10TH CLASS MATH

CHAPTER 1

SOLUTION NOTES

Exercise 1.1

Question No 1:
Write the following quadratic equations in the standard Form and point out pure quadratic equation

$(x+7)(x-3)=-7$

$Solution:(x+7)(x-3)=-7$

$x^2-3x+7x-21=-7$

$x^2+4x-21+7=0$

$x^2+4x-14=0$

The standard form of Quadratic equation is:

$x^2+4x-14=0$

(ii)

$\frac{x^2+4}{3}-\frac{x}{7}=1$

$Solution:\frac{x^2+4}{3}-\frac{x}{7}=1$

$\frac{7(x^2+4)-3x}{21}=1$

$7x^2+28-3x=21$

$7x^2-3x+28-21=0$

$7x^2-3x+7=0$

The standard form of Quadratic equation is:

$$7x^2 3x+7=0$$

(iii)

$\frac{x}{x+1}+\frac{x+1}{x}=6$

$Solution:\frac{x}{x+1}+\frac{x+1}{x}=6$

$\frac{x^2+(x+1)^2}{x(x+1)}=6$

$x^2+x^2+1+2x=6x(x+1)$

$2x^2+2x+1=6x^2+6x$

$0=6x^2+6x-2x^2-2x-1$

$0=4x^2+4x-1$

$4x^2+4x-1=0$

The standard form of Quadratic equation is

$$4x^2+4x-1=0$$

iv)

$\left(\frac{x+4}{x-2}\right)-\left(\frac{x-2}{x}\right)+4=0$

$Solution:\left(\frac{x+4}{x-2}\right)-\left(\frac{x-2}{x}\right)+4=0$

$\frac{(x+4)x-(x-2)^2+4x(x-2)}{(x-2)(x)}=0$

$(x+4)x-\left[(x)^2+(2)^2-2(x)(2)\right]+4x^2-8x=0(x-2)(x)$

$x^2+4x-x^2-4+4x+4x^2-8x=0$

$4x^2+4x+4x-8x-4=0$

$4x^2-4=0$

$4(x^2-1)=0$

$x^2-1=0$

$So, x^2-1=0\, is\ pure\ Quadratic\ equation$

(v) $\frac{x+3}{x+4}-\frac{x-5}{x}=1$

$Solution:\frac{x+3}{x+4}-\frac{x-5}{x}=1$

$\frac{x(x+3)-(x+4)(x-5)}{x(x+4)}=1$

$x^2+3x-(x^2-5x+4x-20)=1x(x+4)$

$x^2+3x-(x^2-1x-20)=x^2+4x$

$x^2+3x-x^2+1x+20=x^2+4x$

$3x+1x+20=x^2+4x$

$4x+20=x^2+4x$

$x^2+4x-4x-20=0$

$x^2-20=0$

$x^2-20=0\ is\ pure\ quadratic\ equation.$

(vi)

$$\frac{x+1}{x+2}+\frac{x+2}{x+3}=\frac{25}{12}$$

$$Solution:\frac{x+1}{x+2}+\frac{x+2}{x+3}=\frac{25}{12}$$

$$\frac{(x+1)(x+3)+(x+2)(x+2)}{(x+2)(x+3)}=\frac{25}{12}$$

$$\frac{(x+3x+1x+3)+(x^2+2x+2x+4)}{x^2+3x+2x+6}=\frac{25}{12}$$

$$\frac{(x^2+4x+3)+(x^2+4x+4)}{x^2+3x+2x+6}=\frac{25}{12}$$

$$\frac{2x^2+8x+7}{x^2+5x+6}=\frac{25}{12}$$

$$12(2x^2+8x+7)=25(x^2+5x+6)$$

$$24x^2+96x+84=25x^2+125x+150$$

$$25x^2-24x^2+125x-96x+150-84=0$$

$$x^2+29x+66=0$$

The standard form of Quadratic equation is:
$x^2+29x+66=0$

Question no. 2 Solve by factorization

$(i)\ x^2-x-20=0$

$x^2-5x+4x-20=0$

$x(x-5)+4(x-5)=0$

$(x-5)(x+4)=0$

$x-5=0 \quad , \quad x+4=0$

$x=5 \quad , \quad x=-4$

$Solution\ set\ is\ \{-4,5\}$

$(ii)\ 3y^2=y(y-5)$

$Solution:\ 3y^2=y^2-5y$

$3y^2-y^2+5y=0$

$2y^2+5y=0$

$y(2y+5)=0$

$y=0 \quad 2y+5=0$

$y=0 \quad 2y=-5$

$y=0 \quad y=\frac{-5}{2}$

$Solution\ set\ is\left\{0,\frac{-5}{2}\right\}$

$(iii)\ 4-32x=-17x^2$

$Solution: 4-32x=-17x^2$

$17x^2+32x-4=0$

$17x^2+34x-2x-4=0$

$17x(x+2)-2(x+2)=0$

$(17x-2)(x+2)=0$

$17x-2=0 \quad x+2=0$

$17x=2 \quad x=-2$

$x=\frac{2}{17} \quad x=-2$

$Solution\ set\ is\left\{-2,\frac{2}{17}\right\}$

$(iv)\ x^2-11x=152$

$Solution:\ x^2-11x=152$

$x^2-19x+8x-152=0$

$x(x-19)+8(x-19)=0$

$x-19=0 \quad x+8=0$

$x=19 \quad x=-8$

$Solution\ set\ is\ \{19,-8\}$

$$(v)\ \frac{x+1}{x}+\frac{x}{x+1}=\frac{25}{12}$$

$$Solution:\frac{x+1}{x}+\frac{x}{x+1}=\frac{25}{12}$$

$$\frac{(x+1)^2+x^2}{x(x+1)}=\frac{25}{12}$$

$$\frac{x^2+1+2x+x^2}{x^2+x}=\frac{25}{12}$$

$12(2x^2+2x+1)=25(x^2+x)$

$24x^2+24x+12=25x^2+25x$

$0=25x^2+25x-24x^2-24x-12$

$0=x^2+x-12$

$x^2+4x-3x-12=0$

$x(x+4)-3(x+4)=0$

$(x+4)(x-3)=0$

$x+4=0 \quad x-3=0$

$x=-4 \quad x=3$

$Solution\ set\ is\ \{3,-4\}$

$$(vi)\ \frac{2}{x-9}=\frac{1}{x-3}-\frac{1}{x-4}$$

$$Solution:\frac{2}{x-9}=\frac{1}{x-3}-\frac{1}{x-4}$$

$$\frac{2}{x-9}=\frac{(x-4)-(x-3)}{(x-4)(x-3)}$$

$$\frac{2}{x-9}=\frac{x-4-x+3}{x^2-4x-3x+12}$$

$$\frac{2}{x-9}=\frac{-1}{x^2-7x+12}$$

$2(x^2-7x+12)=-1(x-9)$

$2x^2-14x+24=-x+9$

$2x^2-14x+x+24-9=0$

$2x^2-13x+15=0$

$2x^2-10x-3x+15=0$

$2x(x-5)-3(x-5)=0$

$(x-5)(2x-3)=0$

$x-5=0 \quad 2x-3=0$

$x=5 \quad x=\frac{3}{2}$

Solution set is $\left\{5,\frac{3}{2}\right\}$

Question no 3: Solve the following equations by using completing square

(i) $7x^2+2x-1=0$

Solution: $7x^2+2x-1=0$

Dividing each term by 7

$$\frac{7}{7}x^2+\frac{2}{7}x-\frac{1}{7}=\frac{0}{7}$$

$$x^2+\frac{2}{7}x-\frac{1}{7}=0 \qquad H\text{int}\left(\frac{2}{7}\times\frac{1}{2}=\frac{1}{7}\right)$$

$$(x)^2+2(x)\left(\frac{1}{7}\right)=\frac{1}{7}$$

Adding $\left(\frac{1}{7}\right)^2$ *on both sides*

$$(x)^2+2(x)\left(\frac{1}{7}\right)+\left(\frac{1}{7}\right)^2=\frac{1}{7}+\left(\frac{1}{7}\right)^2$$

$$\left(x+\frac{1}{7}\right)^2=\frac{1}{7}+\frac{1}{49}$$

$$\left(x+\frac{1}{7}\right)^2=\frac{7+1}{49}$$

$$\left(x+\frac{1}{7}\right)^2=\frac{8}{49}$$

Taking square root on both sides

$$\sqrt{\left(x+\frac{1}{7}\right)^2}=\pm\sqrt{\frac{8}{49}}$$

$$\sqrt{\left(x+\frac{1}{7}\right)^2}=\pm\sqrt{\frac{4\times 2}{49}}$$

$$x+\frac{1}{7}=\pm\frac{2\sqrt{2}}{7}$$

$$x=-\frac{1}{7}\pm\frac{2\sqrt{2}}{7}$$

$$x=\frac{-1\pm 2\sqrt{2}}{7}$$

$$S.S\left\{\frac{-1\pm 2\sqrt{2}}{7}\right\}$$

(ii) $ax^2+4x-a=0$

Solution: $ax^2+4x-a=0$

Dividing each term by a

$$\frac{ax^2}{a}+\frac{4x}{a}-\frac{a}{a}=0$$

$$x^2+\frac{4x}{a}-1=0 \qquad H\text{int}\left(\frac{4}{a}\times\frac{1}{2}=\frac{2}{a}\right)$$

$$(x)^2+2(x)\left(\frac{2}{a}\right)=1$$

Ading $\left(\frac{2}{a}\right)^2$ *on both sides*

$$(x)^2+2(x)\left(\frac{2}{a}\right)+\left(\frac{2}{a}\right)^2=1+\left(\frac{2}{a}\right)^2$$

$$\left(x+\frac{2}{a}\right)^2=1+\frac{4}{a^2}$$

$$\left(x+\frac{2}{a}\right)^2=\frac{a^2+4}{a^2}$$

Taking square root on both sides

$$\sqrt{\left(x+\frac{2}{a}\right)^2}=\sqrt{\frac{a^2+4}{a^2}}$$

$$x+\frac{2}{a}=\pm\frac{\sqrt{a^2+4}}{a^2}$$

$$x=-\frac{2}{a}\pm\frac{\sqrt{a^2+4}}{a^2}$$

$$x=\frac{-2\pm\sqrt{a^2+4}}{a^2}$$

$$S.S\left\{\frac{-2\pm\sqrt{a^2+4}}{a^2}\right\}$$

(iii) $11x^2-34x+5=0$

Solution: $11x^2-34x+5=0$

Dividing each term by 11

$$\frac{11}{11}x^2-\frac{34}{11}x+\frac{3}{11}=0$$

$$x^2-\frac{34}{11}x=-\frac{3}{11} \qquad H\text{int}\left(\frac{34}{11}\times\frac{1}{2}=\frac{17}{11}\right)$$

$$(x)^2-2(x)\left(\frac{17}{11}\right)=-\frac{3}{11}$$

$$\textit{Adding both sides}\left(\frac{17}{11}\right)^2$$

$$(x)^2-2(x)\left(\frac{17}{11}\right)+\left(\frac{17}{11}\right)^2=-\frac{3}{11}+\left(\frac{17}{11}\right)^2$$

$$\left(x-\frac{17}{11}\right)^2=-\frac{3}{11}+\frac{289}{121}$$

$$\left(x-\frac{17}{11}\right)^2=\frac{-33+289}{121}$$

$$\left(x-\frac{17}{11}\right)^2=\frac{256}{121}$$

Taking square root on both sides

$$\sqrt{\left(x-\frac{17}{11}\right)^2}=\sqrt{\frac{256}{121}}$$

$$x-\frac{17}{11}=\pm\frac{16}{11}$$

$$x=\frac{17}{11}\pm\frac{16}{11}$$

$$x=\frac{11\pm16}{11}$$

$$x=\frac{17-16}{11} \qquad x=\frac{17+16}{11}$$

$$x=\frac{1}{11} \qquad x=\frac{33}{11}$$

$$x=\frac{1}{11} \qquad x=3$$

$$S.S\left\{3,\frac{1}{11}\right\}$$

(iv) $lx^2+mx+n=0$

Solution: $lx^2+mx+n=0$

Dividing each term by l

$$\frac{lx^2}{l}+\frac{mx}{l}=-\frac{n}{l}$$

$$x^2+\frac{m}{i}x=-\frac{n}{l} \qquad \therefore\left(\frac{m}{i}\times\frac{1}{2}=\frac{m}{2l}\right)$$

$$\textit{Adding}\left(\frac{m}{2l}\right)^2\textit{ on both sides}$$

$$(x)^2+2(x)\left(\frac{m}{2l}\right)+\left(\frac{m}{2l}\right)^2=-\frac{n}{l}+\left(\frac{m}{2l}\right)^2$$

$$\left(x+\frac{m}{2l}\right)^2=-\frac{n}{l}+\frac{m^2}{4l^2}$$

$$\left(x+\frac{m}{2l}\right)^2=\frac{-4\ln+m^2}{4l^2}$$

Taking square root on both sides

$$\sqrt{\left(x+\frac{m}{2l}\right)^2}=\sqrt{\frac{-4l\,n+m^2}{4l^2}}$$

$$x+\frac{m}{2l}=\pm\frac{\sqrt{m^2-4l\,n}}{2l}$$

$$x=-\frac{m}{2l}\pm\frac{\sqrt{m^2-4l\,n}}{2l}$$

$$x=\frac{-m\pm\sqrt{m^2-4l\,n}}{2l}$$

$$S.S\left\{\frac{-m\pm\sqrt{m^2-4l\,n}}{2l}\right\}$$

(v) $3x^2+7x=0$

Solution: $3x^2+7x=0$

Dividing each term by 3

$$\frac{3x^2}{3}+\frac{7}{3}x=0 \qquad H\text{int}\left(\frac{7}{3}\times\frac{1}{2}=\frac{7}{6}\right)$$

$$(x)^2+2(x)\left(\frac{7}{6}\right)=0$$

$$\textit{Adding}\left(\frac{7}{6}\right)^2\textit{ on both sides}$$

$$(x)^2+2(x)\left(\frac{7}{6}\right)+\left(\frac{7}{6}\right)^2=\left(\frac{7}{6}\right)^2$$

$$\left(x+\frac{7}{6}\right)^2=\frac{49}{36}$$

Taking square root on both sides

$$\sqrt{\left(x+\frac{7}{6}\right)^2}=\sqrt{\frac{49}{36}}$$

$$\left(x+\frac{7}{6}\right)=\pm\frac{7}{6}$$

$$x=-\frac{7}{6}\pm\frac{7}{6}$$

$$x=\frac{-7\pm7}{6}$$

$$x=\frac{-7-7}{6} \qquad x=\frac{-7+7}{6}$$

$x = \frac{-14}{6}$ $\quad$ $x = \frac{0}{6}$

$x = -\frac{7}{3}$ $\quad$ $x = 0$

$s.s\left\{-\frac{7}{3}, 0\right\}$

$(vi)\ x^2 - 2x - 195 = 0$

$Solution: x^2 - 2x - 195 = 0$

$x^2 - 2x = 195 \qquad \therefore \left(2 \times \frac{1}{2} = 1\right)$

$(x)^2 - 2(x)(1) = 195$

Adding $(1)^2$ *on both sides*

$(x)^2 - 2(x)(1) + (1)^2 = 195 + (1)^2$

$(x-1)^2 = 195 + 1$

$(x-1)^2 = 196$

Taking square root on both sides

$\sqrt{(x-1)^2} = \sqrt{196}$

$x - 1 = \pm 14$

$x = 1 \pm 14$

$x = 1 - 14 \qquad x = 1 + 14$

$x = -13 \qquad x = 15$

$S.S\{-13, 15\}$

$(vii)\ -x^2 + \frac{15}{2} = \frac{7}{2}x$

$Solution: -x^2 + \frac{15}{2} = \frac{7}{2}x$

$\frac{15}{2} = x^2 + \frac{7}{2}x$

$x^2 + \frac{7}{2}x = \frac{15}{2} \qquad \therefore \left(\frac{7}{2} \times \frac{1}{2} = \frac{7}{4}\right)$

Adding $\left(\frac{7}{4}\right)^2$ *on both sides*

$(x)^2 + 2(x)\left(\frac{7}{4}\right) + \left(\frac{7}{4}\right)^2 = \frac{15}{2} + \left(\frac{7}{4}\right)^2$

$\left(x + \frac{7}{4}\right)^2 = \frac{15}{2} + \frac{49}{16}$

$\left(x + \frac{7}{4}\right)^2 = \frac{120 + 49}{16}$

$\left(x + \frac{7}{4}\right)^2 = \frac{169}{16}$

Taking square root on both sides

$\sqrt{\left(x + \frac{7}{4}\right)^2} = \sqrt{\frac{169}{16}}$

$x + \frac{7}{4} = \pm \frac{13}{14}$

$x = -\frac{7}{4} \pm \frac{13}{14}$

$x = \frac{-7 \pm 13}{4}$

$x = \frac{-7 - 13}{4} \qquad x = \frac{-7 + 13}{4}$

$x = \frac{-20}{4} \qquad x = \frac{6}{4}$

$x = -5 \qquad x = \frac{3}{2}$

$S.S\left\{-5, \frac{3}{2}\right\}$

$(viii)\ x^2 + 17x + \frac{33}{4} = 0$

$Solution: x^2 + 17x + \frac{33}{4} = 0$

$x^2 + 17x = -\frac{33}{4}$

$(x)^2 + 2(x)\left(\frac{17}{2}\right) = -\frac{33}{4}$

Adding $\left(\frac{17}{2}\right)^2$ *on both sides*

$(x)^2 + 2(x)\left(\frac{17}{2}\right) + \left(\frac{17}{2}\right)^2 = -\frac{33}{4} + \left(\frac{17}{2}\right)^2$

$\left(x + \frac{17}{2}\right)^2 = -\frac{33}{4} + \frac{289}{4}$

$\left(x + \frac{17}{2}\right)^2 = \frac{-33 + 289}{4}$

$\left(x + \frac{17}{2}\right)^2 = \frac{256}{4}$

Taking square root on both sides

$\sqrt{\left(x + \frac{17}{2}\right)^2} = \sqrt{\frac{256}{4}}$

$x+\frac{17}{2}=\pm\frac{16}{2}$

$x=-\frac{17}{2}\pm\frac{16}{2}$

$x=\frac{-17\pm16}{2}$

$x=\frac{-17-16}{2}$ $\quad$ $x=\frac{-17+16}{2}$

$x=-\frac{33}{2}$ $\quad$ $x=-\frac{1}{2}$

$S.S\left\{-\frac{33}{2},x=-\frac{1}{2}\right\}$

(ix) $4-\frac{8}{3x+1}=\frac{3x^2+5}{3x+1}$

Solution: $4-\frac{8}{3x+1}=\frac{3x^2+5}{3x+1}$

$4=\frac{8}{3x+1}+\frac{3x^2+5}{3x+1}$

$4=\frac{3x^2+5+8}{3x+1}$

$4(3x+1)=3x^2+5+8$

$12x+4=3x^2+13$

$3x^2+13-12x-4=0$

$3x^2-12x+9=0$

Dividing each term by 3

$\frac{3}{3}x^2-\frac{12}{3}x+\frac{9}{3}=\frac{0}{3}$

$x^2-4x+3=0$

$x^2-4x=-3$

$(x)^2-2(x)(2)=-3$

Adding $(2)^2$ *on both sides*

$(x)^2-2(x)(2)+(2)^2=-3+(2)^2$

$(x-2)^2=-3+4$

$(x-2)^2=1$

Taking square root on both sides

$\sqrt{(x-2)^2}=\pm\sqrt{1}$

$\sqrt{(x-2)^2}=\pm\sqrt{1}$

$x-2=\pm1$

$x-2=-1$ $\quad$ $x-2=+1$

$x=2+1$ $\quad$ $x=2-1$

$x=3$ $\quad$ $x=1$

$S.S\{3,1\}$

(x) $7(x+2a)^2+3a^2=5a(7x+23a)$

Solution: $7(x+2a)^2+3a^2=5a(7x+23a)$

$7(x^2+4a^2+4ax)+3a^2=35ax+115a^2$

$7x^2+28a^2+28ax+3a^2-35ax-115a^2=0$

$7x^2-35a+28ax+28a^2+3a^2-115a^2=0$

$7x^2-7ax-84a^2=0$

Dividing each term by 7

$\frac{7x^2}{7}-\frac{7ax}{7}=\frac{84a^2}{7}$

$x^2-ax=12a^2$ $\quad$ *Hint* $\left(a\times\frac{1}{2}=\frac{a}{2}\right)$

Adding $\left(\frac{a}{2}\right)^2$ *on both sides*

$(x)^2-2(x)\left(\frac{a}{2}\right)+\left(\frac{a}{2}\right)^2=12a^2+\left(\frac{a}{2}\right)^2$

$\left(x-\frac{a}{2}\right)^2=12a^2+\frac{a^2}{4}$

$\left(x-\frac{a}{2}\right)^2=\frac{48a^2+a^2}{4}$

$\left(x-\frac{a}{2}\right)^2=\frac{49a^2}{4}$

Taking square on both sides

$\sqrt{\left(x-\frac{a}{2}\right)^2}=\sqrt{\frac{49a^2}{4}}$

$\left(x-\frac{a}{2}\right)=\pm\frac{7a}{2}$

$x=\frac{a}{2}\pm\frac{7a}{2}$

$x=\frac{a+7a}{2}$ $\quad$ $x=\frac{a-7a}{2}$

$x=\frac{8a}{2}$ $\quad$ $x=\frac{-6a}{2}$

$x=4a$ $\quad$ $x=-3a$

Solution set is $\{4a,-3a\}$

Uses of Quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Exercise 1.2

Question No.1

$2 - x^2 = 7x$

$2 - x^2 = 7x$

$1x^2 + 7x - 2$

$ax^2 + bx + c = 0$

$a = 1 \quad b = 7 \quad c = -2$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(7) \pm \sqrt{(7)^2 - 4(1)(-2)}}{2(1)}$

$x = \frac{-7 \pm \sqrt{49 + 8}}{2}$

$x = \frac{-7 \pm \sqrt{57}}{2}$

$solution\ set\ is \left\{ \frac{-7 \pm \sqrt{57}}{2} \right\}$

Question No.2

$5x^2 + 8x + 1 = 0$

$5x^2 + 8x + 1 = 0$

$ax^2 + bx + c = 0$

$a = 5 \quad b = 8 \quad c = 1$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-8 \pm \sqrt{(8)^2 - 4(5)(1)}}{2(5)}$

$x = \frac{-8 \pm \sqrt{64 - 20}}{10}$

$x = \frac{-8 \pm \sqrt{44}}{10}$

$x = \frac{-8 \pm \sqrt{4 \times 11}}{10}$

$x = \frac{-8 \pm 2\sqrt{11}}{10}$

$x = \frac{2(-4 \pm \sqrt{11})}{10}$

$x = \frac{-4 \pm \sqrt{11}}{5}$

$\left\{ \frac{-4 \pm \sqrt{11}}{5} \right\}$

Question NO.3

$\sqrt{3}x^2 + x = 4\sqrt{3}$

$\sqrt{3}x^2 + x = 4\sqrt{3}$

$\sqrt{3}x^2 + 1x - 4\sqrt{3} = 0$

$ax^2 + bx + c = 0$

$a = \sqrt{3} \quad b = 1 \quad c = -4\sqrt{3}$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-1 \pm \sqrt{(1)^2 - 4(\sqrt{3})(-4\sqrt{3})}}{2\sqrt{3}}$

$x = \frac{-1 \pm \sqrt{1 + 16(\sqrt{3})^2}}{2\sqrt{3}}$

$x = \frac{-1 \pm \sqrt{1 + 16(3)}}{2\sqrt{3}}$

$x = \frac{-1 \pm \sqrt{1 + 48}}{2\sqrt{3}}$

$x = \frac{-1 \pm \sqrt{49}}{2\sqrt{3}}$

$x = \frac{-1 \pm 7}{2\sqrt{3}}$

$x = \frac{-1 + 7}{2\sqrt{3}} \quad , \quad x = \frac{-1 - 7}{2\sqrt{3}}$

$x = \frac{6}{2\sqrt{3}} \quad , \quad x = \frac{-8}{2\sqrt{3}}$

$x = \frac{3}{\sqrt{3}} \quad , \quad x = \frac{-4}{\sqrt{3}}$

$x = \frac{\sqrt{3}\sqrt{3}}{\sqrt{3}} \quad , \quad x = \frac{-4}{\sqrt{3}}$

$x = \sqrt{3} \quad , \quad x = \frac{-4}{\sqrt{3}}$

$solution\ set \{ \sqrt{3}, \frac{-4}{\sqrt{3}} \}$

Question No.4

$4x^2 - 14 = 3x$

$4x^2 - 14 = 3x$

$4x^2 - 3x - 14 = 0$

$ax^2 + bx + c = 0$

$a = 4 \quad b = -3 \quad c = -14$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x=\frac{-(-3)\pm\sqrt{(-3)^2-4(4)(-14)}}{2(4)}$$
$$x=\frac{3\pm\sqrt{9+224}}{8}$$
$$\left\{\frac{3\pm\sqrt{9+224}}{8}\right\}$$

Question No.5

$$6x^2-3-7x=0$$
$$6x^2-3-7x=0$$
$$6x^2-7x-3=0$$
$$ax^2+bx+c=0$$
$$a=7 \quad b=-7 \quad c=-3$$
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{7\pm\sqrt{49+72}}{12}$$
$$x=\frac{7\pm\sqrt{121}}{12}$$
$$x=\frac{7\pm11}{12}$$
$$x=\frac{7-11}{12} \qquad x=\frac{7+11}{12}$$
$$x=\frac{-4}{12} \qquad x=\frac{18}{12}$$
$$x=\frac{-1}{3} \qquad x=\frac{3}{2}$$
$$\left\{\frac{-1}{3},\frac{3}{2}\right\}$$

Question No.6

$$3x^2+8x+2=0$$
$$3x^2+8x+2=0$$
$$ax^2+bx+c=0$$
$$a=3 \quad b=8 \quad c=2$$
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-8\pm\sqrt{(8)^2-4(3)(2)}}{2(3)}$$
$$x=\frac{-8\pm\sqrt{64-24}}{6}$$
$$x=\frac{-8\pm\sqrt{40}}{6}$$
$$x=\frac{-8\pm\sqrt{4\times10}}{6}$$
$$x=\frac{-8\pm2\sqrt{10}}{6}$$
$$x=\frac{2(-4\pm\sqrt{10})}{6}$$
$$x=\frac{-4\pm\sqrt{10}}{3}$$
$$\left\{\frac{-4\pm\sqrt{10}}{3}\right\}$$

Question No.7

$$\frac{3}{x-6}-\frac{4}{x-5}=1$$
$$\frac{3}{x-6}-\frac{4}{x-5}=1$$
$$\frac{3(x-5)-4(x-6)}{(x-6)(x-5)}=1$$
$$3(x-5)-4(x-6)=(x-6)(x-5)$$
$$3x-15-4x+24=x^2-5x-6x+30$$
$$-1x+9=x^2-11x+30$$
$$x^2-11x=1x=30-9=0$$
$$1x^2-10x+21=0$$
$$ax^2+bx+c=0$$
$$a=1 \quad b=-10 \quad c=21$$
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-(-10)\pm\sqrt{(-10)^2-4(1)(21)}}{2(1)}$$
$$x=\frac{10\pm\sqrt{100-84}}{2}$$
$$x=\frac{10\pm\sqrt{16}}{2}$$
$$x=\frac{10\pm4}{2}$$
$$x=2\frac{(5\pm2)}{2}$$
$$x=5+2 \qquad x=5-2$$
$$x=7 \qquad x=3$$
$$\{3,7\}$$

Question No.8

$$\frac{x+2}{x-1}-\frac{4-x}{2x}=2\frac{1}{3}$$
$$\frac{x+2}{x-1}-\frac{4-x}{2x}=2\frac{1}{3}$$
$$\frac{(x+2)2x-(4-x)(x-1)}{(x-1)(2x)}=\frac{7}{3}$$

$$\frac{(2x^2+4x)-(4x-4-x^2+x)}{2x^2-2x}=\frac{7}{3}$$
$$\frac{2x^2+4x-5x+4+x^2}{2x^2-2x}=\frac{7}{3}$$
$$\frac{3x^2-x+4}{2x^2-2x}=\frac{7}{3}$$
$$3(3x^2-x+4)=7(2x^2-2x)$$
$$9x^2-3x+12=14x^2-14x$$
$$14x^2-9x^2-14x+3x-12=0$$
$$5x^2-11x-12=0$$
$$ax^2+bx+c=0$$
$$a=5 \quad b=-11 \quad c=-12$$
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-(-11)\pm\sqrt{(-11)^2-4(5)(-12)}}{2(5)}$$
$$x=\frac{11\pm\sqrt{121+240}}{10}$$
$$x=\frac{11\pm\sqrt{361}}{10}$$
$$x=\frac{11\pm19}{10}$$
$$x=\frac{11-19}{10} \qquad x=\frac{11+19}{10}$$
$$x=\frac{-8}{10} \qquad x=\frac{30}{10}$$
$$x=\frac{-4}{5} \qquad x=3$$
$$\left\{3,\frac{-4}{5}\right\}$$

Question No.9

$$\frac{a}{x-b}+\frac{b}{x-a}=2$$

Solution:

$$\frac{a}{x-b}+\frac{b}{x-a}=2$$
$$\frac{a(x-a)+b(x-b)}{(x-b)(x-a)}=2$$
$$\frac{ax-a^2+bx-b^2}{x^2-ax-bx+ab}=2$$
$$ax-a^2+bx-b^2=2(x^2-ax-bx+ab)$$
$$ax-a^2+bx-b^2=2x^2-2ax-2bx+2ab$$
$$2x^2-2ax-2bx+2ab-ax+a^2-bx+b^2=0$$
$$2x^2-2ax-ax-2bx-bx+2ab+a^2+b^2=0$$
$$2x^2-3ax-3bx+2ab+a^2+b^2=0$$
$$2x^2-(3a+3b)x+(a^2+b^2+2ab)=0$$
$$2x^2-(3a+3b)x+(a+b)^2=0$$
$$ax^2+bx+c=0$$
$$a=2 \quad b=-(3a+3b) \quad c=(a+b)^2$$
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-\left[-(3a+3b)\right]\pm\sqrt{\left[-(3a+3b)\right]^2-4(2)(a+b)^2}}{2(2)}$$
$$x=\frac{(3a+3b)\pm\sqrt{\left[-3(a+b)\right]^2-8(a+b)^2}}{4}$$
$$x=\frac{3(a+b)\pm\sqrt{\left[(-3)^2(a+b)^2\right]-8(a+b)^2}}{4}$$
$$x=\frac{3(a+b)\pm\sqrt{9(a+b)^2-8(a+b)^2}}{4}$$
$$x=\frac{3(a+b)\pm\sqrt{(a+b)^2}}{4}$$
$$x=\frac{3(a+b)\pm(a+b)}{4}$$
$$x=\frac{3(a+b)-(a+b)}{4} \qquad x=\frac{3(a+b)+(a+b)}{4}$$
$$x=\frac{2(a+b)}{4} \qquad x=\frac{4(a+b)}{4}$$
$$x=\frac{(a+b)}{2} \qquad x=a+b$$
$$x=\frac{1}{2}(a+b) \qquad x=a+b$$
$$\left\{(a+b),\frac{1}{2}(a+b)\right\}$$

Question No.10

$$-(l+m)-lx^2+(2l+m)x=0$$
$$-(l+m)-lx^2+(2l+m)x=0$$
$$(l+m)+lx^2-(2l+m)x=0$$
$$lx^2-(2l+m)x+(l+m)=0$$
$$ax^2+bx+c=0$$
$$a=l \quad b=-(2l+m) \quad c=(l+m)$$
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-\left[-(2l+m)\right]\pm\sqrt{\left[-(2l+m)\right]^2-4(l)(l+m)}}{2(l)}$$
$$x=\frac{(2l+m)\pm\sqrt{(-1)^2(2l+m)^2-4l(l+m)}}{2l}$$

$$x=\frac{(2l+m)\pm\sqrt{(-1)^2(2l+m)^2-4l(l+m)}}{2l}$$
$$x=\frac{(2l+m)\pm\sqrt{(2l)^2+(m)^2+2(2l)(m)-4l^2-4lm}}{2l}$$
$$x=\frac{(2l+m)\pm\sqrt{4l^2+m^2+4lm-4l^2-4lm}}{2l}$$
$$x=\frac{(2l+m)\pm\sqrt{m^2}}{2l}$$
$$x=\frac{(2l+m)\pm m}{2l}$$
$$x=\frac{2l+m-m}{2l} \qquad x=\frac{2l+m+m}{2l}$$
$$x=\frac{2l}{2l} \qquad x=\frac{2l+2m}{2l}$$
$$x=1 \qquad x=\frac{2(l+m)}{2l}$$
$$x=\frac{l+m}{l}$$
$$\left\{1,\frac{l+m}{l}\right\}$$

Exercise 1.3

Question N0.1

$2x^4-11x^2+5=0$

$Solution: 2x^4-11x^2+5=0$

$Let \quad x^2=y \Rightarrow (x^2)^2=y^2$

$x^4=y^2$

$Put \quad x^2=y \;\; and \;\; x^4=y^2$

$2x^4-11x^2+5=0$

$2y^2-11y+5=0$

$2y^2-10y-y+5=0$

$2y(y-5)-1(y-5)$

$(y-5)(2y-1)$

$y-5=0 \quad , \quad 2y-1=0$

$y=5 \quad , \quad 2y=1$

$y=5 \quad , \quad y=\frac{1}{2}$

$Put\,in \quad x^2=y$

$x^2=5 \quad , \quad x^2=\frac{1}{2}$

$\sqrt{x^2}=\pm\sqrt{5} \qquad \sqrt{x^2}=\pm\sqrt{\frac{1}{2}}$

$x=\pm\sqrt{5} \qquad x=\pm\frac{1}{\sqrt{2}}$

$Solution\,Set\,is\left\{\pm\frac{1}{\sqrt{2}},\pm\sqrt{5}\right\}$

Question no: 2

$2x^4 = 9x^2 - 4$

Solution: $2x^4 = 9x^2 - 4$

$2x^4 - 9x^2 + 4 = 0$ ......(1)

Let $x^2 = y$ ......(2)

Taking square on both sides

$(x^2)^2 = y^2$

$x^4 = y^2$

Put $x^2 = y$ *and* $x^4 = y^2$ *in eq*(1)

$2y^2 - 9y + 4 = 0$

$2y^2 - 8y - 1y + 4 = 0$

$2y(y-4) - 1(y-4) = 0$

$(y-4)(2y-1) = 0$

$y - 4 = 0 \quad 2y - 1 = 0$

$y = 4 \quad 2y = 1$

$y = 4 \quad y = \frac{1}{2}$

Put $y = x^2$

$x^2 = 4 \quad x^2 = \frac{1}{2}$

$\sqrt{x^2} = \pm\sqrt{4} \quad \sqrt{x^2} = \sqrt{\frac{1}{2}}$

$x = \pm 2 \quad x = \pm\frac{1}{\sqrt{2}}$

Solution set is $\left\{\pm 2, \pm\frac{1}{\sqrt{2}}\right\}$

Question no: 3

$5x^{\frac{1}{2}} = 7x^{\frac{1}{4}} - 2$

Solution: $5x^{\frac{1}{2}} = 7x^{\frac{1}{4}} - 2 \rightarrow (1)$

Let $x^{\frac{1}{4}} = y \rightarrow (2)$

Taking square on both sides

$\left(x^{\frac{1}{4}}\right)^2 = y^2$

$x^{\frac{1}{2}} = y^2$

Put $x^{\frac{1}{4}} = y$ *and* $x^{\frac{1}{2}} = y^2$

$5x^{\frac{1}{2}} = 7x^{\frac{1}{4}} - 2$

$5y^2 = 7y - 2$

$5y^2 - 7y + 2 = 0$

$5y^2 - 5y - 2y + 2 = 0$

$5y(y-1) - 2(y-1) = 0$

$(y-1)(5y-2) = 0$

$y - 1 = 0 \quad , \quad 5y - 2 = 0$

$y = -1 \quad , \quad 5y = 2$

$y = -1 \quad , \quad y = \frac{2}{5}$

Put $y = x^{\frac{1}{4}}$

$x^{\frac{1}{4}} = 1 \quad , \quad x^{\frac{1}{4}} = \frac{2}{5}$

$\left(x^{\frac{1}{4}}\right)^4 = (1)^4 \quad , \quad \left(x^{\frac{1}{4}}\right)^4 = \left(\frac{2}{5}\right)^4$

$x = 1 \quad , \quad x = \frac{2^4}{5^4}$

$x = \frac{16}{625}$

So, the solution set is $\left\{1, \frac{16}{625}\right\}$

Question No.4

$x^{\frac{2}{3}} + 54 = 15x^{\frac{1}{3}}$

Solution: $x^{\frac{2}{3}} + 54 = 15x^{\frac{1}{3}}$ .....(1)

Let $x^{\frac{1}{3}} = y$ ....(2)

Taking square

$\left(x^{\frac{1}{3}}\right)^2 = y^2 \Rightarrow x^{\frac{2}{3}} = y^2$ *in eq*(1)

$x^{\frac{2}{3}} + 54 = 15x^{\frac{1}{3}}$

$y^2 + 54 = 15y$

$y^2 - 15y + 54 = 0$

$y^2 - 9y - 6y + 54 = 0$

$y(y-9) - 6(y-9) = 0$

$(y-9)(y-6)$

$y-9=0 \quad y-6=0$

$y=9 \quad y=6$

Put $y=x^{\frac{1}{3}}$

$x^{\frac{1}{3}}=9 \quad x^{\frac{1}{3}}=6$

$\left(x^{\frac{1}{3}}\right)^3=(9)^3 \quad \left(x^{\frac{1}{3}}\right)=(6^3)$

$x=9^3 \quad x=6^3$

$x=729 \quad x=216$

So the solution set is $\{729, 216\}$

Question No.5

$3x^{-2}+5=8x^{-1}$

Solution: $3x^{-2}+5=8x^{-1}$........(1)

$x^{-1}=y \quad x^{-2}=y^2$

Put $x^{-1}=y$ and $x^{-2}=y^2$ in eq (1)

$3y^2+5=8y$

$3y^2-8y+5=0$

$3y^2-3y-5y+5=0$

$3y(y-1)-5(y-1)=0$

$(3y-5)(y-1)=0$

$3y-5=0 \quad y-1=0$

$3y=5 \quad y=1$

$y=\frac{5}{3} \quad y=1$

From eq (2) put $y=x^{-1}$

$x^{-1}=\frac{5}{3} \quad x^{-1}=1$

$\frac{1}{x}=\frac{5}{3} \quad \frac{1}{x}=1$

$x=\frac{3}{5} \quad x=1$

S.S $\left\{\frac{3}{5},1\right\}$

Question no: 6

$\left(2x^2+1\right)+\frac{3}{2x^2+1}=4$

Solution:

$\left(2x^2+1\right)+\frac{3}{2x^2+1}=4 \quad \rightarrow (1)$

Let $2x^2+1=y$

$y+\frac{3}{y}=4$

Multiplying both sides by "y"

$y^2+3=4y$

$y^2-4y+3=0$

$y^2-3y-1y+3=0$

$y(y-3)-1(y-3)=0$

$(y-3)(y-1)=0$

$y-1=0 \quad y-3=0$

$y=1 \quad y=3$

Put $y=2x^2+1$

$2x^2+1=3 \quad 2x^2+1=1$

$2x^2=3-1 \quad 2x^2=1-1$

$2x^2=2 \quad 2x^2=0$

$x^2=\frac{2}{2} \quad x^2=\frac{0}{2}$

$x^2=1 \quad x^2=0$

$\sqrt{x^2}=\pm\sqrt{1} \quad \sqrt{x^2}=\pm\sqrt{0}$

$x=\pm 1 \quad x=0$

S.S $\{-1,1,0\}$

Question no 7

$\frac{x}{x-3}+4\left(\frac{x-3}{x}\right)=4$

Solution: $\frac{x}{x-3}+4\left(\frac{x-3}{x}\right)=4$

$$Let\ \frac{x}{x-3}=y \Rightarrow \frac{x-3}{x}=\frac{1}{y}$$

Equcaton become

$$y+4\left(\frac{1}{y}\right)=4$$

Multiplying both side by "y"

$$y^2+4=4y$$
$$y^2-4y+4=0$$
$$y^2-2y-2y+4=0$$
$$y(y-2)-2(y-2)=0$$
$$(y-2)(y-2)$$
$$y-2=0 \Rightarrow y=2$$

Put the value of y

$$\frac{x}{x-3}=2 \Rightarrow x=2(x-3)$$
$$x=2x-6 \Rightarrow 6=2x-x$$
$$x=6$$
$$S.S\{6\}$$

Question no 8

$$\frac{4x+1}{4x-1}+\frac{4x-1}{4x+1}=2\frac{1}{6}$$

$$Solution: \frac{4x+1}{4x-1}+\frac{4x-1}{4x+1}=2\frac{1}{6} \to (1)$$

$$\frac{4x+1}{4x-1}=y....(2) \Rightarrow \frac{4x-1}{4x+1}=\frac{1}{y}$$

Equation (1) become

$$y+\frac{1}{y}=2\frac{1}{6}$$
$$y+\frac{1}{y}=\frac{13}{6}$$

Multiplying both sides by "6y"

$$6y^2+6=13y$$
$$6y^2-13y-6=0$$
$$6y^2-9y-4y-6=0$$
$$3y(2y-3)-2(2y-3)=0$$
$$(2y-3)(3y-2)$$
$$2y-3=0 \quad 3y-2=0$$
$$2y=3 \quad 3y=2$$
$$y=\frac{3}{2} \quad y=\frac{2}{3}$$
$$\frac{4x+1}{4x-1}=\frac{3}{2}$$
$$2(4x+1)=3(4x-1)$$
$$8x+2=12x-3$$
$$2+3=12x-8x$$
$$5=4x$$
$$x=\frac{5}{4}$$
$$\frac{4x+1}{4x-1}=\frac{2}{3}$$
$$3(4x+1)=2(4x-1)$$
$$12x+3=8x-2$$
$$12x-8x=-2-3$$
$$4x=-5$$
$$x=-\frac{5}{4}$$
$$S.S\left\{\pm\frac{5}{4}\right\}$$

Question No.9

$$\frac{x-a}{x+a}-\frac{x+a}{x-a}=\frac{7}{12}$$

$$Solution: \frac{x-a}{x+a}-\frac{x+a}{x-a}=\frac{7}{12}$$

$$Let\ \frac{x-a}{x+a}=y\ or\ \frac{x+a}{x-a}=\frac{1}{y}$$

$$y-\frac{1}{y}=\frac{7}{12}$$
$$\frac{y^2-1}{y}=\frac{7}{12}$$
$$12(y^2-1)=7y$$
$$12y^2-12=7y \Rightarrow 12y^2-7y-12=0$$
$$12y^2-16y+9y-12=0$$
$$4y(3y-4)+3(3y-4)$$

$(3y-4)(4y+3)$

$3y-4=0 \quad 4y+3=0$

$3y=4 \quad 4y=-3$

$y=\frac{4}{3} \quad y=-\frac{3}{4}$

$\frac{x-a}{x+a}=\frac{4}{3} \quad \frac{x+a}{x-a}=-\frac{3}{4}$

$3(x-a)=4(x+a)$

$3x-3a=4x+4a$

$-3a-4a=4x-3x$

$-7a=x$

$x=-7a$

$\frac{x+a}{x-a}=-\frac{3}{4}$

$4(x+a)=-3(x-a)$

$4x+4a=-3x+3a$

$4x+3x=3a-4a$

$7x=-a$

$x=\frac{a}{7}$

$S.S\left\{-7a,\frac{a}{7}\right\}$

Question no: 10

$x^4-2x^3-2x^2+2x+1=0$

Solution:

$x^4-2x^3-2x^2+2x+1=0$

Dividing both sides by "x^2"

$\frac{x^4}{x^2}-\frac{2x^3}{x^2}-\frac{2x^2}{x^2}+\frac{2x}{x^2}+\frac{1}{x^2}=\frac{0}{x^2}$

$x^2-2x-2+\frac{2}{x}+\frac{1}{x^2}=0$

$\left(x^2+\frac{1}{x^2}\right)-2x+\frac{2}{x}-2=0$

$\left(x^2+\frac{1}{x^2}\right)-2\left(x+\frac{1}{x}\right)-2=0 \quad \rightarrow (1)$

Let $x-\frac{1}{x}=y \quad \rightarrow (2)$

Taking square on both sides

$\left(x-\frac{1}{x}\right)^2=(y)^2$

$x^2+\frac{1}{x^2}-2=y^2$

$x^2+\frac{1}{x^2}=y^2+2$

Putting values in eq.(1)

$y^2+2-2(y)-2=0$

$y^2-2y=0$

$y(y-2)=0$

$y=0 \quad y-2=0$

$y=2$

Put $y=x-\frac{1}{x}$ *from eq.*2

$x-\frac{1}{x}=0 \quad x-\frac{1}{x}=2$

$\frac{x^2-1}{x}=0 \quad \frac{x^2-1}{x}=2$

$x^2-1=0 \quad x^2-1=2x$

$x^2=1 \quad x^2-2x-1=0$

$\sqrt{x^2}=\pm\sqrt{1} \quad 1x^2-2x-1=0$

$x=\pm 1$

Solving $1x^2-2x-1=0$ *by quadratic formula*

$a=1 \;\; b=-2 \;\; c=-1$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-(-2)\pm\sqrt{(-2)^2-4a(1)(-1)}}{2(1)}$

$x=\frac{+2\pm\sqrt{4+4}}{2}$

$x=\frac{+2\pm\sqrt{8}}{2}$

$x=\frac{+2\pm\sqrt{4\times 2}}{2}$

$x=\frac{+2\pm 2\sqrt{2}}{2}$

$x=\frac{2(1\pm\sqrt{2})}{2}$

$x=1\pm\sqrt{2}$

$S.S\left\{\pm 1, 1\pm\sqrt{2}\right\}$

Question no: 11

$2x^4+x^3-6x^2+x+2=0$

Solution:

$2x^4+x^3-6x^2+x+2=0$

Dividing each term by "x^2"

$\frac{2x^4}{x^2}+\frac{x^3}{x^2}-\frac{6x^2}{x^2}+\frac{x}{x^2}+\frac{2}{x^2}=\frac{0}{x^2}$

$2x^2+x-6+\frac{1}{x}+\frac{2}{x^2}=0$

$2x^2+\frac{2}{x^2}+x+\frac{1}{x}-6=0$

$$2\left(x^2+\frac{1}{x^2}\right)+\left(x+\frac{1}{x}\right)-6=0 \quad ......(1)$$

$Let\ x+\frac{1}{x}=y \quad ......(2)$

$Taking\ square\ on\ both\ side$

$\left(x+\frac{1}{x}\right)^2=(y)^2$

$x^2+\frac{1}{x^2}+2x\frac{1}{x}=y^2$

$x^2+\frac{1}{x^2}+2=y^2$

$Put\ x+\frac{1}{x}=y\ and\ x^2+\frac{1}{x^2}=y^2-2\ in\ eq(1)$

$2\left(x^2+\frac{1}{x^2}\right)+\left(x+\frac{1}{x}\right)-6=0$

$2(y^2-2)+y-6=0$

$2y^2-4+y-6=0$

$2y^2+y-10=0$

$2y^2+5y-4y-10=0$

$y(2y+5)-2(2y+5)=0$

$2y+5=0 \quad y-2=0$

$2y=-5 \quad y=2$

$y=-\frac{5}{2} \quad y=2$

$Put\ x+\frac{1}{x}=y\ \ in\ eq\ (2)$

$x+\frac{1}{x}=-\frac{5}{2}$

$\frac{x^2+1}{x}=-\frac{5}{2}$

$2(x^2+1)=-5x$

$2x^2+2=-5x$

$2x^2+5x+2=0$

$2x^2+4x+x+2=0$

$2x(x+2)+1(x+2)=0$

$(x+2)(2x+1)$

$x+2=0 \quad 2x+1=0$

$x=-2 \quad 2x=-1$

$x=-2 \quad x=-\frac{1}{2}$

$x+\frac{1}{x}=2$

$\frac{x^2+1}{x}=2$

$x^2+1=2x$

$x^2-2x+1=0$

$x^2-x-x+1=0$

$x(x-1)-1(x-1)=0$

$(x-1)(x-1)=0$

$x-1=0$

$x=1$

$S.S\left\{1,-2,-\frac{1}{2}\right\}$

$Question\ no\ 12.$

$4.2^{2x+1}-9.2^x+1=0$

$Solution:\ 4.2^{2x+1}-9.2^x+1=0$

$4.2^{2x}.2^1-9.2^x+1=0$

$8.(2^x)^2.-9.2^x+1=0$

$Let\ 2^x=y \quad (2^x)^2=y^2$

$8y^2-9y+1=0$

$8y^2-8y-y+1=0$

$8y(y-1)-1(y-1)=0$

$(8y-1)(y-1)=0$

$8y-1=0 \quad y-1=0$

$8y=1 \quad y=1$

$y=\frac{1}{8} \quad y=1$

$Put\ the\ value\ of\ \ y\ in\ above\ equation$

$2^x=y \quad 2^x=1$

$2^x=\frac{1}{8} \quad 2^x=2^0$

$2^x=\frac{1}{2^3} \quad 2^x=2^0$

$2^x=2^{-3} \quad x=0$

$x=-3$

$S.S\ \{-3,0\}$

$Question\ \ no:13$

$3^{2x+2}=12.3^x-3$

$Solution: 3^{2x+2}=12.3^x-3$

$3^{2x}\,3^2-12.(3^x)+3=0$

$9(3^x)^2-12.(3^x)+3=0 \quad ......(1)$

$Let\ \ 3^x=y$

$Put\ \ 3^x=y\ \ in\ \ eq.(1)$

$9y^2-12y+3=0$

$9y^2-9y-3y+3=0$

$9y(y-1)-3(y-1)=0$

$(y-1)(9y-3)=0$

$y-1=0 \quad 9y-3=0$

$y=1 \quad 9y=3$

$y=\frac{3}{9}$

Put $y=3^x$ *from eq.*(2)

$3^x=1 \quad 3^x=\frac{1}{3}$

$3^x=3^0 \quad 3^x=3^{-1}$

$x=0 \quad x=-1$

$S.S\{0,-1\}$

Question no:14

$2^x+64.2^{-x}-20=0$

Solution:

$2^x+64.2^{-x}-20=0$

$2^x+\frac{64}{2^x}-20=0$ ......(1)

Let $2^x=y$ ......(2)

Put $2^x=y$ *in eq.*(1)

$y+\frac{64}{y}-20=0$

Multiply both sides by "y"

$y^2+64-20y=0$

$y^2-20y+64=0$

$y^2-16y-4y+64=0$

$y(y-16)-4(y-16)=0$

$(y-16)(y-4)=0$

$y-16=0 \quad y-4=0$

$y=16 \quad y=4$

Put $y=2^x$ *from eq.*(2)

$2^x=16 \quad 2^x=4$

$2^x=2^4 \quad 2^x=2^2$

$x=4 \quad x=2$

$S.S\{2,4\}$

Question No.15

Solution:

$(x+1)(x+3)(x-5)(x-7)=192$

$\because \; a+b=c+d$

$1-5=3-7$

$-4=-4$

$(x+1)(x-5)(x+3)(x-7)=192$

$(x^2-5x+1x-5)(x^2-7x+3x-21)=192$

$(x^2-4x-5)(x^2-4x-21)=192$ ......(1)

Let $x^2-4x=y$ ......(2)

So, eq.(1) *becomes*

$(y-5)(y-21)=192$

$y^2-21y-5y+105=192$

$y^2-26y+105-192=0$

$y^2-26y-87=0$

$y^2-29y+3y-87=0$

$y(y-29)+3(y-29)=0$

$(y-29)(y+3)=0$

$y-29=0 \;,\; y+3=0$

$y=29 \;,\; y=-3$

Put $y=x^2-4x$ *in eq.*(2)

$x^2-4x=29 \quad x^2-4x=-3$

$x^2-4x-29=0 \quad x^2-4x+3=0$

Solve $x^2-4x-29=0$ *by quadratic formula*

$a=1 \quad b=-4 \quad c=-29$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(-29)}}{2(1)}$

$x=\frac{4\pm\sqrt{16+116}}{2}$

$x=\frac{4\pm\sqrt{132}}{2}$

$x=\frac{4\pm\sqrt{4\times33}}{2}$

$x=\frac{4\pm2\sqrt{33}}{2}$

$x=\frac{2(2\pm\sqrt{33})}{2}$

$x=2\pm\sqrt{33}$

Solve $x^2-4x+3=0$ *by factorization*

$x^2-4x+3=0$

$x^2-3x-x+3=0$

$x(x-3)-1(x-3)=0$

$(x-3)(x-1)=0$

$x-3=0 \quad x-1=0$

$x=3 \quad x=1$

$S.S\{1,3,2\pm\sqrt{33}\}$

Question No.16

$(x-1)(x-2)(x-8)(x+5)+360=0$

Solution:

$(x-1)(x-2)(x-8)(x+5)+360=0$

$[(x-1)(x-2)][(x-8)(x+5)]+360=0$

$[x^2-2x-1x+2][x^2+5x-8x-40]+360=0$

$(x^2-3x+2)(x^2-3x-40)+360=0$ ......(1)

Let $x^2-3x=y$ ......(2)

Put in equation (1)

$(y+2)(y-40)+360=0$

$y^2-40y+2y-80+360=0$

$y^2-38y+280=0$

$y^2-28y-10y+280=0$

$y(y-28)-10(y-28)=0$

$(y-28)(y-10)=0$

$y-28=0 \qquad y-10=0$

$y=28 \qquad y=10$

Put $y=x^2-3x$ from eq.(2)

$x^2-3x=28$

$x^2-3x-28=0$

$x^2-7x+4x-28=0$

$x(x-7)+4(x-7)=0$

$(x-7)(x+4)=0$

$x-7=0 \quad , \quad x+4=0$

$x=7 \quad , \quad x=-4$

$x^2-3x=10$

$x^2-3x-10=0$

$x^2-5x+2x-10=0$

$x(x-5)+2(x-5)=0$

$(x-5)(x+2)=0$

$x-5=0 \qquad x+2=0$

$x=5 \qquad x=-2$

$S.S\{5,-2,7,-4\}$

Exercise 1.4

Question no1:

$2x+5=\sqrt{7x+16}$

Solution: $2x+5=\sqrt{7x+16}$ ....(1)

Taking square on both side

$(2x+5)^2=(\sqrt{7x+16})^2$

$(2x)^2+(5)^2+2(2x)(5)=7x+16$

$4x^2+25+20x=7x+16$

$4x^2+20x-7x+25-16=0$

$4x^2+13x+9=0$

$4x^2+4x+9x+9=0$

$4x(x+1)+9(x+1)=0$

$(x+1)(4x+9)=0$

$x+1=0 \quad , \quad 4x+9=0$

$x=-1 \quad , \quad 4x=-9$

$x=-1 \quad , \quad x=\dfrac{-9}{4}$

Checking:

put $x=-1$ *in the equcation*

$2x+5=\sqrt{7x+16}$

$2(-1)+5=\sqrt{7(-1)+16}$

$-2+5=\sqrt{-7+16}$

$3=\sqrt{9}$

$3=3$ *which is true*

$x=\dfrac{-9}{4}$ *in the equcation*

$2x+5=\sqrt{7x+16}$

$2\left(\dfrac{-9}{4}\right)+5=\sqrt{7\left(\dfrac{-9}{4}\right)+16}$

$\dfrac{-18}{4}+5=\sqrt{\dfrac{-63}{4}+16}$

$$\frac{-18+20}{4}=\sqrt{\frac{-63+64}{4}}$$

$$\frac{2}{4}=\sqrt{\frac{1}{4}} \quad , \quad \frac{1}{2}=\frac{1}{2}\ \textit{which is true}$$

$$\textit{Solution set}\left\{-1,\frac{-9}{4}\right\}$$

Question No2.

$$\sqrt{x+3}=3x-1$$

$$\textit{Solution}:\sqrt{x+3}=3x-1$$

Taking square on both side

$$(\sqrt{x+3})^2=(3x-1)^2$$
$$x+3=(3x)^2+(1)^2-2(3x)(1)$$
$$x+3=9x^2+1-6x=0$$
$$9x^2-6x-x+1-3=0$$
$$9x^2-7x-2=0$$
$$9x^2-9x+2x-2=0$$
$$9x(x-1)+2(x-1)=0$$
$$(x-1)(9x+2)$$
$$x-1=0 \quad , \quad 9x+2=0$$
$$x=1 \quad , \quad 9x=-2$$
$$x=1 \quad , \quad x=-\frac{2}{9}$$

Checking :

put $x=1$ in the equation

$$\sqrt{x+3}=3x-1$$
$$\sqrt{1+3}=3(1)-1$$
$$\sqrt{4}=2 \Rightarrow 2=2\ \textit{which is true}$$

put $x=-\frac{2}{9}$ in the equation

$$\sqrt{x+3}=3x-1$$
$$\sqrt{-\frac{2}{9}+3}=3\left(-\frac{2}{9}\right)-1$$
$$\sqrt{\frac{-2+27}{9}}=-\frac{6}{9}-1$$
$$\sqrt{\frac{25}{9}}=\frac{-6-9}{9}$$
$$\frac{5}{3}=\frac{-15}{9} \quad \Rightarrow \frac{5}{3}=-\frac{5}{3}$$

As $-\frac{2}{9}$ is an extraneous root

So the solution set is $\{1\}$

Question No.3

$$4x=\sqrt{13x+14}-3$$
$$4x=\sqrt{13x+14}-3 \rightarrow (1)$$
$$4x+3=\sqrt{13x+14}$$
$$(4x+3)^2=\left(\sqrt{13x+14}\right)^2$$
$$(4x)^2+(3)^2+2(4x)(3)=13x+14$$
$$16x^2+9+24x=13x+14$$
$$16x^2+9+24x-13x-14=0$$
$$16x^2+11x-5=0$$
$$16x^2+16x-5x-5=0$$
$$16x(x+1)-5(x+1)=0$$
$$(x+1)(16x-5)=0$$
$$x+1=0 \quad , \quad 16x-5=0$$
$$x=-1 \quad , \quad 16x=5$$
$$x=-1 \quad , \quad x=\frac{5}{16}$$
$$x=-1$$
$$4x=\sqrt{13x+14}-3$$
$$4(-1)=\sqrt{13(-1)+14}-3$$
$$-4=\sqrt{-13+14}-3$$
$$-4=\sqrt{1}-3$$
$$-4=1-3$$

$-4\neq-2$ which is not true

Put $x=\frac{5}{16}$ in the equatoin

$$4x=\sqrt{13x+14}-3$$
$$4\left(\frac{5}{16}\right)=\sqrt{13\left(\frac{5}{16}\right)+14}-3$$
$$\frac{20}{16}=\sqrt{\frac{65}{16}+14}-3$$
$$\frac{5}{4}=\sqrt{\frac{65+224}{16}}-3$$
$$\frac{5}{4}=\sqrt{\frac{289}{16}}-3$$
$$\frac{5}{4}=\frac{17}{4}-3$$
$$\frac{5}{4}=\frac{17-12}{4}$$

$\frac{5}{4}=\frac{5}{4}\ which\ is\ true$

So the solution set is $\frac{5}{16}$

Question No.4

$\sqrt{3x+100}-x=4$

$\sqrt{3x+100}-x=4 \quad ......(1)$

$\sqrt{3x+100}=4+x$

Taking square on both side

$\left(\sqrt{3x+100}\right)^2=(4+x)^2$

$3x+100=(4)^2+(x)^2+2(4)(x)$

$3x+100=16+x^2+8x$

$x^2+8x+16-3x-100=0$

$x^2+5x-84=0$

$x^2+12x-7x-84=0$

$x(x+12)-7(x+12)=0$

$(x+12)(x-7)=0$

$x+12=0 \quad , \quad x-7=0$

$x=-12 \quad , \quad x=7$

$x=-12$

$\sqrt{3x+100}-x=4$

$\sqrt{3(-12)+100}-(-12)=4$

$\sqrt{-36+100}+12=4$

$\sqrt{64}+12=4$

$8+12=4$

$20\neq 4$

So -12 is an extraneous root

put $x=7$ in the equation

$\sqrt{3x+100}-x=4$

$\sqrt{3(7)+100}-(7)=4$

$\sqrt{21+100}-7=4$

$\sqrt{121}-7=4$

$11-7=4$

$4=4$ *which is true*

So the solution set is $\{7\}$

Question No.5

$\sqrt{x+5}+\sqrt{x+21}=\sqrt{x+60}$

$\sqrt{x+5}+\sqrt{x+21}=\sqrt{x+60} \quad \to (1)$

Squaring both sides:

$\left(\sqrt{x+5}+\sqrt{x+21}\right)^2=\left(\sqrt{x+60}\right)^2$

$\left(\sqrt{x+5}\right)^2+\left(\sqrt{x+21}\right)^2+2\left(\sqrt{x+5}\right)\left(\sqrt{x+21}\right)=x+60$

$x+5+x+21+2\left(\sqrt{x+5}\right)\left(\sqrt{x+21}\right)=x+60$

$2x+26+2\sqrt{x^2+21x+5x+105}=x+60$

$2\sqrt{x^2+26x+105}=x+60-2x-26$

$2\sqrt{x^2+26x+105}=-x+34$

$2\sqrt{x^2+26x+105}=34-x$

Again squaring both sides:

$\left(2\sqrt{x^2+26x+105}\right)^2=(34-x)^2$

$4(x^2+26x+105)=(34)^2+x^2-2(34)(x)$

$4x^2+104x+420=1156+x^2-68x$

$4x^2+104x+420-1156-x^2+68x=0$

$3x^2+172x-736=0$

$ax^2+bx+c=0$

$a=3 \quad b=172 \quad c=-736$

By using quadratic formula:

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-(172)\pm\sqrt{(172)^2-4(3)(-736)}}{2(3)}$

$x=\frac{-172\pm\sqrt{29584+8832}}{6}$

$x=\frac{-172\pm\sqrt{38416}}{6}$

$x=\frac{-172\pm 196}{6}$

$x=\frac{-172-196}{6} \quad , \quad x=\frac{-172+196}{6}$

$x=\frac{-368}{6} \quad , \quad x=\frac{24}{6}$

$x=\frac{-184}{3} \quad , \quad x=4$

Checking:

Putting $x=4$ in equation (1)

$\sqrt{x+5}+\sqrt{x+21}=\sqrt{x+60}$

$\sqrt{4+5}+\sqrt{4+21}=\sqrt{4+60}$

$\sqrt{9}+\sqrt{25}=\sqrt{64}$

$$2\sqrt{x^2+26x+105}=x+60-2x-26$$
$$2\sqrt{x^2+26x+105}=-x+34$$
$$2\sqrt{x^2+26x+105}=34-x$$
Again squaring both sides:
$$\left(2\sqrt{x^2+26x+105}\right)^2=(34-x)^2$$
$$4(x^2+26x+105)=(34)^2+x^2-2(34)(x)$$
$$4x^2+104x+420=1156+x^2-68x$$
$$4x^2+104x+420-1156-x^2+68x=0$$
$$3x^2+172x-736=0$$
$$ax^2+bx+c=0$$
$$a=3\quad b=172\quad c=-736$$
By using quadratic formula:
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-(172)\pm\sqrt{(172)^2-4(3)(-736)}}{2(3)}$$
$$x=\frac{-172\pm\sqrt{29584+8832}}{6}$$
$$x=\frac{-172\pm\sqrt{38416}}{6}$$
$$x=\frac{-172\pm196}{6}$$
$$x=\frac{-172-196}{6}\quad,\quad x=\frac{-172+196}{6}$$
$$x=\frac{-368}{6}\quad,\quad x=\frac{24}{6}$$
$$x=\frac{-184}{3}\quad,\quad x=4$$
Checking:

Putting $x=4$ *in equation* (1)
$$\sqrt{x+5}+\sqrt{x+21}=\sqrt{x+60}$$
$$\sqrt{4+5}+\sqrt{4+21}=\sqrt{4+60}$$
$$\sqrt{9}+\sqrt{25}=\sqrt{64}$$
$$3+5=8$$
$8=8$ *which is true*

Putting $x=\frac{-184}{3}$
$$\sqrt{x+5}+\sqrt{x+21}=\sqrt{x+60}$$
$$\sqrt{\left(\frac{-184}{3}\right)+5}+\sqrt{\left(\frac{-184}{3}\right)+21}=\sqrt{\left(\frac{-184}{3}\right)+60}$$
$$\sqrt{\frac{-184+15}{3}}+\sqrt{\frac{-184+63}{3}}=\sqrt{\frac{-184+180}{3}}$$
$$\sqrt{\frac{-169}{3}}+\sqrt{\frac{-121}{3}}=\sqrt{\frac{-4}{3}}$$
$$\sqrt{\frac{-1\times169}{3}}+\sqrt{\frac{-1\times121}{3}}=\sqrt{\frac{-1\times4}{3}}$$
$$\frac{13i}{\sqrt3}+\frac{11i}{\sqrt3}=\frac{2i}{\sqrt3}\qquad\because i=\sqrt{-1}$$
$$\frac{13i+11i}{\sqrt3}=\frac{2i}{\sqrt3}$$
$\frac{24i}{\sqrt3}=\frac{2i}{\sqrt3}$ *which is not true*

As $x=\frac{-184}{3}$ *is extraneous root*

Solution set is $\{4\}$

Question no 6:
$$\sqrt{x+1}+\sqrt{x-2}=\sqrt{x+6}$$
Solution:
$$\sqrt{x+1}+\sqrt{x-2}=\sqrt{x+6}\quad......(1)$$
Squaring both sides:
$$\left(\sqrt{x+1}+\sqrt{x-2}\right)^2=\left(\sqrt{x+6}\right)^2$$
$$(x+1)^2+(x-2)^2+2(x+1)(x-2)=x+6$$
$$x+1+x-2+2\sqrt{(x+1)(x-2)}+x+6$$
$$2x-1+2\sqrt{x^2-2x+1x-2}=x+6$$
$$2\sqrt{x^2-x-2}=x+6-2x+1$$
$$2\sqrt{x^2-x-2}=7-x$$

Again squaring both sides

$$\left(2\sqrt{x^2-x-2}\right)^2=(7-x)^2$$

$$4\left(x^2-x-2\right)=(7)^2+x^2-2(7)(x)$$

$$4x^2-4x-8=49+x^2-14x$$

$$4x^2-x-4x+14x-8-49=0$$

$$3x^2+10x-57=0$$

$$3x^2+19x-9x-57=0$$

$$3(3x+19)-3(3x+19)=0$$

$$(3x+19)(x-3)=0$$

$$3x+19=0 \quad , \quad x-3=0$$

$$3x=-19 \quad , \quad x=3$$

$$x=\frac{-19}{3}$$

Checking:

Putting $x=3$ in the equation (1)

$$\sqrt{x+1}+\sqrt{x-2}=\sqrt{x+6}$$

$$\sqrt{3+1}+\sqrt{3-2}=\sqrt{3+6}$$

$$\sqrt{4}+\sqrt{1}=\sqrt{9}$$

$$2+1=3$$

$3=3$ *which is true*

Putting $x=\frac{-19}{3}$ in the equation (1)

$$\sqrt{\left(\frac{-19}{3}\right)+1}+\sqrt{\left(\frac{-19}{3}\right)-2}=\sqrt{\left(\frac{-19}{3}\right)+6}$$

$$\sqrt{\frac{-19+3}{3}}+\sqrt{\frac{-19-6}{3}}=\sqrt{\frac{-19+18}{3}}$$

$$\sqrt{\frac{-1\times16}{3}}+\sqrt{\frac{-1\times25}{3}}=\sqrt{\frac{-1\times1}{3}}$$

$$\frac{4i}{\sqrt3}+\frac{5i}{\sqrt3}=\frac{1i}{\sqrt3}$$

$$\frac{4i+5i}{\sqrt3}=\frac{1i}{\sqrt3}$$

$\frac{9i}{\sqrt3}=\frac{1i}{\sqrt3}$ *which is not true*

As $x=\frac{-19}{3}$ is an extraneous root

So the solution set is $\{3\}$

Question No.7

$$\sqrt{11-x}-\sqrt{6-x}=\sqrt{27-x}$$

Solution:

$$\sqrt{11-x}-\sqrt{6-x}=\sqrt{27-x} \quad ......(1)$$

Squaring both sides:

$$\left(\sqrt{11-x}-\sqrt{6-x}\right)^2=\left(\sqrt{27-x}\right)^2$$

$$\left(\sqrt{11-x}\right)^2+\left(\sqrt{6-x}\right)^2-2\left(\sqrt{11-x}\right)\left(\sqrt{6-x}\right)=27-x$$

$$11-x+6-x-2\sqrt{(11-x)(6-x)}=27-x$$

$$17-2x-2\sqrt{66-11x-6x+x^2}=27-x$$

$$-2\sqrt{66-17x+x^2}=27-x-17+2x$$

$$11-x+6-x-2\sqrt{(11-x)(6-x)}=27-x$$

$$-2\sqrt{66-17x+x^2}=27-x-17+2x$$

$$-2\sqrt{66-17x+x^2}=10+x$$

Again squaring both sides

$$\left(-2\sqrt{66-17x+x^2}\right)^2=(10+x)^2$$

$$(-2)^2\left(\sqrt{66-17x+x^2}\right)^2=(10+x)^2$$

$$4\left(66-17x+x^2\right)=(10)^2+(x)^2+2(10)(x)$$

$$264-68x+4x^2=100+x^2+20x$$

$$4x^2-x^2-68x-20x+264-100=0$$

$$3x^2-88x+164=0$$

By applying quadratic formula

$$a=3 \quad b=-88 \quad c=164$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-88)\pm\sqrt{(-88)^2-4(3)(164)}}{2(3)}$$

$$x=\frac{88\pm\sqrt{7744-1968}}{6}$$

$$x=\frac{88\pm\sqrt{5776}}{6}$$

$$x=\frac{88-76}{6} \quad , \quad x=\frac{88+76}{6}$$

$$x=\frac{12}{6} \quad , \quad x=\frac{164}{6}$$

$$x=2 \quad , \quad x=\frac{82}{3}$$

Checking:

Putting $x=\frac{82}{3}$ *in the equation* (1)

$$\sqrt{11-x}-\sqrt{6-x}=\sqrt{27-x}$$

$$\sqrt{11-\frac{82}{3}}-\sqrt{6-\frac{82}{3}}=\sqrt{27-\frac{82}{3}}$$

$$\sqrt{\frac{33-82}{3}}-\sqrt{\frac{18-82}{3}}=\sqrt{\frac{81-82}{3}}$$

$$\sqrt{\frac{1-\times 49}{3}}-\sqrt{\frac{-1\times 64}{3}}=\sqrt{\frac{-1\times 1}{3}}$$

$$\frac{7i}{\sqrt{3}}-\frac{8i}{\sqrt{3}}=\frac{i}{\sqrt{3}} \qquad \because i=\sqrt{-1}$$

$$\frac{4i-8i}{\sqrt{3}}=\frac{i}{\sqrt{3}}$$

$$\frac{-i}{\sqrt{3}}=\frac{i}{\sqrt{3}} \quad \text{which is not true}$$

Putting $x=2$ *in the equation* (1)

$$\sqrt{11-x}-\sqrt{6-x}=\sqrt{27-x}$$

$$\sqrt{11-2}-\sqrt{6-2}=\sqrt{27-2}$$

$$\sqrt{9}-\sqrt{4}=\sqrt{25}$$

$$3-2=5$$

$$1=5 \quad \text{which is not true}$$

Here 2 *and* $\frac{82}{3}$ *are extraneous roots.*

So the solution set is $\{\ \}$

Question No.8

$$\sqrt{4a+x}-\sqrt{a-x}=\sqrt{a}$$

Solution:

$$\sqrt{4a+x}-\sqrt{a-x}=\sqrt{a}$$

takinig squarinng on both side

$$(\sqrt{4a+x}-\sqrt{a-x})^2=(\sqrt{a})^2$$

$$(\sqrt{4a+x})^2+(\sqrt{a-x})^2-2(\sqrt{4a+x})(\sqrt{a-x})=(\sqrt{a})^2$$

$$4a+x+a-x-2\sqrt{(4a+x)(a-x)}=a$$

$$5a-2\sqrt{4a^2-4ax+ax-x^2}=a$$

$$-2\sqrt{4a^2-3ax-x^2}=a-5a$$

$$-2\sqrt{4a^2-3ax-x^2}=-4a$$

$$\sqrt{4a^2-3ax-x^2}=2a$$

taking again square on both side

$$\left(\sqrt{4a^2-3ax-x^2}\right)^2=(2a)^2$$

$$4a^2-3ax-x^2=4a^2$$

$$4a^2+3ax+x^2-4a^2=0$$

$$3ax+x^2=0$$

$$x(3a+x)=0$$

$$x=0 \qquad 3a+x=0$$

$$x=0 \qquad x=-3a$$

Checking:

Put $x=0$ *in the equation*

$$\sqrt{4a+0}-\sqrt{a-0}=\sqrt{a}$$

$$2\sqrt{a}-\sqrt{a}=\sqrt{a}$$

$$\sqrt{a}=\sqrt{a} \text{ which is true}$$

Put $x=-3a$ *in the equation*

$$\sqrt{4a-3a}-\sqrt{a+3a}=\sqrt{a}$$

$$\sqrt{a}-\sqrt{2a}=\sqrt{a}$$

$$-\sqrt{a}=\sqrt{a} \text{ which is not true}$$

As $-3a$ *is extraneous root*

so the solution set is $\{0\}$

Question No.9

$$\sqrt{x^2+x+1}-\sqrt{x^2+x-1}=1$$

Solution:

$$\sqrt{x^2+x+1}-\sqrt{x^2+x-1}=1 \quad ......(1)$$

Let $x^2+x=y$ ......(2)

Put it in equation (1)

$$\sqrt{y+1}-\sqrt{y-1}=1$$

$$\sqrt{y+1}=1+\sqrt{y-1}$$

$y+1=1+y-1+2\sqrt{y-1}$

$y+1=y+2\sqrt{y-1}$

$y+1-y=2\sqrt{y-1}$

$1=2\sqrt{y-1}$

Taking again squaring both sides

$(1)^2=\left(2\sqrt{y-1}\right)^2$

$1=4(y-1)$

$1=4y-4$

$1+4=4y$

$5=4y$

From equation (2) *put* $y=x^2+x$

$5=4(x^2+x)$

$5=4x^2+4x$

$0=4x^2+4x-5$

$4x^2+4x-5=0$

By applying quadratic formula

$a=4 \quad b=4 \quad c=-5$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-4\pm\sqrt{(4)^2-4(4)(-5)}}{2(4)}$$

$$x=\frac{-4\pm\sqrt{16+80}}{8}$$

$$x=\frac{-4\pm\sqrt{96}}{8}$$

$$x=\frac{-4\pm\sqrt{16\times 6}}{8}$$

$$x=\frac{-4\pm 4\sqrt{6}}{8}$$

$$x=\frac{4(-1\pm\sqrt{6})}{8}$$

$$x=\frac{-1\pm\sqrt{6}}{2}$$

The solution is $\left\{\frac{-1\pm\sqrt{6}}{2}\right\}$

Question No.10

$\sqrt{x^2+3x+8}+\sqrt{x^2+3x+2}=3$

Solution:

$\sqrt{x^2+3x+8}+\sqrt{x^2+3x+2}=3$ ......(1)

Let $x^2+3x=y$ ......(2)

Put it in equation (1)

$\sqrt{y+8}+\sqrt{y+2}=3$

$\sqrt{y+8}=3-\sqrt{y+2}$

Taking square on both sides

$\left(\sqrt{y+8}\right)^2=\left(3-\sqrt{y+2}\right)^2$

$y+8=(3)^2+\left(\sqrt{y+2}\right)^2-2(3)(\sqrt{y+2})$

$y+8=9+y+2-6\sqrt{y+2}$

$y+8=11+y-6\sqrt{y+2}$

$y-y+8-11=6\sqrt{y+2}$

$-3=6\sqrt{y+2}$

Again squaring both sides

$(-3)^2=\left(6\sqrt{y+2}\right)^2$

$9=36(y+2)$

$9=36y+72$

$9-72=36y$

$-63=36y$

$36y=-63$

Put $x^2+3x=y$

$36(x^2+3x)=-63$

$36x^2+108x+63=0$

Solving by quadratic formula

$a=36 \quad b=108 \quad c=63$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-108\pm\sqrt{(108)^2-4(36)(63)}}{2(36)}$$

$$x=\frac{-108\pm\sqrt{11664-9072}}{72}$$

$$x=\frac{-108\pm\sqrt{2592}}{72}$$

$$x=\frac{-108\pm\sqrt{4\times 4\times 9\times 9\times 2}}{72}$$

$$x=\frac{-108\pm\sqrt{4^2\times 9^2\times 2}}{72}$$

$$x=\frac{-108\pm 4\times 9\sqrt{2}}{72}$$

$$x=\frac{-108\pm 36\sqrt{2}}{72}$$

$$x=\frac{36(-3\pm\sqrt{2})}{72}$$
$$x=\frac{-3\pm\sqrt{2}}{72}$$
$$So\ the\ solution\ set\ is\ \left(\frac{-3\pm\sqrt{2}}{72}\right)$$

Question no 11:

$$\sqrt{x^2+3x+9}+\sqrt{x^2+3x+4}=5$$
$$Solution:$$
$$\sqrt{x^2+3x+9}+\sqrt{x^2+3x+4}=5$$
$$Put\ x^2+3x=y\ \ldots\ldots\ldots(1)$$
$$\sqrt{y+9}+\sqrt{y+4}=5$$
$$\sqrt{y+9}=5-\sqrt{y+4}$$
$$Taking\ square\ on\ both\ side$$
$$\left(\sqrt{y+9}\right)^2=\left(5-\sqrt{y+4}\right)^2$$
$$y+9=(5)^2+y+4-2(5)\sqrt{y+4}$$
$$y+9=25+y+4-10\sqrt{y+4}$$
$$y+9-25-y-4=-10\sqrt{y+4}$$
$$-20=-10\sqrt{y+4}$$
$$2=\sqrt{y+4}$$
$$Taking\ again\ square\ on\ both\ side$$
$$4=y+4\ \Rightarrow\ y=0$$
$$Put\ y=0\ in\ equation\ 1$$
$$x^2+3x=0\ \Rightarrow x(x+3)=0$$
$$x=0\quad or\quad x+3=0$$
$$x=0\quad or\quad x=-3$$
$$So\ the\ solution\ set\ is\ \{0,-3\}$$