x+3m-5=0$

$ax^2+bx+c=0$

$a=1,\ b=-7,\ c=3m-5$

Sum of roots $=S=\alpha+\beta=\frac{-b}{a}$

$=-\left(\frac{-7}{1}\right)=7 \quad ...(i)$

Product of roots $=\alpha\beta=\frac{c}{a}=\frac{3m-5}{1}$

$=3m-5 \quad ...(ii)$

Since $3\alpha+2\beta=4 \quad ...(iii)$

From equation (i)

$\alpha+\beta=7$

$\beta=7-\alpha$

Put $\beta=7-\alpha$ in equation (iii)

$3\alpha+2\beta=4$

$3\alpha+2(7-\alpha)=4$

$3\alpha+14-2\alpha=4$

$\alpha+14=4$

$\alpha=4-14$

$\alpha=-10$

Put $\alpha=-10$ in $\beta=7-\alpha$

$\beta=7-(-10)$

$\beta=7+10 \Rightarrow \beta=17$

Put $\alpha=-10$ and $\beta=17$ in equation (ii)

$\alpha\beta=3m-5$

$(-10)(17)=3m-5$

$-170=3m-5$

$-170+5=3m$

$-165=3m$

$\frac{-165}{3}=m$

$-55=m \Rightarrow m=-55$

(ii) Find m if the roots of the equation $x^2-7x+3m-5=0$ satisfy the relation $3\alpha-2\beta=4$.

Solution: $x^2-7x+3m-5=0$

Let α,β *be the roots of given equation*

$1x^2+7x+3m-5=0$

$ax^2+bx+c=0$

$a=1,\ b=7,\ c=3m-5$

Sum of roots $=S=\alpha+\beta=\frac{-b}{a}$

$=\frac{-7}{1}=-7 \quad ...(i)$

Product of roots$=\alpha\beta=\frac{c}{a}=\frac{3m-5}{1}$

$=3m-5 \quad ...(ii)$

Since $3\alpha+2\beta=4 \quad ...(iii)$

From equation (i)

$\alpha+\beta=-7$

$\beta=-7-\alpha$

Put $\beta=-7-\alpha$ in equation (iii)

$3\alpha-2\beta=4$

$3\alpha-2(-7-\alpha)=4$

$3\alpha+14+2\alpha=4$

$5\alpha+14=4$

$5\alpha=4-14$

$5\alpha=-10$

$\alpha=\frac{-10}{5}$

$\alpha=-2$

Put $\alpha=-2$ in equation (i)

$\alpha+\beta=-7$

$-2+\beta=-7$

$\beta=-7+2 \Rightarrow \beta=-5$

Put $\alpha=-2$ and $\beta=-5$ in equation (ii)

$\alpha\beta=3m-5$

$(-2)(-5)=3m-5$

$10=3m-5$

$10+5=3m$

$15=3m$

$\frac{15}{3}=m \Rightarrow m=5$

(iii) Find m if the roots of the equation $3x^2-2x+7m+2=0$ satisfy the relation $7\alpha-3\beta=18$.

Solution: $3x^2-2x+7m+2=0$

Let α,β *be the roots of given equation*

$3x^2-2x+7m+2=0$

$ax^2+bx+c=0$

$a=3,\ b=-2,\ c=7m+2$

Sum of roots $=S=\alpha+\beta=\frac{-b}{a}$

$=\frac{-(-2)}{3}=\frac{2}{3} \quad ...(i)$

Product of roots$=\alpha\beta=\frac{c}{a}$

$=\frac{7m+2}{3} \quad ...(ii)$

Since $7\alpha-3\beta=18 \quad ...(iii)$

From equation (i)

$\alpha+\beta=\frac{2}{3}$

$\beta=\frac{2}{3}-\alpha$

Put $\beta=\frac{2}{3}-\alpha$ in equation (iii)

$7\alpha-3\beta=18$

$7\alpha-3\left(\frac{2}{3}-\alpha\right)=18$

$7\alpha-\frac{6}{3}+3\alpha=18$

$7\alpha-2+3\alpha=18$

$10\alpha = 18 + 2$

$10\alpha = 20$

$\alpha = \frac{20}{10}$

$\alpha = 2$

Put $\alpha = 2$ in equation (i)

$\alpha + \beta = \frac{2}{3}$

$2 + \beta = \frac{2}{3}$

$\beta = \frac{2}{3} - 2 = \frac{-4}{3} \quad \Rightarrow \beta = \frac{-4}{3}$

Put $\alpha = 2$ and $\beta = \frac{-4}{3}$ in equation (ii)

$\alpha\beta = \frac{7m+2}{3}$

$2\left(\frac{-4}{3}\right) = \frac{7m+2}{3}$

$\frac{-8}{3} \times 3 = 7m + 2$

$7m + 2 = -8$

$7m = -8 - 2$

$7m = -10 \Rightarrow m = \frac{-10}{7}$

Q.6 Find m if sum and product of the roots of the following equations is equal to given number λ.

(i) $(2m+3)x^2 + (7m-5)x + (3m-10) = 0$

Solution: $(2m+3)x^2 + (7m-5)x + (3m-10) = 0$

$ax^2 + bx + c = 0$

$a = (2m+3),\ b = (7m-5),\ c = (3m-10)$

Let α, β are the roots of the equation, then

Sum of roots $= S = \alpha + \beta = \frac{-b}{a}$

$= -\frac{(7m-5)}{2m+3} = \frac{5-7m}{2m+3}$

Product of roots $= P = \frac{c}{a} = \frac{3m-10}{2m+3}$

As given that

$\alpha + \beta = \lambda \ ...(i) \quad \alpha\beta = \lambda ...(ii)$

From (i) and (ii)

$\alpha + \beta = \alpha\beta$

$\frac{5-7m}{2m+3} = \frac{3m-10}{2m+3}$

$5 - 7m = \frac{3m-10}{2m+3} \times 2m + 3$

$5 - 7m = 3m - 10$

$5 + 10 = 3m + 7m$

$15 = 10m$

$\frac{15}{10} = m \quad \Rightarrow m = \frac{3}{2}$

(ii) $4x^2 - (3+5m)x - (9m-17) = 0$

Solution: $4x^2 - (3+5m)x - (9m-17) = 0$

Let α, β are the roots of the equation, then

$4x^2 - (3+5m)x - (9m-17) = 0$

$ax^2 + bx + c = 0$

$a = 4,\ b = -(3+5m),\ c = -(9m-17)$

Sum of roots $= S = \alpha + \beta = \frac{-b}{a}$

$= -\left(-\frac{(3+5m)}{4}\right) = \frac{3+5m}{4}$

Product of roots $= P = \alpha\beta = \frac{c}{a} = -\left(\frac{9m-17}{4}\right)$

Let $\alpha + \beta = \lambda \ ...(i) \quad \alpha\beta = \lambda ...(ii)$

From (i) and (ii)

$\alpha + \beta = \alpha\beta$

$\frac{3+5m}{4} = \frac{-(9m-17)}{4}$

$3 + 5m = -9m + 17$

$9m + 5m = 17 - 3$

$14m = 14$

$\frac{14}{14} = m \quad \Rightarrow m = 1$

Exercise 2.4

$Q.1\ If\ \alpha,\beta\ are\ the\ roots\ of\ the\ equations$

$x^2+px+q=0\ then\ evaluate$

$Solution: x^2+px+q=0$

$a=1,\ b=p,\ c=q$

$Sum\ of\ roots,$

$$\alpha+\beta=\frac{-b}{a}=\frac{-p}{1}=-p$$

$$\alpha+\beta=-p$$

Product of roots,

$$\alpha\beta=\frac{c}{a}=\frac{q}{1}=q$$

$$\alpha\beta=q$$

$(i)\alpha^2+\beta^2$

$Solution:$

$$(\alpha+\beta)^2=\alpha^2+\beta^2+2\alpha\beta$$

$$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$$

$$=(-p)^2-2(q)$$

$$=p^2-2q$$

$(ii)\alpha^3\beta+\alpha\beta^3$

$Solution: \alpha^3\beta+\alpha\beta^3$

$$=\alpha\beta\left(\alpha^2+\beta^2\right)$$

$$=\alpha\beta\left[(\alpha+\beta)^2-2\alpha\beta\right]$$

$$=q\left[(-p)^2-2q\right]$$

$$=q\left(p^2-2q\right)$$

$(iii)\dfrac{\alpha}{\beta}+\dfrac{\beta}{\alpha}$

$Solution: \dfrac{\alpha}{\beta}+\dfrac{\beta}{\alpha}$

$$=\frac{\alpha^2+\beta^2}{\alpha\beta}$$

$$=\frac{1}{\alpha\beta}\left(\alpha^2+\beta^2\right)$$

$$=\frac{1}{q}\left[(-p)^2-2\times q\right]$$

$$=\frac{1}{q}\left(p^2-2q\right)$$

$Q.2\ If\ \alpha,\beta\ are\ the\ roots\ of\ the\ equation$

$4x^2-5x+6=0, then\ find\ the\ value$

$Solution: 4x^2-5x+6=0$

$ax^2+bx+c=0$

$a=4,\ b=-5,\ c=6$

$Sum\ of\ roots,$

$$\alpha+\beta=\frac{-b}{a}=\frac{-(-5)}{4}=\frac{5}{4}$$

Product of roots,

$$\alpha\beta=\frac{c}{a}=\frac{6}{4}$$

$(i)\dfrac{1}{\alpha}+\dfrac{1}{\beta}$

$Solution: \dfrac{1}{\alpha}+\dfrac{1}{\beta}$

$$=\frac{\alpha+\beta}{\alpha\beta}=\frac{5/4}{6/4}=\frac{5}{6}$$

$(ii)\alpha^2\beta^2$

$Solution: \alpha^2\beta^2$

$$=(\alpha\beta)^2=\left(\frac{6}{4}\right)^2=\left(\frac{3}{2}\right)^2=\frac{9}{4}$$

$(iii)\dfrac{1}{\alpha^2\beta}+\dfrac{1}{\alpha\beta^2}$

$Solution: \dfrac{1}{\alpha^2\beta}+\dfrac{1}{\alpha\beta^2}$

$$=\frac{\beta+\alpha}{\alpha^2\beta^2}=\frac{\alpha+\beta}{(\alpha\beta)^2}=\frac{5/4}{(6/4)^2}$$

$$=\frac{5/4}{36/16}=\frac{5}{4}\times\frac{16}{36}=\frac{5}{9}$$

(iv) $\dfrac{\alpha^2}{\beta}+\dfrac{\beta^2}{\alpha}$

$Solution: \dfrac{\alpha^2}{\beta}+\dfrac{\beta^2}{\alpha}=\dfrac{\alpha^3+\beta^3}{\alpha\beta}$

Using formula: $(\alpha+\beta)^3=\alpha^3+\beta^3+3(\alpha\beta)(\alpha+\beta)$

$$\alpha^3+\beta^3=(\alpha+\beta)^3-3(\alpha\beta)(\alpha+\beta)$$

$$=\frac{(\alpha+\beta)^3-3(\alpha\beta)(\alpha+\beta)}{\alpha\beta}$$

$$=\frac{(5/4)^3-3(6/4)(5/4)}{(6/4)}$$

$$=\left(\frac{125}{64}-\frac{90}{16}\right)\frac{4}{6}=\left(\frac{125-360}{64}\right)\frac{4}{6}$$

$$=\frac{-235}{96}$$

Q.3 If α,β are the roots of the equation $lx^2+mx+n=0$ then find the value

$Solution: lx^2+mx+n=0$

$$ax^2+bx+c=0$$

$$a=l,\ b=m,\ c=n$$

f α,β are the roots of given equation

Sum of roots $=\dfrac{-b}{a}$

$$\alpha+\beta=\frac{-m}{l}$$

Product of roots= $\dfrac{c}{a}$

$$\alpha\beta=\frac{n}{l}$$

(i) $\alpha^3\beta^2+\alpha^2\beta^3$

$Solution: \alpha^3\beta^2+\alpha^2\beta^3$

$$=\alpha^2\beta^2(\alpha+\beta)$$

$$=(\alpha\beta)^2(\alpha\beta)^2=\left(\frac{n}{l}\right)^2\left(\frac{-m}{l}\right)$$

$$=\left(\frac{n^2}{l^2}\right)\left(\frac{-m}{l}\right)=\frac{-mn^2}{l^3}$$

(ii) $\dfrac{1}{\alpha^2}+\dfrac{1}{\beta^2}$

$Solution: \dfrac{1}{\alpha^2}+\dfrac{1}{\beta^2}$

$$=\frac{\alpha^2+\beta^2}{\alpha^2\beta^2}=\frac{(\alpha+\beta)^2-2\alpha\beta}{(\alpha\beta)^2}$$

$$=(\alpha+\beta)^2-2\alpha\beta\div(\alpha\beta)^2$$

$$=\left(\frac{-m}{l}\right)^2-2\left(\frac{n}{l}\right)\div\left(\frac{n}{l}\right)^2$$

$$=\frac{m^2}{l^2}-\frac{2n}{l}\div\frac{n^2}{l^2}=\left(\frac{m^2-2nl}{l^2}\right)\times\frac{l^2}{n^2}$$

$$=\frac{1}{n^2}(m^2-2nl)$$

Exercise 2.5

Question No.1 Write the quadratic equation having following roots.

Write the Quadratic equation having following roots.

a) $\mathbf{1, 5}$

Solution:

Since 1 and 5 are the roots of the required quadratic equation, therefore

Sum of roots= $\boldsymbol{S = 1 + 5 = 6}$

$\boldsymbol{product\ of\ roots = P = 1 \times 5 = 5}$

As $x^2 - Sx + P = 0$ $so\ the\ required\ equation\ is$

$$x^2 - 6x + 5 = 0$$

b) 4,9

Solution:

Since 4 and 9 are the roots of the required quadratic equation, therefore

Sum of roots= $\boldsymbol{S = 4 + 9 = 13}$

$\boldsymbol{product\ of\ roots = P = 4 \times 9 = 36}$

As $x^2 - Sx + P = 0$ $so\ the\ required\ equation\ is$

$$x^2 - 13x + 36 = 0$$

c) $(-2,3)$

$solution$:

Since -2 and 3 are the roots of the required quadratic equation, therefore

Sum of roots= $\boldsymbol{S = -2 + 3 = 1}$

$\boldsymbol{product\ of\ roots = P = -2 \times 3 = -6}$

As $x^2 - Sx + P = 0$ $so\ the\ required\ equation\ is$

$$x^2 - x - 6 = 0$$

d) $0, -3$

Solution:

Since 0 and -3 are the roots of the required quadratic equation, therefore

Sum of roots= $\boldsymbol{S = 0 - 3 = -3}$

$\boldsymbol{product\ of\ roots = P = 0 \times -3 = 0}$

As $x^2 - Sx + P = 0$ $so\ the\ required\ equation\ is$

$$x^2 + 3x + 0 = 0$$

⇨ $x^2 + 3x + 0$

e) $2, -6$

Solution:

Since 2 and -6 are the roots of the required quadratic equation, therefore

Sum of roots= $\boldsymbol{S = 2 + (-6) = 2 - 6 = -4}$

$\boldsymbol{product\ of\ roots = P = 2 \times (-6) = -12}$

As $x^2 - Sx + P = 0$ $so\ the\ required\ equation\ is$

$$x^2 + 4x - 12 = 0$$

f) $-1, -7$

$Solution$:

Since -1 and -7 are the roots of the required quadratic equation, therefore

Sum of roots= $\boldsymbol{S = (-1) + (-7) = -1 - 7 = -8}$

$\boldsymbol{product\ of\ roots = P = -1 \times -7 = 7}$

As $x^2 - Sx + P = 0$ $so\ the\ required\ equation\ is$

$$x^2 + 8x + 7 = 0$$

g) $(1 + i, 1 - i)$

Solution:

Since $\boldsymbol{1 + i}$ **and** $\boldsymbol{1 - i}$ **are the roots of the required quadratic equation, therefore**

Sum of roots= $\boldsymbol{S = 1 + i + 1 - i = 2}$

$$\boldsymbol{product\ of\ roots = P = (1 + i) \times (1 - i)}$$
$$\boldsymbol{= P = (1)^2 - (i)^2}$$
$$\boldsymbol{P = 1 - (-1)}$$
$$\boldsymbol{P = 1 + 1 = 2}$$

As $x^2 - Sx + P = 0$ $so\ the\ required\ equation\ is$

$$x^2 - 2x + 2 = 0$$

h) $3 + \sqrt{2},\ 3 - \sqrt{2}$

Solution:

Since $\boldsymbol{3 + \sqrt{2}}$ **and** $\boldsymbol{3 - \sqrt{2}}$ **are the roots of the required quadratic equation, therefore**

Sum of roots= $\boldsymbol{S = 3 + \sqrt{2} + 3 - \sqrt{2} = 6}$

$$\boldsymbol{product\ of\ roots = P = \left(3 + \sqrt{2}\right)\left(3 - \sqrt{2}\right)}$$
$$\boldsymbol{P = (3)^2 - \left(\sqrt{2}\right)^2}$$
$$P = 9 - 2$$
$$P = 7$$

As $x^2 - Sx + P = 0$ $so\ the\ required\ equation\ is$

$$x^2 - 6x + 7 = 0$$

QuestionNo.2 if α, β are the roots of the equation $x^2 - 3x + 6 = 0$ $from\ equation\ whoes$roots are

Solution:

As α, β $are\ the\ roots$ of the equation

$$x^2 - 3x + 6 = 0$$
$$a = 1, b = -3\ , c = 6$$

Therefore

$$\alpha + \beta = -\frac{b}{a} = -\frac{-3}{1} = 3$$
$$\Rightarrow \alpha + \beta = 3$$
$$\alpha\beta = \frac{c}{a} = \frac{6}{1} = 6$$
$$\alpha\beta = 6$$

a) $2\alpha + 1, 2\beta + 1$

Solution:

Sum of the roots

$$S = 2\alpha + 1 + 2\beta + 1$$
$$S = 2\alpha + 2\beta + 2$$
$$S = 2(\alpha + \beta) + 2$$
$$S = 2(3) + 2 = 6 + 2 = 8$$
$$S = 8$$

Product of the roots

$$P = (2\alpha + 1)(2\beta + !)$$
$$P = 4\alpha\beta + 2\alpha + 2\beta + 1$$
$$P = 4\alpha\beta + 2(\alpha + \beta) + 1$$
$$P = 4(6) + 2(3) + 1$$
$$P = 24 + 6 + 1 = 31$$
$$P = 31$$

Using $x^2 - SX + P = 0$ $w\ e\ have$ $x^2 - 8x + 31 = 0$

b) α^2, β^2

Solution:

As α, β are the roots of the equation $x^2 - 3x + 6 = 0$

$$a = 1, b = -3, c = 6$$

Therefore

$$\alpha + \beta = -\frac{b}{a} = \frac{-3(-3)}{1} = 3$$
$$\alpha + \beta = 3$$

$$\alpha\beta=\frac{c}{a}=\frac{6}{1}=6$$

⇨ $\alpha\beta=6$

Sum of the roots= $S=\alpha^2+\beta^2$

$$S=(\alpha+\beta)^2-2\alpha\beta$$
$$S=(3)^2-2(6)$$
$$S=9-12=-3$$
$$S=-3$$
$$Product\ of\ roots=P=\alpha^2\beta^2$$
$$P=(\alpha\beta)^2$$
$$P=(6)^2=36$$
$$P=36$$
$$Using\ x^2-Sx+P=0, we\ have$$
$$x^2+3x+36=0$$

c) $\frac{1}{\alpha},\frac{1}{\beta}$

Solution:

As α,β $are\ the\ roots$ of the equation

$$x^2-3x+6=0$$
$$a=1, b=-3\ , c=6$$

Therefore

$$\alpha+\beta=-\frac{b}{a}=-\frac{-3}{1}=3$$
$$\Rightarrow\alpha+\beta=3$$
$$\alpha\beta=\frac{c}{a}=\frac{6}{1}=6$$
$$\alpha\beta=6$$

Sum of roots $=S=\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\beta+\alpha}{\alpha\beta}$

$$=s=(\alpha+\beta)\left(\frac{1}{\alpha\beta}\right)$$
$$S=3\times\frac{1}{6}=\frac{3}{6}=\frac{1}{2}$$
$$S=\frac{1}{2}$$
$$Product\ of\ roots=P=\left(\frac{1}{\alpha}\right)\left(\frac{1}{\beta}\right)$$
$$P=\frac{1}{\alpha\beta}=\frac{1}{6}$$
$$P=\frac{1}{6}$$
$$using\ x^2-Sx+P=0\ we\ have$$
$$x^2-\frac{1}{2}x+\frac{1}{6}=0$$
$$Multiplying\ by\ 6\ on\ both\ sides, we\ have$$
$$6x^2-3x+1=0$$

d) $\frac{\alpha}{\beta},\frac{\beta}{\alpha}$

Solution:

As α,β $are\ the\ roots$ of the equation

$$x^2-3x+6=0$$
$$a=1, b=-3\ , c=6$$

Therefore

$$\alpha+\beta=-\frac{b}{a}=-\frac{-3}{1}=3$$
$$\Rightarrow\alpha+\beta=3$$
$$\alpha\beta=\frac{c}{a}=\frac{6}{1}=6$$
$$\alpha\beta=6$$

Sum of the roots $S=\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$

$$S=\frac{\alpha^2+\beta^2}{\alpha\ \beta}$$
$$S=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$$
$$S=\frac{(3)^2-2(6)}{6}$$
$$S=\frac{9-12}{6}$$
$$S=-\frac{3}{6}$$
$$S=-\frac{1}{2}$$
$$Product\ of\ roots=P=\left(\frac{\alpha}{\beta}\right)\left(\frac{\beta}{\alpha}\right)=1$$
$$using\ x^2-Sx+P=0\ we\ have\ x^2+\frac{1}{2}x+1=0$$

$we\ have$

$$2x^2+x+2=0$$

e) $\alpha+\beta,\frac{1}{\alpha}+\frac{1}{\beta}$

$Solution:$

As α,β $are\ the\ roots$ of the equation

$$x^2-3x+6=0$$
$$a=1, b=-3\ , c=6$$

Therefore

$$\alpha+\beta=-\frac{b}{a}=-\frac{-3}{1}=3$$
$$\Rightarrow\alpha+\beta=3$$
$$\alpha\beta=\frac{c}{a}=\frac{6}{1}=6$$
$$\alpha\beta=6$$
$$Sum\ of\ roots=S=(\alpha+\beta)+\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)$$
$$S=(\alpha+\beta)+\left(\frac{\beta+\alpha}{\alpha\beta}\right)$$
$$S=3+\frac{3}{6}$$
$$S=3+\frac{1}{2}$$
$$S=\frac{6+1}{2}$$
$$S=\frac{7}{2}$$
$$S=\frac{7}{2}$$

Product of roots$P=(\alpha+\beta)\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)$

$$(\alpha+\beta)\times\left(\frac{\beta+\alpha}{\alpha\beta}\right)$$
$$=3\left(\frac{3}{6}\right)$$
$$P=\frac{3}{2}$$

Using $x^2-Sx+P=0$ $we\ have$

$$x^2-\frac{7}{2}x+\frac{3}{2}=0$$

Multiplying both sides by 2 we have

$$2x^2-7x+3=0$$

Question No.3

if α, β *are the roots of the equation*

$x^2 + px + q = 0$ *from* equation whoes roots are

(a) α^2, β^2

Solution:

Since α, β are the roots of the equation

$$x^2 + px + q = 0$$
$$ax^2 + bx + c = 0$$

By comparing the coefficients of these equations, we have

$$a = 1, b = p, c = q$$
$$\alpha + \beta = -\frac{b}{a} = -\frac{P}{1} = -P$$
$$\alpha + \beta = -P$$
$$\alpha\beta = \frac{c}{a} = \frac{q}{1} = q$$
$$\alpha\beta = q$$

(a) α^2, β^2

Solution:

$$Sum\ of\ roots = S = \alpha^2 + \beta^2$$
$$S = (\alpha + \beta)^2 - 2\alpha\beta$$
$$S = (-P)^2 - 2q$$
$$S = p^2 - 2q$$

Product of roots= $P = \alpha^2\beta^2$

$$P = (\alpha\beta)^2$$
$$P = q^2$$

Using $x^2 - Sx + P = 0$, *we have*

$$x^2 - (P^2 - 2q)x + q^2 = 0$$

(b) $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$

Solution:

Since α, β are the roots of the equation

$$x^2 + px + q = 0$$
$$ax^2 + bx + c = 0$$

By comparing the coefficients of these equations, we have

$$a = 1, b = p, c = q$$
$$\alpha + \beta = -\frac{b}{a} = -\frac{P}{1} = -P$$
$$\alpha + \beta = -P$$
$$\alpha\beta = \frac{c}{a} = \frac{q}{1} = q$$
$$\alpha\beta = q$$

Sum of roots $= S = \frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

$$S = \frac{\alpha^2 + \beta^2}{\alpha\beta}$$
$$S = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$$
$$S = \frac{(-P)^2 - 2(q)}{q}$$
$$S = \frac{p^2 - 2q}{q}$$

Product of roots $= P = \left(\frac{\alpha}{\beta}\right)\left(\frac{\beta}{\alpha}\right) = 1$

Using $x^2 - Sx + P = 0$ *we have*

$$x^2 - Sx + P = 0 \text{ we have}$$
$$x^2 - \left(\frac{p^2 - 2q}{q}\right)x + 1 = 0$$

Multiplying by q

$$qx^2 - (p^2 - 2q)x + q = 0$$

Exercise 2.6

1. Uses synthetic division to find the quotient and the remainder, when

(i) $(x^2 + 7x - 1) \div (x + 1)$

Solution:

$P(x) = x^2 + 7x - 1$

$x + 1 = x - (-1)$

⇨ $a = -1$

	1	7	-1
−1	↓	-1	-6
	1	6	-7

$Quotient = Q(x) = x + 6$

$Remainder = -7$

(ii) $(4x^3) - 5x + 15) \div (x - 3)$

Solution:

$P(x) = 4x^3 - 5x + 15) \div (x + 3)$

$(x + 3) = x - (-3) \Rightarrow a = -3$

	4	0	-5	15
−3	↓	-12	36	-93
	1	-12	31	-78

$Quotient = 4x^2 - 12x + 31$

$Remainder = -78$

(iii) $(x^3 + x^2 - 3x + 2) \div (x - 2)$

Solution:

$P(x) = (x^3 + x^2 - 3x + 2)$

$x - 2 = x - (2) \Rightarrow a = 2$

	1	1	-3	2
2	↓	2	6	6
	1	3	3	8

$Quotient = x^2 + 3x + 3$

$Remainder = 8$

2. Find the value of h using synthetic division, if

(i) 3 *is the zero of the polynomial* $2x^3 - 3hx^2 + 9$

Solution:

$P(x) = 2x^3 - 3hx^2 + 9$ *and its root is* 3

3	2	$-3h$	0	9
	$\downarrow$	6	$3(6-3h)$	$9(6-3h)$
	1	$(6-3h)$	$3(6-3h)$	$9+9(6-3h)$

$$Quotient = Q(x) = 2x^2 + (6-3h)x + 3(6-3h)$$
$$Remainder = 9 + 9(6-3h)$$

$$9 + 9(6-3h) = 0$$
$$9 + 9(6-3h) = 0$$
$$9 + 54 - 27h = 0$$
$$63 - 27h = 0$$
$$63 - 27h = 0$$
$$-27h = -63$$
$$h = \frac{63}{27}$$
$$h = \frac{7}{3}$$

(ii) 1 *is the zero of the polynomial* $2x^3 - 2hx^2 + 11$

Solution:

$P(x) = 2x^3 - 2hx^2 + 11$ *and its root is* 1

1	1	$-2h$	0	11
	$\downarrow$	1	$(1-2h)$	$(1-2h)$
	1	$(1-2h)$	$(1-2h)$	$11+(1-2h)$

$$Quotient = Q(x) = x^2 + (1-2h)x + (1-2h)$$
$$Remainder = 11 + (1-2h)$$

$$11 + (1-2h) = 0$$
$$11 + 1 - 2h = 0$$
$$12 - 2h = 0$$
$$-2h = -12$$
$$h = 6$$

(iii) -1 *is the zero of the polynomial* $2x^3 + 5hx - 23$

Solution:

$P(x) = 2x^3 + 5hx - 23$ and *its root is* -1

$$l = -\frac{3}{2}, m = -18$$

-1	2	0	$5h$	-23
	$\downarrow$	-2	2	$-(5h+2)$
	2	-2	$(5h+2)$	$-23-(5h+2)$

$$Quotient = Q(x) = 2x^2 - 2x + (5h+2)$$
$$Remainder = -23 - (5h+2)$$

$$-23 - (5h+2) = 0$$
$$-23 - 5h - 2 = 0$$
$$-23 - 2 - 5h = 0$$
$$-25 - 5h = 0$$
$$-5h = 25$$
$$h = -5$$

3. *uses synthetic division to find the valves of* l *and* m

(i) $(x+3)$*and* $(x-2)$*are the factors of the polynomial* $x^3 - 4x^2 + 2lx + m$

Solution:

$x = -3$ *and* $x = 2$ *are two roots for* $x = -3$

$Q(x) = x^3 - 4x^2 + 2lx + m$

-3	1	4	$2l$	m
	$\downarrow$	-3	-3	$-3(2l-3)$
	1	-2	$(2l-3)$	$m-3(2l-3)$

$$Quotient = Q(x) = x^2 + x + (2l-3)$$
$$Remainder = m - 3(2l-3)$$

$m - 3(2l-3) = 0$

$m - 6l + 9 = 0 \rightarrow (i)$

For $x = 2$

2	1	4	$2l$	m
	$\downarrow$	2	12	$2(2l+12)$
	1	6	$(2l-3)$	$m+2(2l+12)$

$Quotient = Q(x) = x^2 + 6x + (2l+12)$

$Remainder = m + 2(2l+12)$

$$m + 2(2l+12) = 0$$
$$m + 2l + 24 = 0 \rightarrow (ii)$$

$eq(i) - eq(ii)$

$$m - 6l + 9 = 0$$
$$-m \pm 4l \pm 24 = 0$$
$$-10l - 15 = 0$$
$$-10l = 15$$
$$l = \frac{15}{-10}$$
$$l = -\frac{3}{2}$$

put $l = -\frac{3}{2}$ *put in eq* (i)

$$m - 6l + 9 = 0$$
$$m - 6\left(-\frac{3}{2}\right) + 9 = 0$$
$$m - 9 + 9 = 0$$
$$m + 18 = 0$$
$$m = -18$$

(iv) $(x-1)$*and* $x + 1)$ *are the factors of the polynomial* $x^3 - 3lx^2 + 2mx + 6$

Solution:

$P(x) = x^3 - 3lx^2 + 2mx + 6$

are two roots $(x-1)$ *and* $(x+1)$

$for\ x = 1$

1	1	$-3l$	$2m$	6
	↓	1	$1-3l$	$2m+(1-2l)$
	1	$(1-3l)$	$2m+(l-3l)$	$6+2m+(1-3l)$

$Quotient = Q(x) = x^2 + (1-3l)x + 2m + (1-3l)$
$Remainder = 6 + 2m + (1-3l)$

$6 + 2m + (1-3l) = 0$
$6 + 2m + 1 - 3l = 0$
$7 + 2m - 3l = 0 \rightarrow (i)$
$for\ x = -1$

-1	1	$-3l$	$2m$	6
	↓	-1	$-(-3l-1)$	$-(2m-(3l-1)$
	1	$(-3l-1)$	$(1-2h)$	$6-(2m-(-3L-1))$

$Quotient = Q(x) = x^2 - (3l+1)x + 2m + (3l+1)$
$Remainder = 6 - 2m - (-3l-1)$

$6-(2m+3l+1) = 0$
$6 - 2m - (3l+1) = 0$
$6 - 2m - 3l - 1 = 0$
$5 - 2m - 3l = 0 \rightarrow (ii)$
$eq(i) + eq(ii) we\ get$

$7 + 2m - 3l = 0$
$5 - 2m - 3l = 0$
$12 - 6l = 0$
$-6l = -12$
$l = 2$
$l = 2\ put\ in\ eq(i)$
$7 + 2m - 3l = 0$
$7 + 2m - 3(2) = 0$
$7 + 2m - 6 = 0$
$1 + 2m = 0$
$2m = -1$
$m = \frac{-1}{2}$

$l = 2, m = \frac{-1}{2}$

4. Solve by using synthetic division, if
(i) $2\ is\ the\ root\ of\ the\ equation x^3 - 28x + 48 = 0$
Solution:
$P(x) = x^3 - 28x + 48 = 0$

2	1	0	-28	48
	↓	2	4	-48
	1	6	-24	0

$Quotient = Q(x) = x^2 + 2x - 24$
$the\ depressed\ equation\ is\ x^2 + 2x - 24 = 0$
$x^2 + 2x - 24 = 0$
$x^2 + 6x - 4x - 24 = 0$
$x(x+6) - 4(x+6) = 0$
$(x-4)(x+6) = 0$
$(x-4) = 0 \qquad (x+6) = 0$
⇨ $x = 4 \qquad x = -6$
Hence2,
$4, -6\ are\ the\ roots\ of\ the\ given\ equation.$

(ii)
$3\ is\ the\ root\ of\ the\ equation\ 2x^3 - 3x^2 - 11x + 6 = 0$
Solution:
$P(x) = 2x^3 - 3x^2 - 11x + 6 = 0$

3	2	-3	-11	6
	↓	6	9	-6
	1	3	-2	0

$Quotient = Q(x) = 2x^2 + 3x - 2 = 0$
$the\ depressed\ equation\ is\ 2x^2 + 3x - 2 = 0$
$2x^2 + 3x - 2 = 0$
$2x^2 + 4x - x - 2 = 0$
$2x(x+2) - 1(x+2) = 0$
$(2x-1)(x+2) = 0$
$(2x-1) = 0 \quad (x+2) = 0$
$2x = 1 \qquad x = -2$
$x = \frac{1}{2} \qquad x = -2$
$hence 3, \frac{1}{2}, -2\ are\ the\ roots\ of\ the\ given\ eqution.$

(iii)
$-1\ is\ the\ root\ of\ the\ equation\ 4x^3 - x^2 - 11x - 6 = 0$
$solution:$
$P(x) = 4x^3 - x^2 - 11x - 6 = 0$
$the\ depressed\ equation\ is\ 4x^3 - x^2 - 11x - 6 = 0$

-1	4	-1	-11	-6
	↓	-4	5	6
	4	-5	-6	0

$Quotient = Q(x) = 4x^2 - 5x - 6$
$the\ depressed\ equation\ is 4x^2 - 5x - 6 = 0$
$4x^2 - 5x - 6 = 0$
$4x^2 - 8x + 3x - 6 = 0$
$4x(x-2) + 3(x-2) = 0$

$$(4x+3)(x-2)=0$$
$$4x+3=0 \quad x-2=0$$
$$4x=-3 \quad x=2$$
$$x=\frac{-3}{4} \quad , \quad x=2$$

hence

$-1,-\frac{3}{4},2$ *are the roots of the given equation.*

5.

(i) 1 na 3 are the roots of the eqution $x^4-10x^2+9=0$

Solution:

$x^4-10x^2+9=0$

$$P(x)=x^4-10x^2+9=0$$

	1	0	−10	0	9
1	↓	1	1	−9	−9
	1	1	−9	−9	0
3	↓	3	12	9	
	1	4	3	0	

$Quotient = Q(x) = x^2+4x+3$

$Remainder = 0$

the depresses equation is $x^2+4x+3=0$

$$x^2+4x+3=0$$
$$x^2+3x+x+3=0$$
$$x(x+3)+1(x+3)=0$$
$$(x+1)(x+3)=0$$
$$(x+3)=0 \quad (x+1)=0$$
$$x=-3 \quad x=-1$$

hence $1,3,-1,-3$ *ar the roots of the given equation.*

ii)

3 *and* -4 *are the roots of the equation*

$$x^4+2x^3-13x^2-14x+24=0$$

Solution:

$$x^4+2x^3-13x^2-14x+24=0$$
$$P(x)= x^4+2x^3-13x^2-14x+24$$

	1	2	−13	-14	24
3	↓	3	15	6	−24
	1	5	2	−8	0
−4	↓	−4	−4	8	
	1	1	−2	0	

$Quotient = Q(x) = x^2+x-2$

$Remainder = 0$

The depressed eqution is $x^2+x-2=0$

$x^2+x-2=0$

$x^2+2x-x-2=0$

$x(x+2)-1(x+2)=0$

$(x-1)(x+2)=0$

$$x-1=0 \quad x+2=0$$
$$x=1 \quad x=-2$$

Hence $3,-4,1,-2$ *are the roots of the given equation.*

Exercise 2.7

Solve the following simultaneous equations.

Q.1 $x+y=5$

$x^2-2y-14=0$

Solution: $x+y=5$ .....(i)

$x^2-2y-14=0$ .....(ii)

From eq. (i)

$x+y=5$

$x=5-y$

Put it in eq. (ii)

$(5-y)^2-2y-14=0$

$25+y^2-10y-2y-14=0$

$y^2-12y+11=0$

$y^2-11y-y+11=0$

$y(y-11)-1(y-11)=0$

$(y-1)(y-11)=0$

$y-1=0$ or $y-11=0$

$y=1$ or $y=11$

Putting the values of y in eq. (i)

$y=1$	$y=11$
$x+y=5$	$x+y=5$
$x+1=5$	$x+11=5$
$x=5-1$	$x=5-11$
$\boxed{x=4}$	$\boxed{x=-6}$

Solution set is $\{(-6,11),(4,1)\}$

Q.2 $3x-2y=1$

$x^2+xy-y^2=1$

Solution: $3x-2y=1$ .....(i)

$x^2+xy-y^2=1$ .....(ii)

From eq. (i)

$3x = 1 + 2y$

$x = \frac{1+2y}{3}$ .....(iii)

Put it in eq. (ii)

$\left(\frac{1+2y}{3}\right)^2 + \left(\frac{1+2y}{3}\right)y - y^2 = 1$

$\frac{1+4y^2+4y}{9} + \frac{y+2y^2}{3} - y^2 = 1$

Multiplying by '9' on both sides

$\frac{9(1+4y^2+4y)}{9} + \frac{9(y+2y^2)}{3} - 9(y^2) = 1 \times 9$

$1 + 4y^2 + 4y + 3y + 6y^2 - 9y^2 = 9$

$y^2 + 7y - 8 = 0$

$y^2 + 8y - y - 8 = 0$

$y(y+8) - 1(y+8) = 0$

$(y+8)(y-1) = 0$

$y + 8 = 0$ or $y - 1 = 0$

$y = -8$ or $y = 1$

Putting these values in eq. (iii)

$y = -8$ $y = 1$

$y = -8$	$y = 1$
$x = \frac{1+2y}{3}$	$x = \frac{1+2y}{3}$
$x = \frac{1+2(-8)}{3}$	$x = \frac{1+2(1)}{3}$
$x = \frac{1-16}{3}$	$x = \frac{1+2}{3}$
$x = \frac{-15}{3} = \boxed{-5}$	$x = \frac{3}{3} = \boxed{1}$

$Solution\ set = \{(-5,-8),(1,1)\}$

Q.3 $x - y = 7$

$\frac{2}{x} - \frac{5}{y} = 2$

Solution: $x - y = 7$ .....(i)

$\frac{2}{x} - \frac{5}{y} = 2$ .....(ii)

Multiply eq. (ii) by "xy"

$2y - 5x = 2xy$ .....(iii)

From eq. (i)

$x = 7 + y$

Put it in eq. (iii)

$2y - 5(7+y) = 2(7+y)y$

$2y - 35 - 5y = 14y + 2y^2$

$y^2 + 17y + 35 = 0$

$2y^2 + 10y + 7y + 35 = 0$

$2y(y+5) + 7(y+5) = 0$

$(y+5)(2y+7) = 0$

$y + 5 = 0$ or $2y + 7 = 0$

$y = -5$ or $2y = -7$

$y = -5$ or $y = \frac{-7}{2}$

Now putting values of y in eq. (i)

$y = -5$	$y = \frac{-7}{2}$
$x = 7 + y$	$x = 7 + y$
$x = 7 + (-5)$	$x = 7 + \left(\frac{-7}{2}\right)$
$x = 7 - 5$	$x = 7 - \frac{7}{2}$
$x = 2$	$x = \frac{14-7}{2}$
$\boxed{x = 2}$	$\boxed{x = \frac{7}{2}}$

$Solution\ set = \left\{(2,-5),\left(\frac{7}{2},-\frac{7}{2}\right)\right\}$

Q.4 $x + y = a - b$

$\frac{a}{x} - \frac{b}{y} = 2$

Solution: $x + y = a - b$ .....(i)

$\frac{a}{x} - \frac{b}{y} = 2$ .....(ii)

Multiplying eq. (ii) by "xy"

$ay - bx = 2xy$ .....(iii)

From equation (i)

$x = a - b - y$.........(iii)

Put it in equation (ii)

$ay - bx = 2xy$

$$ay-b(a-b-y)=2(a-b-y)y$$
$$ay-ba+b^2+by=2a-2by-2y^2$$
$$2y^2+2ay+ay+2by+by+b^2-ab=0$$
$$2y^2-ay+3by+b^2-ab=0$$
$$2y^2-y(a-3b)+(b^2-ab)=0$$

By using quadratic formula

$$a=2,\ b=-(a-3b),\ c=(b^2-ab)$$
$$y=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$y=\frac{-[-(a-3b)]\pm\sqrt{[-(a-3b)]^2-4(2)(b^2-ab)}}{2(2)}$$
$$y=\frac{(a-3b)\pm\sqrt{a^2+9b^2-6ab-8b^2+8ab}}{4}$$
$$y=\frac{(a-3b)\pm\sqrt{a^2+b^2+2ab}}{4}$$
$$y=\frac{(a-3b)\pm\sqrt{(a+b)^2}}{4}$$
$$y=\frac{(a-3b)\pm(a+b)}{4}$$
$$y=\frac{a-3b-a-b}{4}\quad\text{or}\quad y=\frac{a-3b+a+b}{4}$$
$$y=\frac{-4b}{4}\quad\text{or}\quad y=\frac{2a-2b}{4}$$
$$y=-b\quad\text{or}\quad y=\frac{2(a-b)}{4}$$
$$y=-b\quad\text{or}\quad y=\frac{a-b}{2}$$

Putting the values of y in eq. (iii)

$y=-b$	$y=\frac{a-b}{2}$
$x=a-b-y$	$x=a-b-y$
$x=a-b-(-b)$	$x=a-b-\left(\frac{a-b}{2}\right)$
$x=a-b+b$	$x=\frac{2a-2b-a+b}{2}$
$\boxed{x=a}$	$\boxed{x=\frac{a-b}{2}}$

Solution set is $\left\{(a,-b),\left(\frac{a-b}{2},\frac{a-b}{2}\right)\right\}$

Q.5 $x^2+(y-1)^2=10$

$x^2+y^2+4x=1$

Solution: $x^2+(y-1)^2=10$ .....(i)

$x^2+y^2+4x=1$ .....(ii)

Subtracting eq. (ii) from (i)

$$x^2+y^2+1-2y=10$$
$$\pm x^2\pm y^2\qquad \pm 4x=1$$
$$1-2y-4x=9$$
$$-4x-2y=9-1$$
$$-4x-2y=8$$
$$-2(2x+y)=8$$
$$2x+y=\frac{8}{-2}$$
$$2x+y=-4$$

$y=-4-2x$ .....(iii)

Put in eq. (ii)

$$x^2+(-4-2x)^2+4x=1$$
$$x^2+[-(4+2x)]^2+4x=1$$
$$x^2+[16+4x^2+16x]+4x=1$$
$$5x^2+20x+16-1=0$$
$$5x^2+20x+15=0$$
$$5(x^2+4x+3)=0$$
$$x^2+4x+3=0\quad(\because 5\neq 0)$$
$$x^2+3x+x+3=0$$
$$x(x+3)+1(x+3)=0$$
$$(x+3)(x+1)=0$$
$$x+3=0\quad\text{or}\quad x+1=0$$
$$x=-3\quad\text{or}\quad x=-1$$

Putting the values of x in eq. (iii)

$x=-3$	$x=-1$
$y=-4-2x$	$y=-4-2x$
$y=-4-2(-3)$	$y=-4-2(-1)$
$y=-4+6$	$y=-4+2$
$\boxed{y=2}$	$\boxed{y=-2}$

Solution set is $\{(-3,2),(-1,-2)\}$

Q.6 $(x+1)^2+(y+1)^2=5$

$(x+2)^2+y^2=5$

Solution: $(x+1)^2+(y+1)^2=5$ .....(i)

$(x+2)^2+y^2=5$ .....(ii)

From eq. (i)

$$x^2+1+2x+y^2+1+2y=5$$
$$x^2+y^2+2x+2y+2=5$$
$$x^2+y^2+2x+2y=5-2$$
$$x^2+y^2+2x+2y=3 \quad .....(iii)$$

From eq. (ii)

$$(x+2)^2+y^2=5$$
$$x^2+4+4x+y^2=5$$
$$x^2+y^2+4x=5-4$$
$$x^2+y^2+4x=1 \quad .....(iv)$$

Subtracting eq. (iv) from (iii)

$$x^2+y^2+2x+2y=3$$
$$\pm x^2 \pm y^2 \pm 4x \quad = \pm 1$$
$$-2x+2y=2$$
$$-2(x-y)=2$$
$$x-y=\frac{2}{-2}$$
$$x-y=-1$$
$$x=y-1 \quad .....(v)$$

Put it in eq. (iv)

$$(y-1)^2+y^2+4(y-1)=1$$
$$y^2+1-2y+y^2+4y-4=1$$
$$2y^2+2y-4+1-1=0$$
$$2y^2+2y-4=0$$
$$2(y^2+y-2)=0$$
$$y^2+y-2=0 \quad (\because 2\neq 0)$$
$$y^2+y-2=0$$
$$y^2+2y-y-2=0$$
$$y(y+2)-1(y+2)=0$$
$$(y+2)(y-1)=0$$

$y+2=0$ or $y-1=0$

$y=-2$ or $y=1$

Putting the values of y in eq. (v)

$y=-2$	$y=1$
$x=y-1$	$x=y-1$
$x=-2-1$	$x=1-1$
$\boxed{x=-3}$	$\boxed{x=0}$

Solution set is $\{(-3,-2),(0,1)\}$

Q.7 $x^2+2y^2=22$

$5x^2+y^2=29$

Solution: $x^2+2y^2=22$ .....(i)

$5x^2+y^2=29$ .....(ii)

Multiplying eq. (ii) by '2'

$$10x^2+2y^2=58 \quad .....(iii)$$

Subtracting eq. (i) from eq. (iii)

$$10x^2+2y^2=58$$
$$\pm \quad x^2+2y^2=22$$
$$9x^2 \quad =36$$

$$x^2=\frac{36}{9}$$
$$x^2=4$$

Taking square root on both sides

$$\sqrt{x^2}=\sqrt{4}$$
$$x=\pm 2$$
$$\Rightarrow x=-2 \text{ or } x=2$$

Putting the values of x in eq. (i)

$x=-2$	$x=2$
$x^2+2y^2=22$	$x^2+2y^2=22$
$(-2)^2+2y^2=22$	$(2)^2+2y^2=22$
$4+2y^2=22$	$4+2y^2=22$
$2y^2=18$	$2y^2=18$
$y^2=\frac{18}{2}$	$y^2=\frac{18}{2}$
$y^2=9$	$y^2=9$
$\boxed{y=\pm 3}$	$\boxed{y=\pm 3}$

Solution set is $\{(\pm 2, \pm 3)\}$

Q.8 $4x^2 - 5y^2 = 6$

$3x^2 + y^2 = 14$

Solution: $4x^2 - 5y^2 = 6$ .....(i)

$3x^2 + y^2 = 14$ .....(ii)

Multiplying eq. (ii) by '5'

$15x^2 + 5y^2 = 70$ .....(iii)

Adding equation (i) and (iii)

$$4x^2 - 5y^2 = 6$$
$$15x^2 + 5y^2 = 70$$
$$19x^2 = 76$$

$x^2 = \frac{76}{19}$

$x^2 = 4$

Taking square root on both sides

$\sqrt{x^2} = \sqrt{4}$

$x = \pm 2$

$\Rightarrow x = -2$ or $x = 2$

Putting the values of x in eq. (ii)

$x = -2$	$x = 2$
$3x^2 + y^2 = 14$	$3x^2 + y^2 = 14$
$3(-2)^2 + y^2 = 14$	$3(2)^2 + y^2 = 14$
$3(4) + y^2 = 14$	$3(4) + y^2 = 14$
$12 + y^2 = 14$	$12 + y^2 = 14$
$y^2 = 14 - 12$	$y^2 = 14 - 12$
$y^2 = 2$	$y^2 = 2$
$\boxed{y = \pm\sqrt{2}}$	$\boxed{y = \pm\sqrt{2}}$

Solution set is $\{(\pm 2, \pm\sqrt{2})\}$

Q.9 $7x^2 - 3y^2 = 4$

$2x^2 + 5y^2 = 7$

Solution: $7x^2 - 3y^2 = 4$ .....(i)

$2x^2 + 5y^2 = 7$ .....(ii)

Multiply eq. (i) by 5

$35x^2 - 15y^2 = 20$ .....(iii)

Adding equation (ii) and (iii)

$$35x^2 - 15y^2 = 20$$
$$6x^2 + 15y^2 = 21$$
$$41x^2 = 41$$

$x^2 = \frac{41}{41}$

$x^2 = 1$

$x = \pm\sqrt{1}$

$x = \pm 1$

$x = 1$ or $x = -1$

Putting the values of x in eq. (i)

$x = 1$	$x = -1$
$7x^2 - 3y^2 = 4$	$7x^2 - 3y^2 = 4$
$7(1)^2 - 3y^2 = 4$	$7(-1)^2 - 3y^2 = 4$
$7(1) - 3y^2 = 4$	$7(1) - 3y^2 = 4$
$7 - 3y^2 = 4$	$7 - 3y^2 = 4$
$-3y^2 = 4 - 7$	$-3y^2 = 4 - 7$
$-3y^2 = -3$	$-3y^2 = -3$
$y^2 = \frac{-3}{-3}$	$y^2 = \frac{-3}{-3}$
$y^2 = 1$	$y^2 = 1$
$y = \pm\sqrt{1}$	$y = \pm\sqrt{1}$
$\boxed{y = \pm 1}$	$\boxed{y = \pm 1}$

Solution set is $\{(\pm 1, \pm 1)\}$

Q.10 $x^2 + 2y^2 = 3$

$x^2 + 4xy - 5y^2 = 0$

Solution: $x^2 + 2y^2 = 3$ .....(i)

$x^2 + 4xy - 5y^2 = 0$ .....(ii)

Factorizing eq. (ii)

$x^2 + 4xy - 5y^2 = 0$

$x^2 + 5xy - xy - 5y^2 = 0$

$x(x + 5y) - y(x + 5y) = 0$

$(x + 5y)(x - y) = 0$

$x+5y=0 \qquad x-y=0$

$x=-5y \quad .....(iii) \qquad x=y \quad .....(iv)$

Putting value of x in eq. (i)

$x=-5y \qquad x=y$

$(-5y)^2+2y^2=3 \qquad (y)^2+2y^2=3$

$25y^2+2y^2=3 \qquad y^2+2y^2=3$

$27y^2=3 \qquad 3y^2=3$

$y^2=\frac{3}{27} \qquad y^2=\frac{3}{3}$

$y^2=\frac{1}{9} \qquad y^2=1$

$y=\pm\sqrt{\frac{1}{9}} \qquad y=\pm 1$

$y=\pm\frac{1}{3} \qquad y=1 \text{ or } y=-1$

$y=\frac{1}{3} \text{ or } y=\frac{-1}{3}$

Putting values of $y=\pm\frac{1}{3}$ in eq. (iii)

$y=\frac{1}{3} \qquad y=\frac{-1}{3}$

$x=-5y \qquad x=-5y$

$x=-5\left(\frac{1}{3}\right) \qquad x=-5\left(\frac{-1}{3}\right)$

$\boxed{x=\frac{-5}{3}} \qquad \boxed{x=\frac{5}{3}}$

Now putting values of $y=\pm 1$ in eq. (iv)

$y=1 \qquad y=-1$

$x=y \qquad x=y$

$\boxed{x=1} \qquad \boxed{x=-1}$

Solution set is $\left\{(-1,-1),(1,1),\left(\frac{5}{3},\frac{-1}{3}\right),\left(\frac{-5}{3},\frac{1}{3}\right)\right\}$

Q.11 $3x^2-y^2=26$

$3x^2-5xy-12y^2=0$

Solution: $3x^2-y^2=26 \quad .....(i)$

$3x^2-5xy-12y^2=0 \quad .....(ii)$

Factorizing equation (ii)

$3x^2-5xy-12y^2=0$

$3x^2-9xy+4xy-12y^2=0$

$3x(x-3y)+4y(x-3y)=0$

$(x-3y)(3x+4y)=0$

$x-3y=0 \qquad 3x+4y=0$

$x=0+3y \qquad 3x=-4y$

$x=3y \quad .....(iii) \qquad x=\frac{-4y}{3} \quad .....(iv)$

Putting value of x in eq. (i) from eq. (iii)

$3(3y)^2-y^2=26$

$3(9y)^2-y^2=26$

$27y^2-y^2=26$

$26y^2=26$

$y^2=\frac{26}{26}$

$y^2=1$

$y=\pm 1$

$y=1 \quad \text{or} \quad y=-1$

Putting the value of y in eq. (iii)

$y=1 \qquad y=-1$

$x=3y \qquad x=3y$

$x=3(1) \qquad x=3(-1)$

$x=3 \qquad x=-3$

$(x,y)=(3,1) \qquad (x,y)=(-3,-1)$

Putting the value of x in eq. (iv) from eq. (i)

$3\left(\frac{-4y}{3}\right)^2-y^2=26$

$3\times\frac{16y^2}{9}-y^2=26$

$\frac{48y^2-9y^2}{9}=26$

$39y^2=26\times 9$

$y^2=\frac{234}{39}$

$y^2=6$

$y = \pm\sqrt{6}$

$y = \sqrt{6}$ or $y = -\sqrt{6}$

Putting the value of y in eq. (iv)

$y = \sqrt{6}$

$x = \frac{-4y}{3}$

$x = \frac{-4\sqrt{6}}{3}$

$(x, y) = \left(\frac{-4\sqrt{6}}{3}, \sqrt{6}\right)$

$y = -\sqrt{6}$

$x = \frac{-4y}{3}$

$x = \frac{-4(-\sqrt{6})}{3}$

$x = \frac{4\sqrt{6}}{3}$

$\boxed{(x, y) = \left(\frac{-4\sqrt{6}}{3}, \sqrt{6}\right)}$ $\boxed{(x, y) = \left(\frac{4\sqrt{6}}{3}, -\sqrt{6}\right)}$

Solution set is

$$\left\{(3,1), (-3,-1), \left(\frac{-4\sqrt{6}}{3}, \sqrt{6}\right), \left(\frac{4\sqrt{6}}{3}, -\sqrt{6}\right)\right\}$$

Q.12 $x^2 + xy = 5$

$y^2 + xy = 3$

Solution: $x^2 + xy = 5$ .....(i)

$y^2 + xy = 3$ .....(ii)

Multiply eq. (i) by '3' and eq. (ii) by '5'

$3x^2 + 3xy = 15$ .....(iii)

$5y^2 + 5xy = 15$ .....(iv)

Subtracting eq. (iv) from eq. (iii)

$$\begin{array}{l} 3x^2 + 3xy \quad\quad = 15 \\ \quad\quad \pm 5xy \pm 5y^2 = -15 \\ \hline 3x^2 - 2xy - 5y^2 = 0 \end{array}$$

$3x^2 - 5xy + 3xy - 5y^2 = 0$

$x(3x - 5y) + y(3x - 5y) = 0$

$(3x - 5y)(x + y) = 0$

$3x - 5y = 0$ $x + y = 0$

$3x = 5y$ $x = -y$ .....(vi)

$x = \frac{5y}{3}$ .....(v)

From eq. (vi) put y=-x in eq. (i)

$x^2 + xy = 5$

$(-y)^2 + (-y)y = 5$

$y^2 - y^2 = 5$

$0 \neq 5$ impossible

Now from eq. (v) put $x = \frac{5y}{3}$ in eq. (i)

$\left(x = \frac{5y}{3}\right)^2 + \frac{5y}{3} \times y = 5$

$\frac{25y^2}{9} + \frac{5y^2}{3} = 5$

Multiplying both sides by 9

$9 \times \frac{25y^2}{9} + 9 \times \frac{5y^2}{3} = 9 \times 5$

$25y^2 + 15y^2 = 45$

$40y^2 = 45$

$y^2 = \frac{45}{40} \Rightarrow y^2 = \frac{9}{8}$

$y = \pm\sqrt{\frac{9}{8}} \Rightarrow y = \sqrt{\frac{3^2}{4 \times 2}}$

$y = \pm\frac{3}{2\sqrt{2}}$

$y = \frac{3}{2\sqrt{2}}$ or $y = \frac{-3}{2\sqrt{2}}$

Now putting the value of y in eq. (v)

$x = \frac{5y}{3}$

$y = \frac{3}{2\sqrt{2}}$

$x = \frac{5}{3} \times \frac{3}{2\sqrt{2}}$

$x = \frac{5}{2\sqrt{2}}$

$x = \frac{5y}{3}$

$y = \frac{-3}{2\sqrt{2}}$

$x = \frac{5}{3} \times \left(\frac{-3}{2\sqrt{2}}\right)$

$x = \frac{-5}{2\sqrt{2}}$

$\boxed{x = \left(\frac{5}{2\sqrt{2}}, \frac{3}{2\sqrt{2}}\right)}$ $\boxed{x = \left(\frac{-5}{2\sqrt{2}}, \frac{-3}{2\sqrt{2}}\right)}$

Solution set is $\left\{\left(\frac{5}{2\sqrt{2}}, \frac{3}{2\sqrt{2}}\right), \left(\frac{-5}{2\sqrt{2}}, \frac{-3}{2\sqrt{2}}\right)\right\}$

Q.13 $x^2 - 2xy = 7$

$xy + 3y^2 = 2$

Solution: $x^2 - 2xy = 7$ .....(i)

$xy + 3y^2 = 2$ .....(ii)

Multiplying eq. (i) by 2 and eq. (ii) by 7

$2x^2 - 4xy = 14$ .....(iii)

$7xy + 21y^2 = 14$ .....(iv)

Subtract eq. (iv) from eq. (iii)

$$\begin{array}{l} 2x^2 - 4xy \qquad = 14 \\ \quad \pm 7xy \pm 21y^2 = -14 \\ \hline 2x^2 - 11xy - 21y^2 = 0 \end{array}$$

$2x^2 - 14xy + 3xy - 21y^2 = 0$

$2x(x-7y) + 3y(x-7y) = 0$

$(x-7y)(2x+3y) = 0$

$x - 7y = 0$ $\quad$ $2x + 3y = 0$

$x = 7y$ $\quad$ $2x = -3y$

$x = 7y$ ...(v) $\quad$ $x = \frac{-3y}{2}$ ...(vi)

Put x=7y in eq. (i)

$(7y)^2 - 2(7y)y = 7$

$49y^2 - 14y^2 = 7$

$35y^2 = 7 \quad \Rightarrow y^2 = \frac{7}{35}$

$y^2 = \frac{1}{5} \quad \Rightarrow y = \pm\frac{1}{\sqrt{5}}$

$y = \frac{1}{\sqrt{5}} \quad y = -\frac{1}{\sqrt{5}}$

Putting values of y in eq. (v)

$x = 7y$ $\quad$ $x = 7y$

$y = \frac{1}{\sqrt{5}}$ $\quad$ $y = -\frac{1}{\sqrt{5}}$

$x = 7\left(\frac{1}{\sqrt{5}}\right)$ $\quad$ $x = 7\left(-\frac{1}{\sqrt{5}}\right)$

$x = \frac{7}{\sqrt{5}}$ $\quad$ $x = \frac{-7}{\sqrt{5}}$

$(x,y) = \left(\frac{7}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right)$ $\quad$ $(x,y) = \left(\frac{-7}{\sqrt{5}}, \frac{-1}{\sqrt{5}}\right)$

From eq. (vi) put value of x in eq. (i)

$\left(\frac{-3}{2}y\right)^2 - 2\left(\frac{-3}{2}y\right)y = 7$

$\frac{9}{4}y^2 + 3y^2 = 7$

$9y^2 + 12y^2 = 28$

$21y^2 = 28$

$y^2 = \frac{28}{21} \quad y^2 = \frac{4}{3}$

$\sqrt{y^2} = \sqrt{\frac{4}{3}} \quad y = \pm\frac{2}{\sqrt{3}}$

$y = \frac{2}{\sqrt{3}} \quad y = -\frac{2}{\sqrt{3}}$

Putting values of y in eq. (vi)

$y = \frac{2}{\sqrt{3}}$ $\quad$ $y = -\frac{2}{\sqrt{3}}$

$x = \frac{-3}{2}\left(\frac{2}{\sqrt{3}}\right)$ $\quad$ $x = \frac{-3}{2}\left(\frac{-2}{\sqrt{3}}\right)$

$x = -\sqrt{3}$ $\quad$ $x = \sqrt{3}$

$\boxed{(x,y) = \left(-\sqrt{3}, \frac{2}{\sqrt{3}}\right)}$ $\quad$ $\boxed{(x,y) = \left(\sqrt{3}, \frac{-2}{\sqrt{3}}\right)}$

Solution set is

$$\left\{\left(\frac{7}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right), \left(\frac{-7}{\sqrt{5}}, \frac{-1}{\sqrt{5}}\right), \left(-\sqrt{3}, \frac{2}{\sqrt{3}}\right), \left(\sqrt{3}, \frac{-2}{\sqrt{3}}\right)\right\}$$

Exercise 2.8

Q.1 The product of two positive consecutive numbers is 182. Find the numbers.

Solution:

Suppose first positive number=x

Second positive number=x+1

According to given condition:

$x(x+1) = 182$

$x^2 + x - 182 = 0$

$x^2 + 14x - 13x - 182 = 0$

$x(x+14) - 13(x+14) = 0$

$(x+14)(x-13) = 0$

$x + 14 = 0 \quad \text{or} \quad x - 13 = 0$

$x = -14 \quad \text{or} \quad x = -13$

As x is positive number therefore we neglect the negative value, So x=13

Then first positive number=x=13

Second positive number=x+1

=13+1=14

So, 13 and 14 are two required consecutive positive numbers.

Q.2 the sum of the square of three positive consecutive numbersis77.Find them.

Solution:

Let x, (x+1) and (x+2) be the three consecutive positive number

According to given condition:

$$x^2+(x+1)^2+(x+2)^2=77$$
$$x^2+\left[x^2+(1)^2+2(1)(x)\right]+\left[(x)^2+(2)^2+2(x)(2)\right]=77$$
$$x^2+x^2+1+2x+x^2+4+4x=77$$
$$3x^2+6x+5-77=0$$
$$3x^2+6x-72=0$$
$$3\left[x^2+2x-24\right]=0$$
$$x^2+2x-24=0 \quad \because 3\neq 0$$
$$x^2+6x-4x-24=0$$
$$x(x+6)-4(x+6)=0$$
$$(x+6)(x-4)=0$$
$$x+6=0 \quad \text{or} \quad x-4=0$$
$$x=-6 \quad \text{or} \quad x=4$$

As x is a positive number therefore we neglect the negative value and we take positive value of x like x=4

First positive number=x=4

Second positive number=x+1=4+1=5

Third positive number=x+2=4+2=6

So, 4, 5 and 6 are three required positive numbers.

Q.3 The sum of five times a number and the square of the number is 204. Find the number.

Solution: Let required number=x

Five times the number=5x

Square of number= x^2

According to given condition:

$$x^2+5x=204 \Rightarrow x^2+5x-204=0$$
$$x^2+17x-12x-204=0$$
$$x(x+17)-12(x+17)=0$$
$$(x+17)(x-12)=0$$
$$x+17=0 \quad \text{or} \quad x-12=0$$
$$\boxed{x=-17} \quad \text{or} \quad \boxed{x=12}$$

So, required number is -17 and 12.

Q.4 The product of five less than three times a certain number and one less than four times the number is 7. Find the number.

Solution: Let required number=x

Five less than three times the number=3x-5

One less than four times the number=4x-1

According to given condition:

$$(3x-5)(4x-1)=7$$
$$12x^2-3x-20x+5-7=0$$
$$12x^2-23x-2=0$$
$$12x^2-24x+x-2=0$$
$$12x(x-2)+1(x-2)=0$$
$$(x-2)(12x+1)=0$$
$$x-2=0 \quad \text{or} \quad 12x+1=0$$
$$\boxed{x=2} \quad \text{or} \quad 12x=-1$$
$$\boxed{x=\frac{-1}{12}}$$

So, required number is 2 or $\frac{-1}{12}$

Q.5 The difference of a number and its reciprocal is $\frac{15}{4}$. Find the number.

Solution: Let required number=x

Reciprocal of the number= $\frac{1}{x}$

According to given condition:

$$x-\frac{1}{x}=\frac{15}{4}$$
$$\frac{x^2-1}{x}=\frac{15}{4}$$
$$4(x^2-1)=15x$$
$$4x^2-4-15x=0$$
$$4x^2-15x-4=0$$
$$4x^2-16x+1x-4=0$$
$$4x(x-4)+1(x-4)=0$$
$$(x-4)(4x+1)=0$$
$$x-4=0 \quad \text{or} \quad 4x+1=0$$
$$x=4 \quad \text{or} \quad 4x=-1$$
$$\boxed{x=4} \quad \text{or} \quad \boxed{x=\frac{-1}{4}}$$

So, required number is 4 or $\frac{-1}{4}$

Q.6 The sum of squares of two digits of a positive integral number is 65 and the number is 9 times the sum of its digits. Find the number.

Solution: Let

Digits at unit's place of a number=x

Digits at ten's place of a number=y

Required number=10y+x

According to first condition:

$$x^2+y^2=65 \quad .....(i)$$

According to second condition:

$$10y+x=9(x+y)$$
$$10y+x=9x+9y$$
$$10y-9y=9x-x$$
$$y=8x \quad .....(ii)$$

Put value of y in eq. (i)

$$x^2+(8x)^2=65$$
$$x^2+64x^2=65$$
$$65x^2=65$$
$$x^2=1$$
$$\sqrt{x^2}=\pm\sqrt{1}$$
$$x=\pm1$$
$$x=1 \quad \text{or} \quad x=-1$$

As x is a digit at unit's place which is always positive therefore we neglect the negative value and take the positive value like x=1

Put x=1 in eq. (i)

$$y=8(1)$$
$$y=8$$

So, required number $=10y+x$
$=10(8)+1$
$=80+1=\boxed{80}$

Q .7 The sum of the co-ordinates of a point is 9 and sum of their squares is 45. Find the co-ordinates of the point.

Solution: Let (x, y) are co-ordinates of required point.

According to given condition:

$$x+y=9 \quad .....(i)$$
$$x^2+y^2=45 \quad .....(ii)$$

From equation (i)

$$x+y=9$$
$$x=9-y \quad .....(iii)$$

Putting this in eq. (ii), we get

$$(9-y)^2+y^2=45$$
$$(9)^2-2(9)(y)+(y)^2+(y)^2=45$$
$$81-18y+y^2+y^2=45$$
$$2y^2-18y+81-45=0$$
$$2y^2-18y+36=0$$
$$2(y^2-9y+8)=0$$
$$\therefore y^2-9y^2+18=0 \qquad \because 2\neq 0$$
$$y^2-6y-3y+18=0$$
$$y(y-6)-3(y-6)=0$$
$$(y-6)(y-3)=0$$
$$y-6=0 \quad \text{or} \quad y-3=0$$
$$y=6 \quad \text{or} \quad y=3$$

Putting values of y in eq. (iii)

$$y=6 \quad \text{or} \quad y=3$$
$$x=9-6 \quad \text{or} \quad x=9-6$$
$$\boxed{x=3} \quad \text{or} \quad \boxed{x=6}$$

The co-ordinates of the point are either (3,6) or (6,3).

Q.8 Find two integers whose sum is 9 and the difference of their squares is also 9.

Solution: Suppose x and y are two integers

According to given condition:

$$x+y=9 \quad .....(i)$$
$$x^2-y^2=9 \quad .....(ii)$$

From eq. (i)

$$x+y=9$$
$$x=9-y \quad .....(iii)$$

Putting value of x in eq. (ii)

$$(9-y)^2-y^2=9$$
$$(9)^2+(y)^2-2(9)(y)-y^2=0$$
$$81+y^2-18y-y^2-9=0$$
$$72-18y=0$$
$$-18y=-72$$
$$y=\frac{-72}{-18} \Rightarrow \boxed{y=4}$$

Putting the value of y in eq. (iii)

$$x=9-y$$
$$x=9-4 \Rightarrow \boxed{x=5}$$

So, 4 and 5 are required integers.

Q.9 Find two integers whose difference is 4 and whose squares differ by 72.

Solution: Let x and y are two integers

According to given condition:

$x - y = 4$ .....(i)

$x^2 - y^2 = 72$ .....(ii)

From eq. (i)

$x = 4 + y$ .....(iii)

Putting the value of x in eq. (ii)

$(4+y)^2 - y^2 = 72$

$\left[(4)^2 + (y)^2 + 2(4)(y)\right] - y^2 = 72$

$16 + y^2 + 8y - y^2 = 72$

$16 + 8y = 72$

$8y = 72 - 16$

$8y = 56$

$y = \frac{56}{8} \Rightarrow \boxed{y = 7}$

Putting the value of y in eq. (iii)

$x = 4 + y$

$x = 4 + 7 \Rightarrow \boxed{x = 11}$

So, required integers are 7 and 11.

Q.10 Find the dimensions of a rectangle, whose perimeter is 80cm and its area is 375 cm^2

Solution:

Let width of a rectangle=x cm

Length of rectangle =y cm

Perimeter of rectangle=80cm

Area of rectangle=375 cm^2

$\because$ 2(L+W) =P

2(x+ y) =80

x+ y= $\frac{80}{2}$

x+ y=40 .....(i)

Area=Length $\times$ Width

375= $x \times y$

xy =375 .....(ii)

From eq. (i)

$x + y = 40$

$x = 40 - x$

Put it in eq. (ii)

$x(40 - x) = 375$

$40x - x^2 = 375$

$0 = x^2 - 40x + 375$

$x^2 - 40x + 375 = 0$

$x^2 - 25x - 15x + 375 = 0$

$x(x-25) - 15(x-25) = 0$

$(x-15)(x-25) = 0$

$x - 15 = 0$ or $x - 25 = 0$

$x = 15$ or $x = 25$

Putting the value of x in eq. (i)

$x = 15$ or $x = 25$

$15 + y = 40$ or $25 + y = 40$

$y = 40 - 15$ or $y = 40 - 25$

$\boxed{y = 25}$ or $\boxed{y = 15}$

If x=15 then y=25 and x=25 then y=15

So, dimensions of rectangle are either 25cm by 15cm or 15cm by 25cm.